

Heterotic NS5-branes and closed-string K-theory

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Heterotic NS5-branes from closed string tachyon condensation,
I. García-Etxebarria, M. Montero, A. Uranga, hep-th/1405.0009

Heterotic NS5 and closed-string tachyons

- D branes in type I, II = open string tachyon solitons Sen '98, Witten '98.
- Some understanding of NS5 branes from closed-string tachyons in type II Adams, Polchinski, Silverstein '01.
- We can describe the heterotic NS5-brane as a tachyon soliton using **supercritical heterotic strings**.

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- We can describe the heterotic NS5-brane as a tachyon soliton using **supercritical heterotic strings**.

Heterotic supercritical theory $HO^{(+n)}$

Field content: Hellerman '04

- Left-movers: $32 + n$ fermions λ^a in the vector of $SO(32 + n)$
- Right-movers: $10 + n$ embedding functions X^μ +superpartners ψ^μ ($\mu = 1, \dots, 10$) and Y^m, ψ^m ($m = 1 \dots n$).

Extra ingredient: Z_2 orbifold acting as

$$(\lambda^a, Y^m, \psi^m) \rightarrow (-\lambda^a, -Y^m, -\psi^m).$$

Massless spectrum:

- $(10 + n)$ d fields: $\phi, g, B + SO(32 + n)$ gauge field+ \mathcal{T}^a in the fundamental of the gauge field.
- 10d fields: Gravitino+fermions charged under $V \otimes V_{\text{rot.}}$, $V \equiv SO(32 + n)$ bundle, $V_{\text{rot.}} \equiv SO(n)$ normal bundle.

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Tachyon condensation: The idea

Hellerman & Swanson, '07

- The tachyon \mathcal{T}^a couples to the worldsheet as a superpotential,

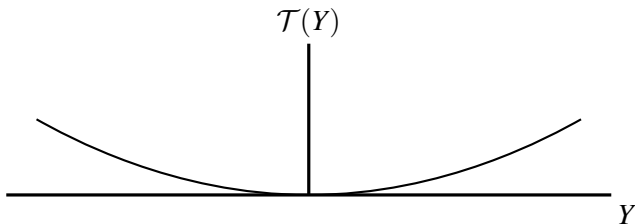
$$\Delta\mathcal{L} = -\frac{1}{2\pi} \int d\theta_+ \sum_a \lambda^a \mathcal{T}^a(X, Y).$$

After eliminating auxiliary fields,

$$\Delta\mathcal{L} = -\frac{1}{8\pi} \left(\sum_a (T^a)^2 \right) + \frac{i}{2\pi} \sqrt{\frac{\alpha'}{2}} [\partial_\mu \mathcal{T}^a \lambda^a \psi^\mu + \partial_m \mathcal{T}^a \lambda^a \psi^m]$$

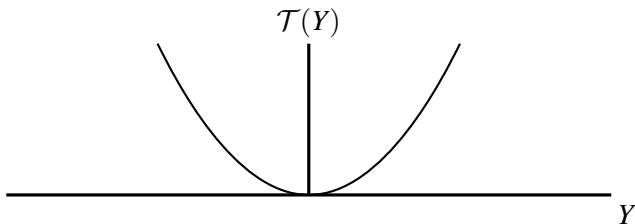
Tachyon condensation: Dynamics

Orbifold action takes $\mathcal{T}^a(X, Y) \rightarrow \mathcal{T}^a(X, -Y)$. Locally,
 $\mathcal{T}^a \sim M_m^a \exp(\beta X^+) Y^m$. X^+ -dependence to obey \mathcal{T}^a e.o.m.
Potential $\sim \exp(\beta X^+) Y^2$ grows without bound so dynamics is
confined to $Y = 0$ slice.



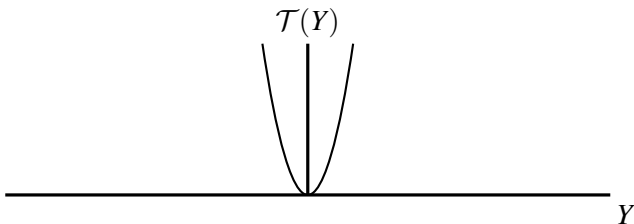
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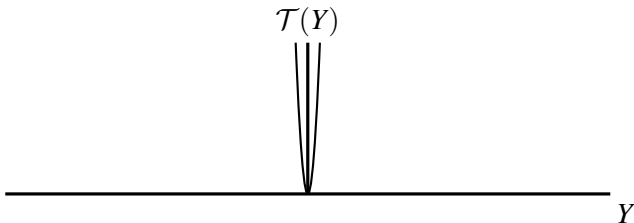
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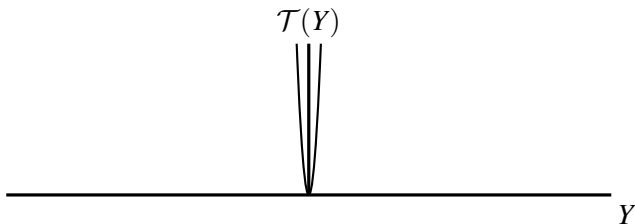
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Fermions couple to $\partial_m \mathcal{T}^a$.

Tachyon condensation

Allow some dependence of \mathcal{T}^a on X . At some particular points, it might be that $\partial_m \mathcal{T}^a = 0$.

To ensure it, protect it topologically: Nontrivial V, V_{rot} .

- Tachyon profile $\partial_m \mathcal{T}^a$ is a map $V_{\text{rot}} \rightarrow V$, isomorphism at infinity.
- ($n = 4$) Nontrivial $\text{tr}(F_{SO(32+n)}^2) - \text{tr}(R_{\text{rot.}}^2)$ results in $\partial_m \mathcal{T}^a = 0$ somewhere:

$$\partial_m \mathcal{T}^a \sim \vec{\Gamma}_{ma} \cdot \vec{X}$$

- This leads to a codimension 4 defect after tachyon condensation.

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Tachyon condensation

We have found a 6d object located at a representative of $P.D\{\text{tr}(F_{SO(32+n)}^2) - \text{tr}(R_{\text{rot.}}^2)\}$. For it to be an NS5, we should have

$$dH = \dots + \text{tr}(F_{SO(32+n)}^2) - \text{tr}(R_{\text{rot.}}^2)$$

We must find a modified Bianchi identity in the $HO^{(+n)}$ theory.

- Anomaly cancellation in 2d
- Anomaly cancellation in 10d

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- Anomaly cancellation in 10d

Anomalies in 2d worldsheet

- λ^a are a section of the $SO(32 + n)$ gauge bundle.

$$I_4^{\lambda^a} = \text{tr}(F_{SO(32+n)}^2)$$

- ψ^μ are sections of the critical slice tangent bundle

$$I_4^{\psi^\mu} = -\text{tr}(R^2)$$

- ψ^m are sections of the normal bundle.

$$I_4^{\psi^m} = -\text{tr}(R_{\text{rot.}}^2)$$

This anomaly is cancelled by inflow from a coupling

$$\int_{10+n} *H_3 \wedge Q_3$$

with $dQ_3 = I_4^{\lambda^a} + I_4^{\psi^\mu} + I_4^{\psi^m}$. This leads to a modified Bianchi identity

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Anomalies in 10d

- Heterotic anomaly cancellation is modified. The anomaly cancellation conditions are those of type I with n extra $D9 - \overline{D9}$ pairs Sugimoto '99, Schwarz-Witten '01.
- Consistent with S-duality Hellerman '04.
- Anomaly polynomial factorizes as $I_4 I_8$, with

$$I_4 = \text{tr}(F_{SO(32+n)}^2) - \text{tr}(R^2) - \text{tr}(R_{\text{rot.}}^2).$$

Usual GS mechanism implies modified Bianchi id.

The 6d heterotic soliton carries the H charge of an NS5 brane

CFT analysis

To see if tachyon soliton really corresponds to NS5 brane, analyze CFT near the zero of the tachyon:

$$\mathcal{T}^a \sim B_{\mu m}^a X^\mu Y^m$$

Deform profile such that $B_{\mu m}^a = \Gamma_{na}^\mu$, where Γ^μ are $SO(4)$ Dirac matrices.

This is a $(0, 4)$ singular CFT which arises in the small instanton limit of a standard sigma model description of instantons

Witten '95

Tachyon condensation with nontrivial $\text{tr}(F_{SO(32+n)}^2) - \text{tr}(R_{\text{rot.}}^2)$ yields an NS5 in the critical theory.

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Closed string K theory

NS5 brane charge is represented by the cohomology class $\text{tr}(F_{SO(32+n)}^2) - \text{tr}(R_{\text{rot.}}^2)$. This suggests a lift to a K theory class $(V, V_{\text{rot.}})$.

- This would be S-dual to type I with extra $D9 - \overline{D9}$ pairs.
- NS5 charge is related to $KO(S^4) = \mathbb{Z}$.
- Other nontrivial K-theory groups:
 - $KO(S^8) = \mathbb{Z}$ Fundamental string
 - $KO(S^2) = \mathbb{Z}_2$ 7-brane [Berasaluce, M. M, Retolaza, Uranga '13]
 - $KO(S^9) = \mathbb{Z}_2$ Massive heterotic spinor
 - $KO(S^1) = \mathbb{Z}_2$ domain wall
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Summary

- Supercritical heterotic strings allow for the construction of critical NS5 as a closed tachyon soliton.
- Modified Bianchi identity in $10 + n$ dimensions.
- NS5 brane charge unveils real K-theory charge classification in heterotic string theory.

Outlook: Type II!

Summary

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Outlook: Type II!

Thank you very much!

Type II preview

Supercritical type II theories can be analysed by D -brane probes. Supercritical D -branes contain a Chern-Simons coupling

$$\int_{\Sigma} ch(F) \sqrt{\frac{\hat{A}(N\Sigma)}{\hat{A}(T\Sigma)}}$$

- Tachyon condensation “erases” part of the normal bundle.
- Suggests that the final state in the critical theory is a gravitational instanton bot in IIA and IIB.