Torus partition functions and spectra of gauged linear sigma models

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Overview of this talk

1. Motivation
2. Gauged linear sigma models
3. Full one-loop partition function
4. Modular invariance
5. Spectra
6. Conclusion

Calabi-Yau compactification with vector bundles

The basic requirement is that one obtains an effective 4D field theory with $\mathcal{N} = 1$ SUSY from the heterotic string:
Candelas,Horowitz,Strominger,Witten'85

$$\mathcal{M}^{1,9} \to \mathcal{M}^{1,3} \times \mathcal{M}^{6}$$

- a six dimensional Calabi-Yau manifold $\mathcal{M}^6$ with vanishing first Chern class
- a gauge background satisfying the Hermitean Yang–Mills equations characterized by a vector bundle

Toroidal orbifold geometries

The idea of orbifolds is that they are very simple geometries yet shared the main property of Calabi–Yau manifolds namely that only 4D $\mathcal{N} = 1$ SUSY survives.

Dixon,Harvey,Vafa,Witten'85, Ibanez,Mas,Nilles,Quevedo'87

Toroidal orbifolds are defined as

$$T^6/G$$

with some six dimensional torus T^6 and a finite group G, like $\mathbb{Z}_N$

Comparison Calabi-Yaus and Orbifolds

Calabi-Yaus with vector bundles are SUGRA compactifications, hence constitute only a low energy limit of string theory:

- only access to the massless spectrum at best
- important worldsheet quantum effects may be missed:,e.g. Wilson line consistency conditions Blaszczyk, et al.'09
- not every SUGRA background might have a string lift

Orbifolds which admit exact SCFT descriptions:

- full spectrum can be computed
- stringy conditions (e.g. modular invariance) can be checked
- but only account for special points in the moduli spaces of a minor set of all known CYs

Gauged Linear Sigma Models

Gauged linear sigma models (GLSMs) provide an approximate worldsheet description:

- Compact CYs and vector bundles: Witten'93,Distler'95
  - they define symplectic quotients related to projective spaces
  - their super potential lead to hyper surfaces constraints to describe complete intersection CYs with monad bundles
- Non-compact CYs with line bundles SGN'10
  - these in particular include non-compact orbifold resolutions

GLSMs are not SCFTs themselves, but are commonly believed to flow to genuine CFTs in the IR Silverstein,Witten'94

Recent developments:

Computation of GLSM partition functions

Using localization techniques it is possible to compute the full path integral for a GLSM on a given worldsheet: Witten'88

Sphere partition functions Benini,Cremonesi'12, Doroud,Gomis, Le Foch,Lee'12

- which can be related to exact moduli Kähler potentials
  Jockers,Kumar,Lapan,Morrison'12, Gomis, Lee'12

and torus partition functions to the elliptic genera
Grassi,Policastro,Scheidegger'07, Benini,Eager,Hori,Tachikawa'13, Gadde,Gukov,Putrov'12

(0,2) superfields

Chiral and Fermi matter superfields:

Super-field	Chiral		Fermi	
	$\mathcal{Z}^a, z^a$	ψ^a	$\Lambda^\alpha, \lambda^\alpha$	f^α
$U(1)$	q_a	q_a	Q_α	Q_α
$U(1)_\text{R}$	r_a	r_a-1	R_α	R_α-1
$U(1)_\text{L}$	l_a	l_a	L_α	L_α

Bosonic and fermionic gauge multiplets:

Super-field	Bos. gauge			Fermi gauge	
	$\mathcal{V}, \mathcal{A}, \mathfrak{a}$	χ	D	Σ, φ	$\mathfrak{s}$
$U(1)_\text{R}$	0	-1	0	0	-1
$U(1)_\text{L}$	0	0	0	-1	-1

Worldsheet action

The (0,2) worldsheet action contains the following elements:

- Gauge coupling: $1/e^2$

  conformal limit $e \to \infty$; free limit $e \to 0$.

- Fayet-Iliopoulos (FI)-term: $\mathcal{W}_{\text{FI}} = \rho\, F$

  encoding the Kahler parameters of the geometry

- Superpotential: $\mu\, \mathcal{W}(\mathcal{Z}, \Lambda)$

  encoding the hyper surface and monad bundle

Torus partition function

We define the GLSM path integral:

$$Z = \int \mathcal{D}(\mathcal{V},\mathcal{A})\mathcal{D}\Sigma\mathcal{D}\mathcal{Z}\mathcal{D}\Lambda\, e^{-S}\,.$$

- on the WS torus with complex structure τ
- with a gauge background determined by its holonomies
  Benini,Eager,Hori,Tachikawa'13

$$\nu = \tau\, a + a'\,, \quad a = \oint_{\mathcal{C}_\tau} \mathfrak{a}\,, \quad a' = \oint_{\mathcal{C}_1} \mathfrak{a}\,,$$

- and the L- and R-symmetry holonomies encode the left- and right-moving spin-structures

$$\nu_L = \frac{t'}{2} + \tau\frac{t}{2}\,, \qquad \nu_R = \frac{s'}{2} + \tau\frac{s}{2}$$

Supersymmetric localization

Consider the deformed path integral: Witten'88

$$Z(t) = \int \mathcal{D}\Phi\, e^{-S - t\, S_{\text{exact}}}\,, \qquad Q\, S = 0\,, \qquad S_{\text{exact}} = Q\, U\,,$$

where S is the original action and S_{exact} a Q-exact modification.

$$Q^2 = 0\,, \qquad Q = \int d^2\sigma\, \psi(\sigma) \frac{\delta}{\delta z(\sigma)} + \ldots$$

Then $Z(t)$ is independent of the parameter t:

$$-\frac{dZ(t)}{dt} = \int \mathcal{D}\Phi\, Q\Big[U\, e^{-S - t\, S_{\text{exact}}}\Big] = 0$$

Up to global subtleties on the worldsheet and in target space.

Supersymmetric localization

In particular, in the limit, $e \to 0$, the kinetic terms of the non-zero modes completely dominate over their interactions:

- The non-zero mode path integral reduces to a free theory one, giving rise to one-loop determinant factors
- The instanton contributions result from non-constant zero modes and e.g. encode Kaluza-Klein and winding numbers

The path integral becomes a finite-dimensional integral:

$$Z = \int \mathrm{d}^2\nu \int \mathrm{d}\Phi_0 \, Z_{\text{determinants}} \, Z_{\text{instantons}}$$

One-loop determinants

Chiral and Fermi superfield partition functions:

$$Z_{\text{chiral}} = \frac{\overline{Z_d\begin{bmatrix} v\text{-}e_d a_{\text{R}} \\ v'\text{-}e_d a'_{\text{R}} \end{bmatrix}}}{\left| Z_d\begin{bmatrix} v \\ v' \end{bmatrix} \right|^2}\,, \qquad Z_{\text{fermi}} = Z_D\begin{bmatrix} V \\ V' \end{bmatrix} = \frac{\theta\begin{bmatrix} \frac{1}{2}e_D - V \\ \frac{1}{2}e_D - V' \end{bmatrix}}{\eta^V}$$

with $v = q \cdot a + l\, a_{\text{L}} + r\, a_{\text{R}}$ and $V = Q \cdot a + L\, a_{\text{L}} + R\, a_{\text{R}}$

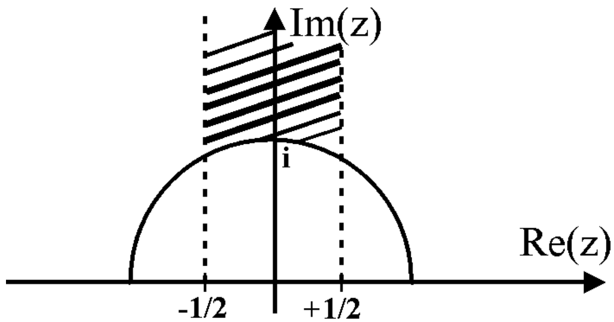
Gauge and Fermi gauge multiplet partition functions:

$$Z_{\text{bos. g.}} = Z_1^{D_{\text{b}}}\begin{bmatrix} \text{-}a_{\text{R}} \\ \text{-}a'_{\text{R}} \end{bmatrix}\,, \qquad Z_{\text{fer. g.}} = \frac{\overline{Z_1^{D_{\text{f}}}\begin{bmatrix} \text{-}a_{\text{L}} \\ \text{-}a'_{\text{L}} \end{bmatrix}}}{\left| Z_1^{D_{\text{f}}}\begin{bmatrix} \text{-}a_{\text{R}}\text{-}a_{\text{L}} \\ \text{-}a'_{\text{R}}\text{-}a'_{\text{L}} \end{bmatrix} \right|^2}$$

Modular transformations

The full one-loop partition function should be invariant under the modular transformation:

$$T:\ \tau \to \tau + 1\ , \qquad S:\ \tau \to -\frac{1}{\tau}$$



Conditions for modular invariance

This leads to conditions like:

$$\tfrac{a_k}{2}(Q_k^T Q_l - q_k^T q_l) \equiv 0$$

can be thought of as:

- **Orbifold CFT modular invariance condition:**

  In the sector $\nu_R = 0 \Rightarrow a_k = 1/N$. With $V = Q/N$ this gives:

  $$\frac{N}{2}(V^2 - v^2) \equiv 0$$

- **GLSM gauge anomaly conditions:**

  In the sectors $\nu_R \neq 0 \Rightarrow a_k \in [0, 1[$, we find:

  $$Q_k^T Q_l - q_k^T q_l = 0$$

Determining the spectrum

The conventional massless spectrum computation of GLSMs makes use of $\overline{Q}$-cohomology see e.g. Kachru,Witten'93, Adams'09, Bertolini,Melnikov,Plesser'14

We outline an alternative way to determine not only the massless but the *full* spectrum of GLSMs:

Perform $q, \bar{q}$-expansions of the full partition function:

$$Z = \sum q^{M_L^2} \bar{q}^{M_R^2} = \dots$$

Mass formulas

For example the left–moving mass read

$$M_\mathrm{L}^2 \;=\; \tfrac{1}{2}W_\mathrm{L}^2 + \tfrac{1}{2}P_\mathrm{L}^2 - 1 + \delta c + N_\mathrm{L}$$

- where W_L encodes the instanton contributions
- and P_L is the so-called shifted momenta

$$P_\mathrm{L} = \binom{P_D + V - \frac{1}{2}e_D}{\mathbf{P_{D_b}} - (\mathbf{a}_\mathrm{R} + \mathbf{\frac{1}{2}})\mathbf{e_{D_b}}}\,, \qquad p_d \in \mathbb{Z}^d$$

The vacuum shift

$$\delta c = \tfrac{1}{2}\tilde{v}^T(e_d - \tilde{v}) - \frac{\mathbf{D_b + D_f}}{\mathbf{8}} + \frac{\mathbf{D_f}}{\mathbf{2}}\tilde{\mathbf{v}}_\mathrm{f}(\mathbf{1} - \tilde{\mathbf{v}}_\mathrm{f})$$

These are **generalizations** of corresponding orbifold formulae.

Summary

GLSMs provide a worldsheet framework to study supersymmetric heterotic string on

- non-compact CYs with line bundles
- compact CY with non-trivial vector bundles

We used supersymmetric localization to determine their one-loop partition functions

Based on these results we:

- derived conditions to ensure one-loop modular invariance
- proposed a way to determine the full target space spectrum

Potential caveats in the localization

There are two situations in which localization might be obstructed because of global issues:

Target space boundary contributions:

The final step to show that the path integral did not depend on t was:

$$-\frac{dZ(t)}{dt} = \int \mathcal{D}\Phi \int d^2\sigma \left\{ \psi(\sigma) \frac{\delta}{\delta z(\sigma)} + \ldots \right\} \Big[U\, e^{-S - t\, S_{\text{exact}}} \Big] \overset{?}{=} 0$$

Hence this depends on

- the topology of the target space
- how the actions S and S_{exact} behave at these boundaries

Potential caveats in the localization

Worldsheet boundary contributions:

For the localization to work one needs $Q\,S = 0$.

However, in the worldsheet susy breaking sectors, i.e. $\nu_R \neq 0$, the super potential $\mathcal{W}$ is not Q-closed:

$$QS_{\mathcal{W}} = \int d^2\sigma\, \partial\overline{\mathcal{W}} \neq 0$$

because the boundary conditions of $\mathcal{W}$ are anti-periodic:

$$\mathcal{W}(\sigma+1) = (-)^{s}\,\mathcal{W}(\sigma)\,, \qquad \mathcal{W}(\sigma-\tau) = (-)^{s'}\,\mathcal{W}(\sigma)$$

Hence in the following we assume that we are either in the sector $\nu_R = 0$ or the super potential is absent.