Higher Curvature Corrections and Non-Minimally Coupled Scalars

Mahdi Torabian
Institute for Research in Fundamental Sciences (IPM), Tehran

String Phenomenology 2014
ICTP, Trieste

Higher-Curvature Corrections in String Theory

✔ Stringy tree-level (α') and 1-loop level (g_s) corrections to 10 dimensional SUGRA has been computed in type II theories.

[Green-Schwarz 1982] [Gross-Witten 1986] [Grisaru-Zanon 1986] [Freeman-Pope 1986]

✔ Corrections are at 8-derivative level, Quartic in the Riemann Tensor

✔ Corrections are uplifted to 11 dimensional SUGRA

[Green-Gutperle-Vanhofe 3x1997] [Antoniadis-Ferrara etal 1997] [Russo-Tseytlin 1997]

$$S = (2\pi)^{-8} m_{11}^9 \int g^{1/2} \mathrm{d}^{11}x \Big[R - \frac{1}{2\cdot 4!} FF - \frac{g^{-1/2}}{(6\cdot 4!)^2} \epsilon AFF + \frac{\pi^2}{9\cdot 2^{11} m_{11}^6} \Big(tt - \frac{1}{4!}\epsilon\epsilon - \frac{1}{6} A\epsilon t \Big) RRRR \Big]$$

Higher-Curvature Corrections in String Theory

$$S = (2\pi)^{-8} m_{11}^9 \int g^{1/2} \mathrm{d}^{11}x \Big[R - \frac{1}{2\cdot 4!} FF - \frac{g^{-1/2}}{(6\cdot 4!)^2} \epsilon AFF + \frac{\pi^2}{9\cdot 2^{11} m_{11}^6} \Big(tt - \frac{1}{4!}\epsilon\epsilon - \frac{1}{6} A\epsilon t \Big) RRRR \Big]$$

$$\begin{aligned} ttRRRR = &+ 12(R_{mnpq}R_{mnpq})^2 + 24 R_{mnpq}R_{mnrs}R_{tupq}R_{turs} - 96 R_{mnpq}R_{mnrs}R_{tups}R_{turq} \\ &- 192 R_{mnpq}R_{mnqr}R_{turs}R_{tusp} + 192 R_{mnpq}R_{ntqr}R_{turs}R_{umsp} + 384 R_{mnpq}R_{tuqr}R_{ntrs}R_{umsp}, \end{aligned}$$

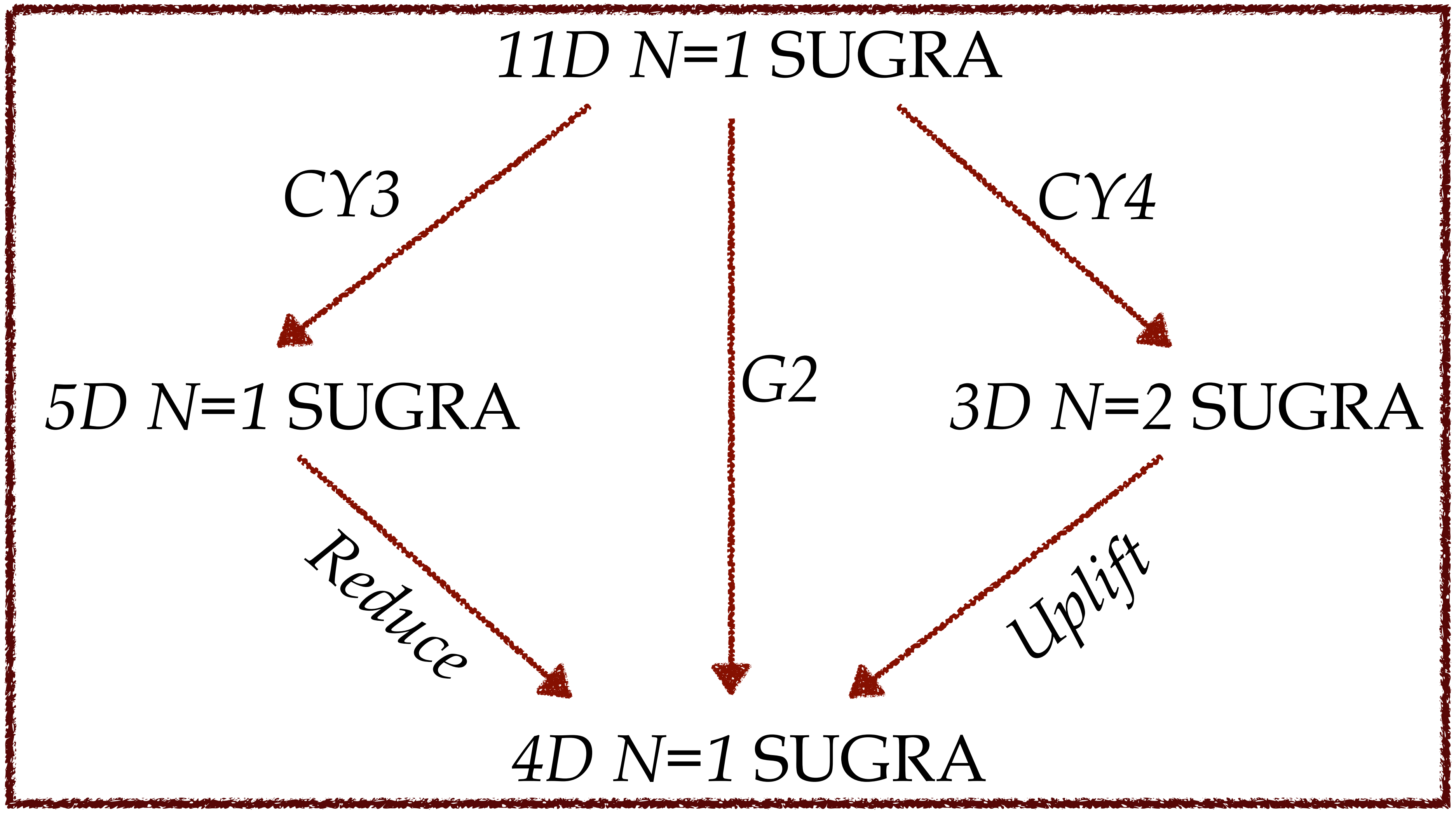
$$\epsilon\epsilon RRRR = -3!8!\delta^{m_1m_2m_3m_4m_5m_6m_7m_8}_{[n_1n_2n_3n_4n_5n_6n_7n_8]} R_{m_1m_2n_1n_2}R_{m_3m_4n_3n_4}R_{m_5m_6n_5n_6}R_{m_7m_8n_7n_8},$$

$$\begin{aligned} A\epsilon tRRRR = 24 A_{m_1m_2m_3}\epsilon^{m_1m_2m_3\cdots m_{11}} \Big(& R_{m_4m_5np}R_{m_6m_7pq}R_{m_8m_9qs}R_{m_{10}m_{11}sn} \\ & - \frac{1}{4} R_{m_4m_5np}R_{m_6m_7pn}R_{m_8m_9qs}R_{m_{10}m_{11}sq} \Big) \end{aligned}$$

Higher-Curvature Corrections in String Theory

✔ KK Reduction on compact manifolds to get 4D minimal SUGRA +Corrections

✔ The Road Map:



[Antoniadis-Ferrara etal 1997] [Grimm-Savelli etal 2013]

Higher-Curvature Corrections in String Theory

- ✔ A glimpse of what we get

$$S_{11} \supset \int_{11} R\wedge R\wedge R\wedge R\wedge e\wedge e\wedge e$$

$$\to a\int_4 \underbrace{R\wedge R}_{R^2 \text{ in FRW geometry}} + b\int_4 R\wedge e\wedge e\wedge e$$

$$a \equiv \int_7 R\wedge R\wedge e\wedge e\wedge e\wedge e\wedge e$$

$$b \equiv \int_7 R\wedge R\wedge R\wedge e$$

- ✔ Note we also have

$$S_{11} \supset \int_{11} R\wedge *(e\wedge e) \to V\int_4 R\wedge e\wedge e$$

$$V \equiv \int_7 e\wedge e\wedge e\wedge e\wedge e\wedge e\wedge e \qquad \mathcal{V} \equiv V_{cl.} + b$$

The 4-dimensional Effective Action

A phenomenological view:

✔ A scalar field non-minimally coupled to a higher-derivative gravity:

$$S = \int \mathrm{d}^4 x (-g)^{1/2} \Big(\frac{1}{2} m_{\mathrm{Pl}}^2 f(\varphi, R) - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \Big),$$

$$f(\varphi, R) = (1 + \xi m_{\mathrm{Pl}}^{-2} \varphi^2) R + \alpha m_{\mathrm{Pl}}^{-2} R^2$$

If not at tree-level, will be loop-induced anyway

✔ A Weyl transformation from the Jordan to the Einstein frame:

$$g^E_{\mu\nu} = f_R g_{\mu\nu} = (1 + \xi m_{\mathrm{Pl}}^{-2} \varphi^2 + 2\alpha m_{\mathrm{Pl}}^{-2} R) g_{\mu\nu} \equiv e^{\tilde{\chi}} g_{\mu\nu},$$

The 4-dimensional Effective Action

✔ The action in the Einstein frame:

Weyl scalar

$$S_E = \int \mathrm{d}^4x(-g_E)^{1/2}\Big(\frac{1}{2}m_{\mathrm{Pl}}^2 R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}e^{-\tilde{\chi}}g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V_E(\varphi,\chi)\Big)$$

non-canonical kinetic term

✔ The scalar potential:

$$V_E(\varphi,\chi) = e^{-2\tilde{\chi}}V(\varphi) + \frac{1}{8}\alpha^{-1}m_{\mathrm{Pl}}^4\big(1 - e^{-\tilde{\chi}}(1+\xi m_{\mathrm{Pl}}^{-2}\varphi^2)\big)^2,$$

$(\chi = (3/2)^{1/2} m_{\mathrm{Pl}}\tilde{\chi})$

The Equations of Motion

✔ Embed this model in a *cosmological* setup

✔ EOMs for the scalars and metric in an isotropic/homogenous b.g.

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)\mathrm{d}\mathbf{x}, \qquad \varphi = \varphi(t), \qquad \chi = \chi(t).$$

$$\ddot{\chi} + 3H\dot{\chi} + 6^{-1/2}e^{-\tilde{\chi}}\dot{\varphi}^2 + V_{\chi}^{E} = 0,$$

$$\ddot{\varphi} + 3H\dot{\varphi} - (2/3)^{1/2}m_{\mathrm{Pl}}^{-1}\dot{\chi}\dot{\varphi} + V_{\varphi}^{E} = 0,$$

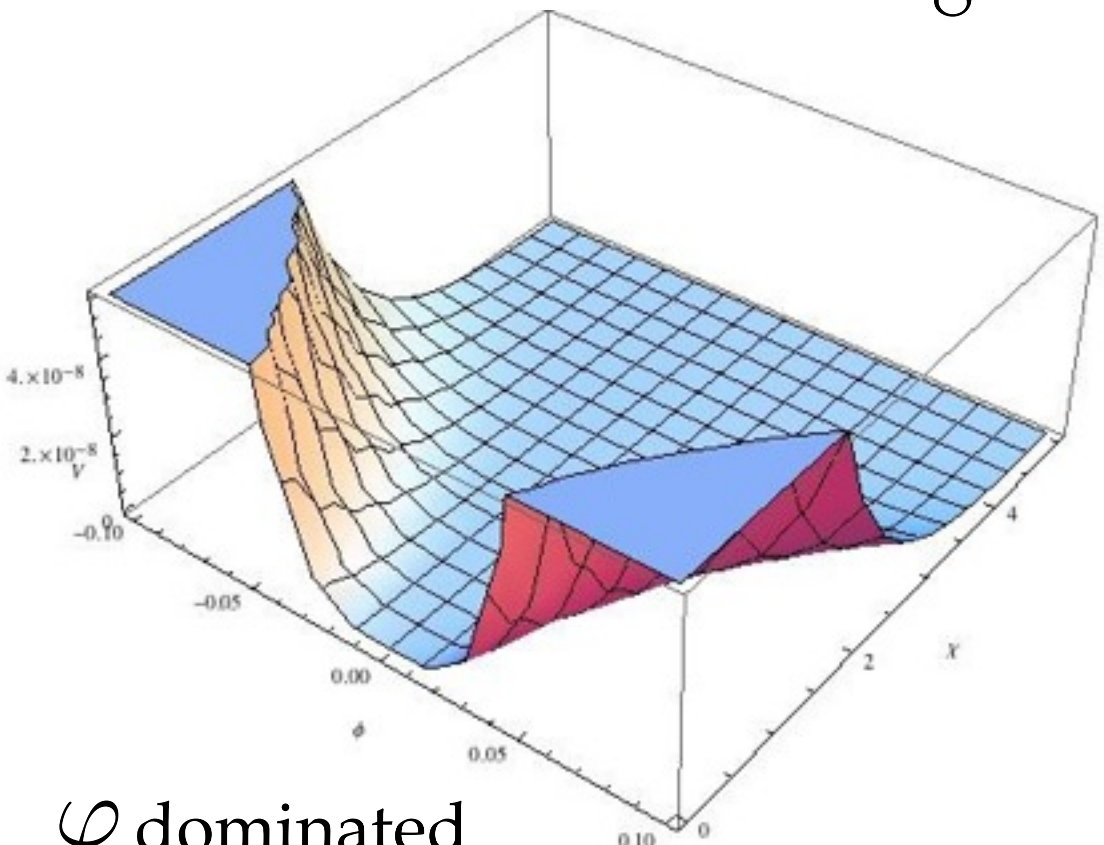
$$3H^2 m_{\mathrm{Pl}}^2 = \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}e^{-\tilde{\chi}}\dot{\varphi}^2 + V_E$$

✔ Numerically solve the equations for varieties of potentials…..

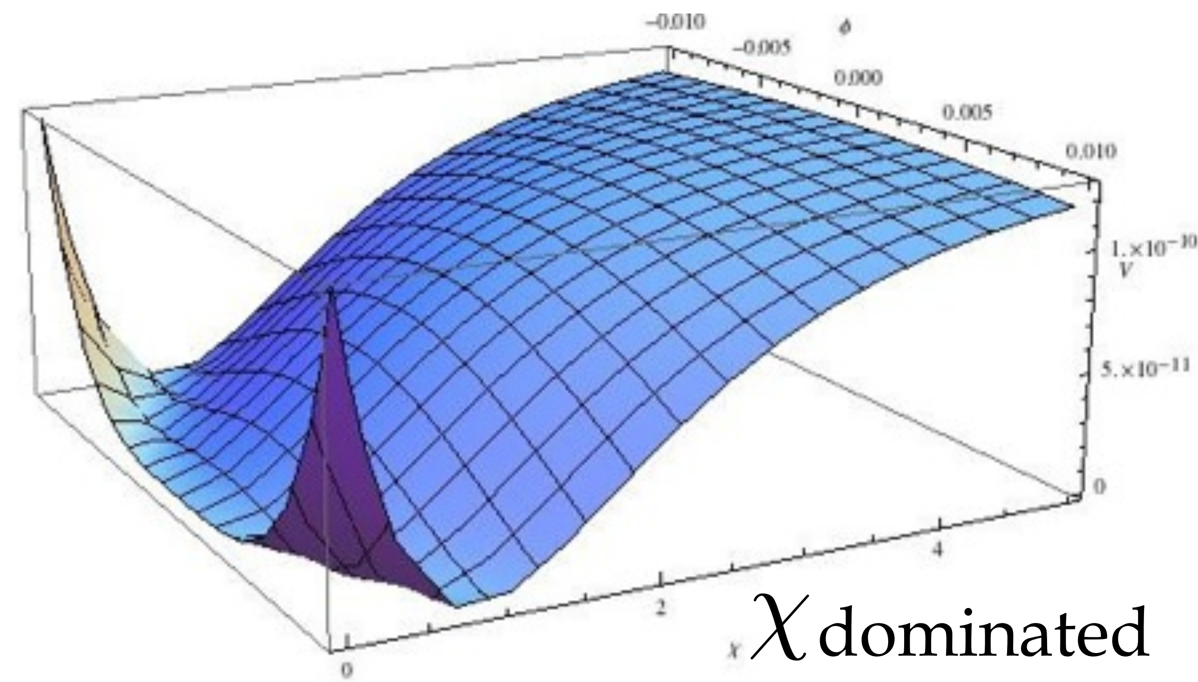
No Scalar Potential in the Jordan Frame

✔ A potential is induced in the Jordan frame:

$$V_E(\varphi,\chi) = \frac{1}{8}\alpha^{-1} m_{\rm Pl}^4 \left(1 - e^{-\tilde{\chi}}(1+\xi m_{\rm Pl}^{-2}\varphi^2)\right)^2$$



φ dominated



χ dominated

$$V_E(\chi,\varphi) = V(\chi) - \frac{1}{4\alpha}e^{-2\tilde{\chi}}(e^{\tilde{\chi}}-1)\xi\varphi^2 + \frac{1}{8\alpha}e^{-2\tilde{\chi}}\xi^2\varphi^4$$

$$V(\chi) = \frac{1}{8\alpha}\left(1-e^{-\tilde{\chi}}\right)^2 m_{\rm Pl}^4.$$

$$\tilde{\chi}_0 = \ln\left[(1+\xi\varphi_0^2)\right] \to 0$$

$$m_\chi^2 = \frac{1}{6\alpha} m_{\rm Pl}^2$$

$$\varphi_0^2 = \xi^{-1} m_{\rm Pl}^2 (e^{\tilde{\chi}_0} - 1) \to 0$$

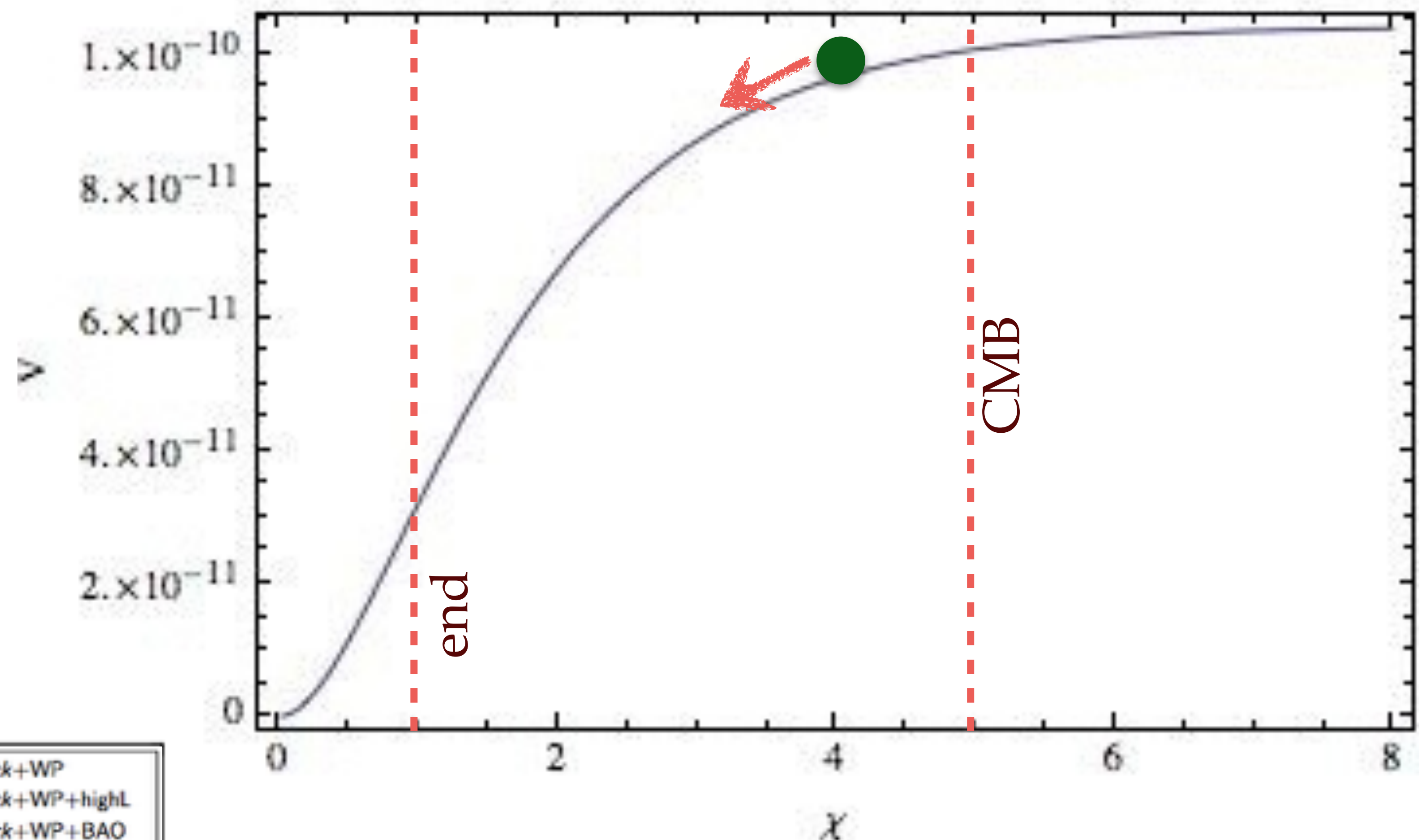
$$m_\varphi^2 = e^{-2\tilde{\chi}_0} m_{\rm Pl}^2 (e^{\tilde{\chi}_0}-1)\xi/\alpha = e^{-2\tilde{\chi}_0}(\xi^2/\alpha)\varphi_0^2 \to 0$$

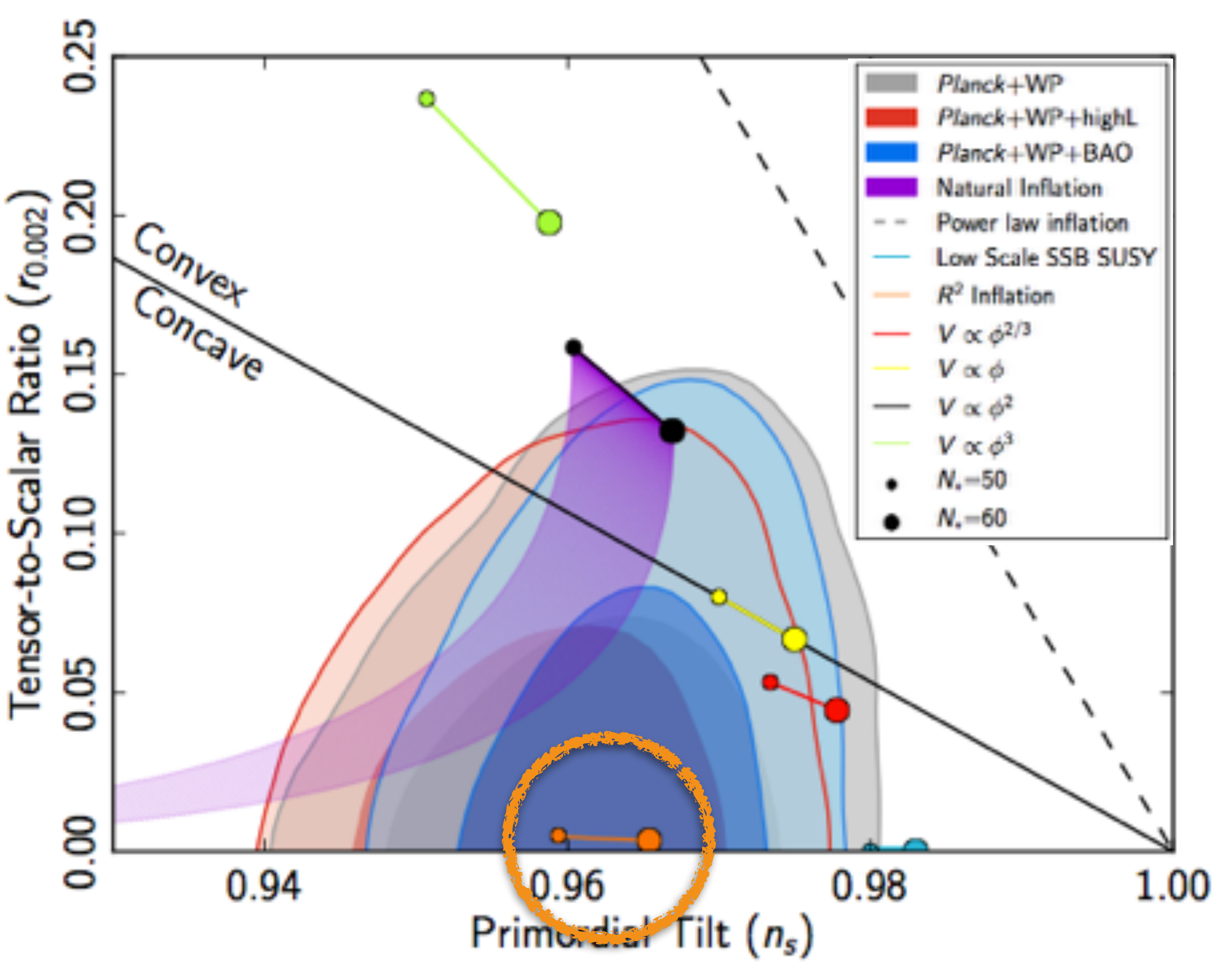
Starobinsky Curvature-Driven Inflation

[Starobinsky '80]

$$V(\chi) = \frac{1}{8\alpha}\left(1 - e^{-\tilde{\chi}}\right)^2 m_{\rm Pl}^4$$

$$\alpha \sim 10^9$$





✔ The simplest model used to be fascinating in the pre-BICEP era!

Starobinsky Curvature-Driven Inflation

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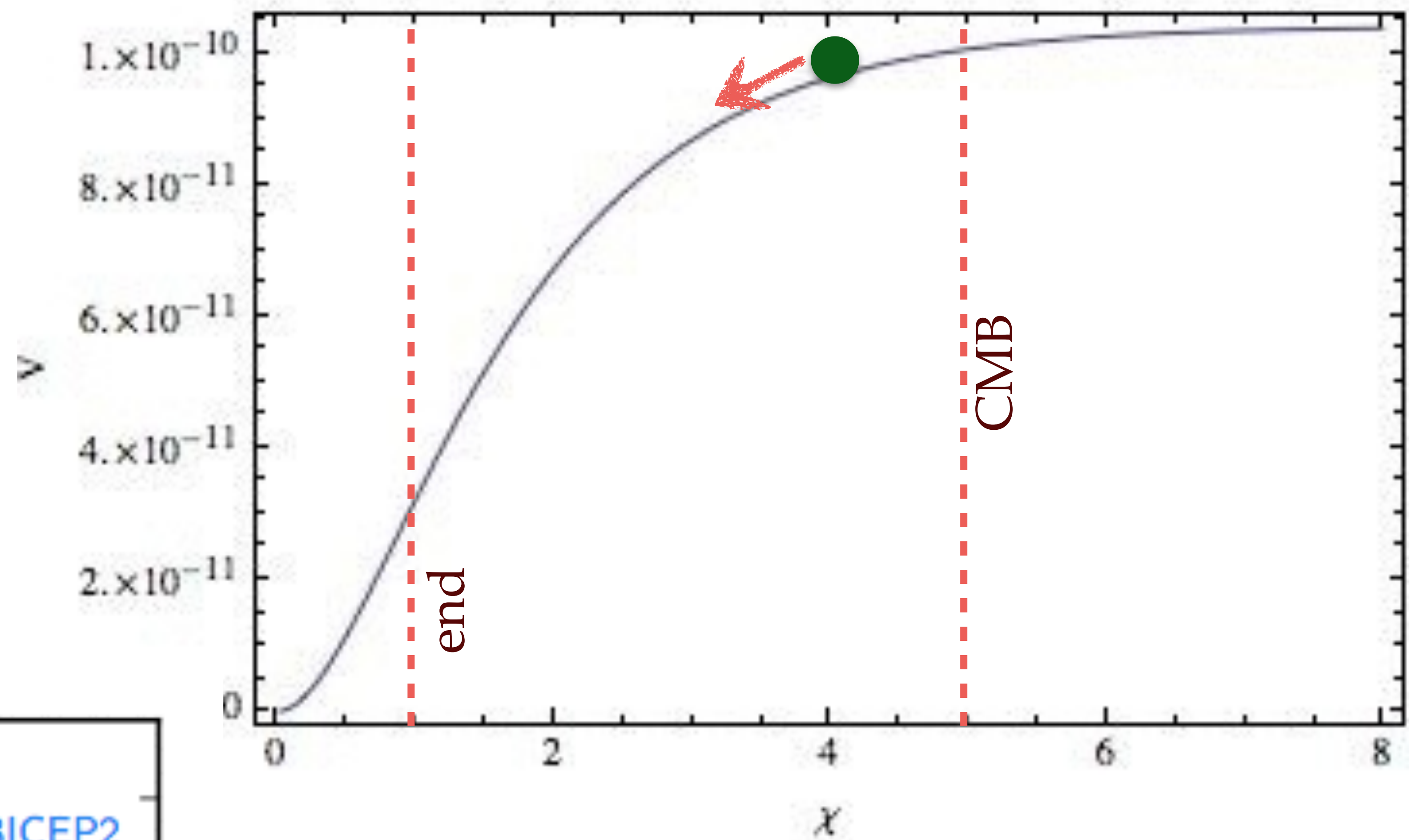
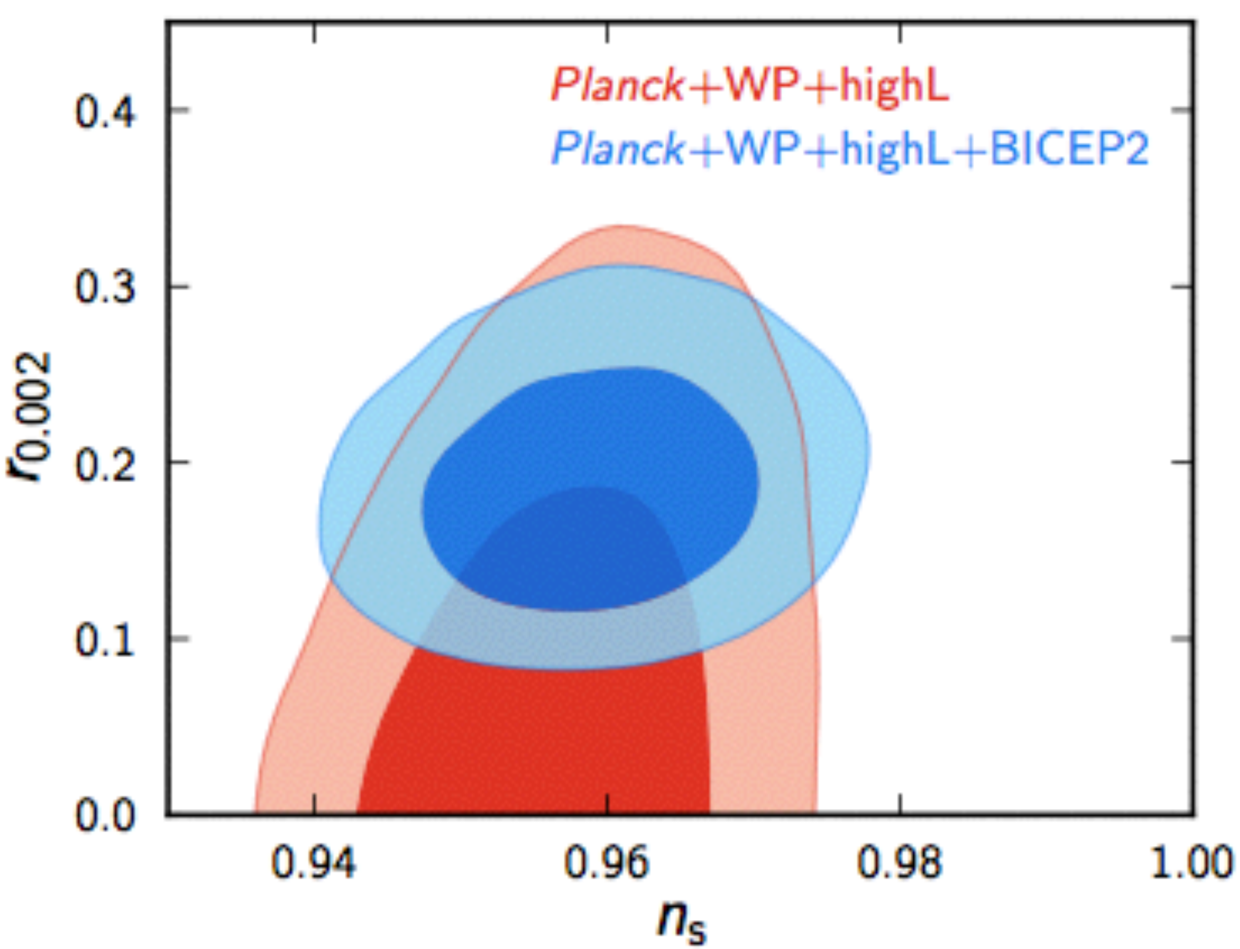
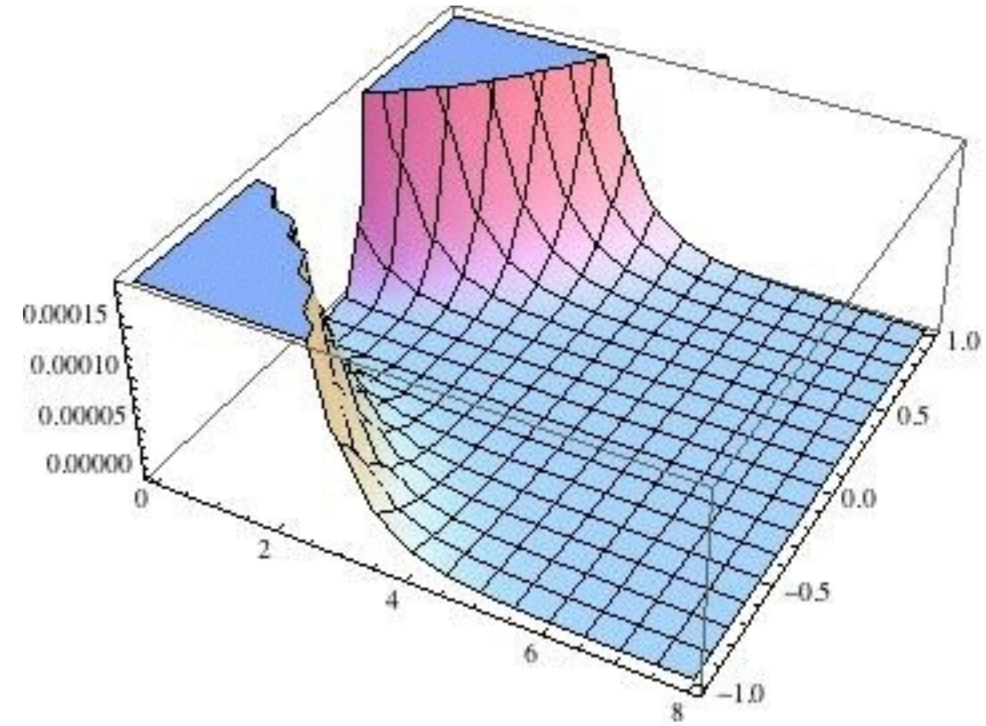




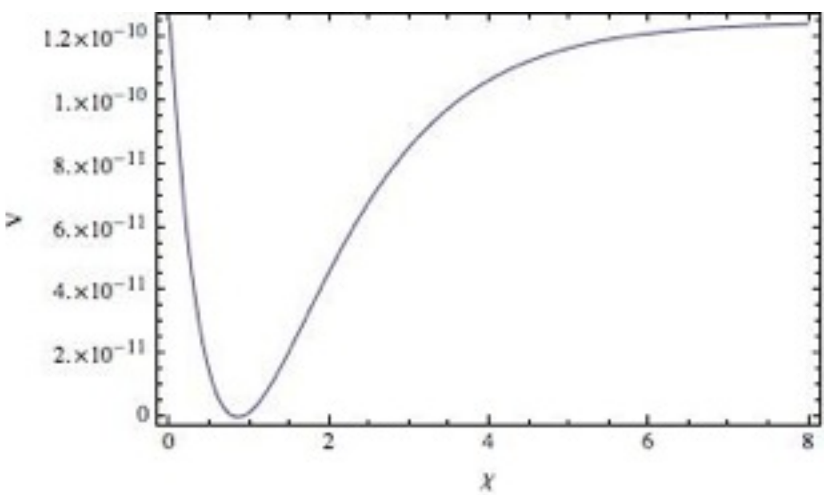
✔ Some variations are consistent with large tensor modes

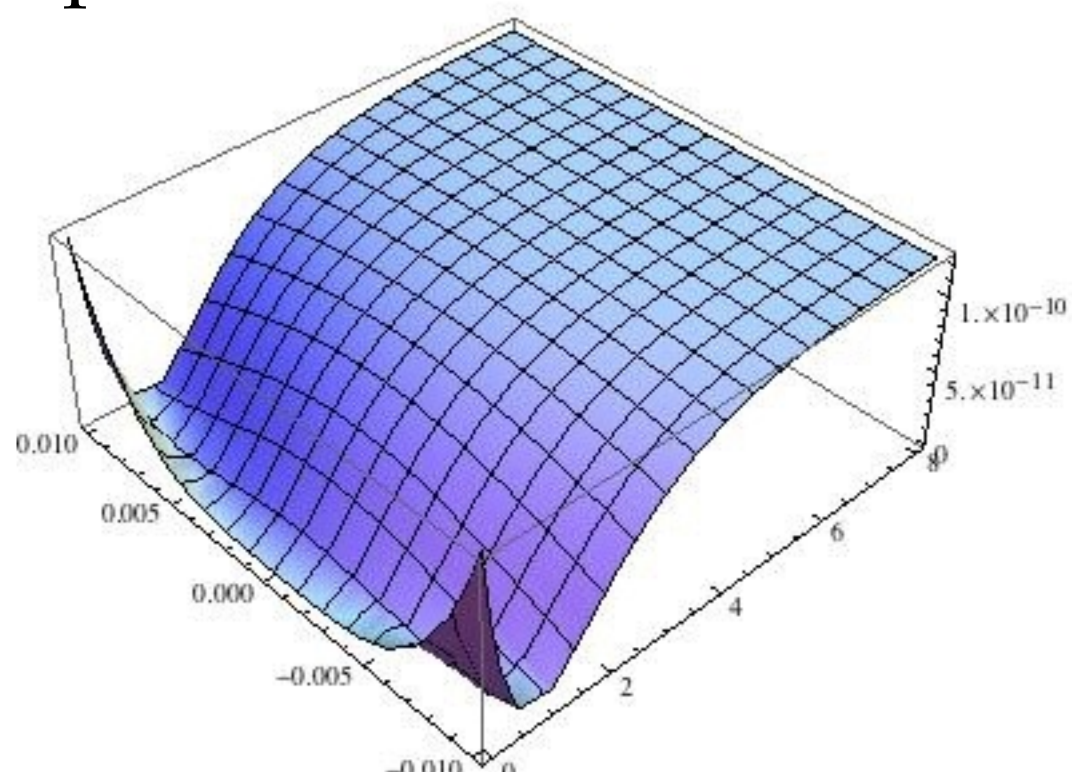
Massive Scalar in the Jordan Frame

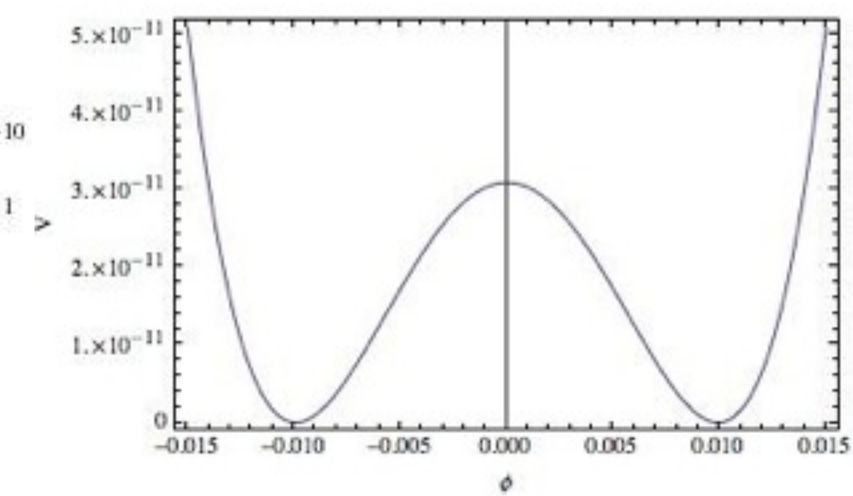
$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$



✔ The minimum of the potential:







$$e^{\tilde{\chi}_0} = 2(1 + (m/m_{\rm Pl})^2 \alpha/\xi)$$

$$\varphi_0^2 = \xi^{-1} m_{\rm Pl}^2$$

$$m_\chi^2 = \frac{m_{\rm Pl}^2}{6\alpha(1 + (m/m_{\rm Pl})^2\alpha/\xi)}$$

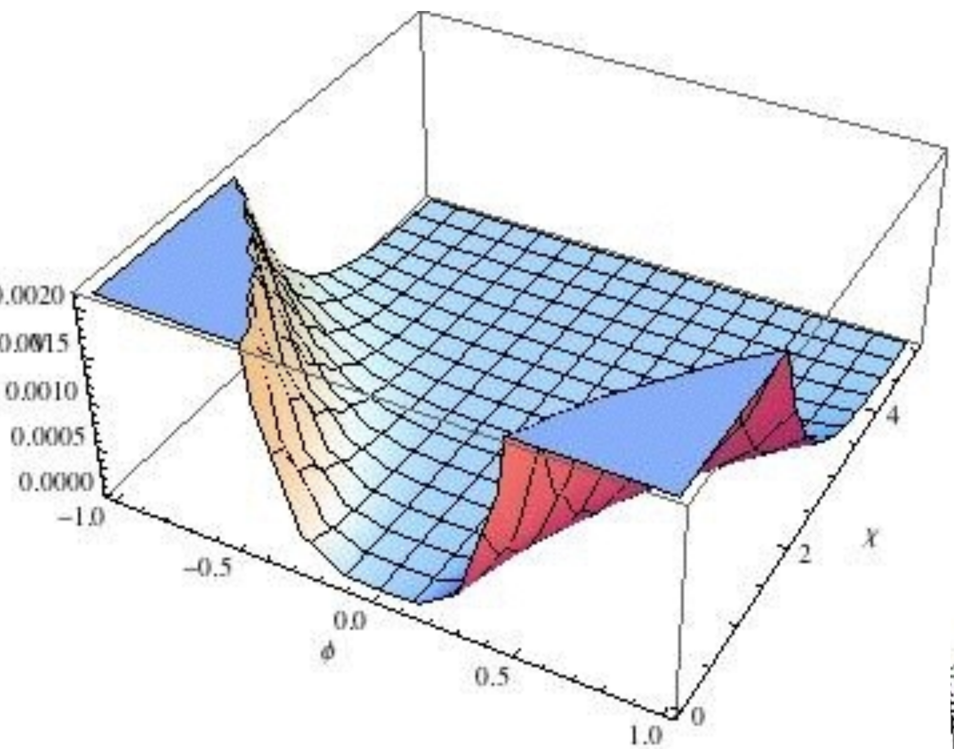
$$m_\varphi^2 = \frac{\xi m_{\rm Pl}^2}{4\alpha(1 + (m/m_{\rm Pl})^2\alpha/\xi)}$$

Higgs-like Field in the Jordan Frame

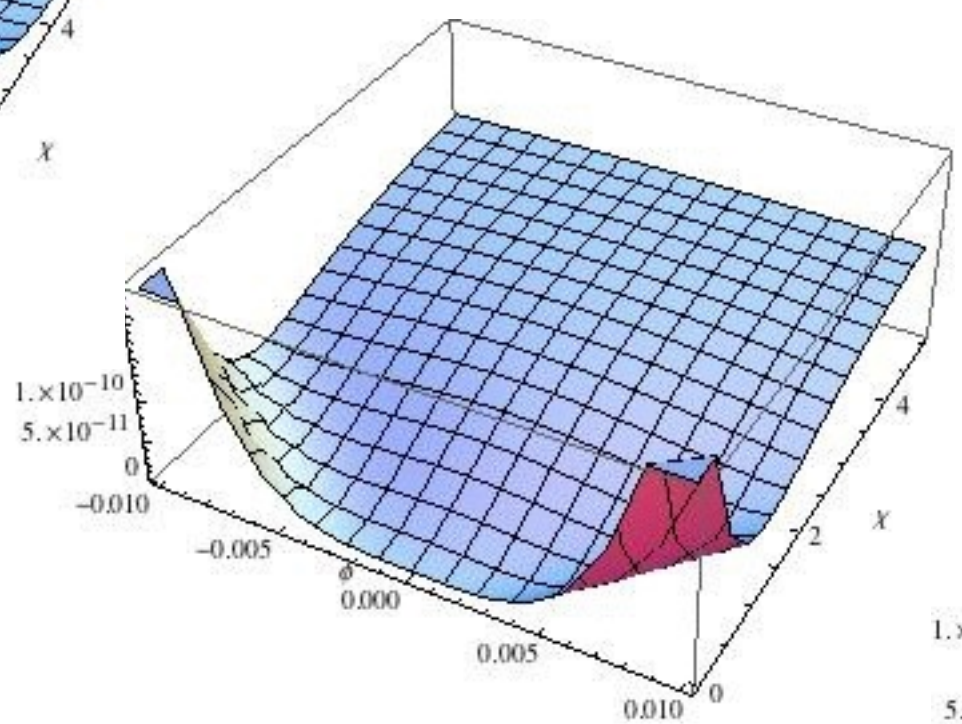
✔ The scalar potential:

$$V_E(\varphi,\chi)=e^{-2\tilde{\chi}}V(\varphi)+\frac{1}{8}\alpha^{-1}m_{\rm Pl}^4\left(1-e^{-\tilde{\chi}}(1+\xi m_{\rm Pl}^{-2}\varphi^2)\right)^2,$$

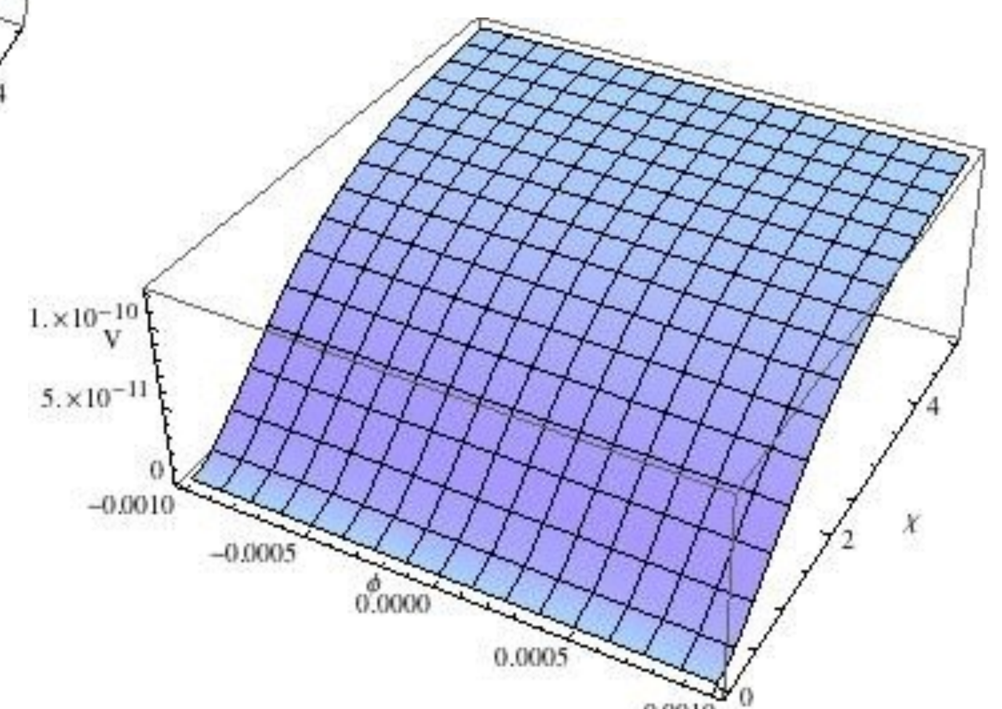
$$V(\varphi)=\frac{1}{2}m^2\varphi^2+\frac{1}{4}\lambda\varphi^4$$



Higgs dominated



Inflaton dominated



Higgs-like Fields in the Jordan Frame

- ✔ The minimum of the potential:

$$\varphi_0^2 \approx \frac{-m^2/\lambda}{1-(m/m_{\rm Pl})^2\xi/\lambda} \sim (246\ \text{GeV})^2$$

$$m_\varphi^2 \approx 2\varphi_0^2(\lambda + \xi^2/2\alpha) \sim (125\ \text{GeV})^2$$

$$\chi_0 \approx \xi\varphi_0^2 m_{\rm Pl}^{-1} \sim 10^{-5}\xi\ \text{eV}$$

$$m_\chi^2 \approx \frac{1}{6\alpha} m_{\rm Pl}^2 \sim (10^{13}\ \text{GeV})^2$$

- ✔ 2 parameters get determined

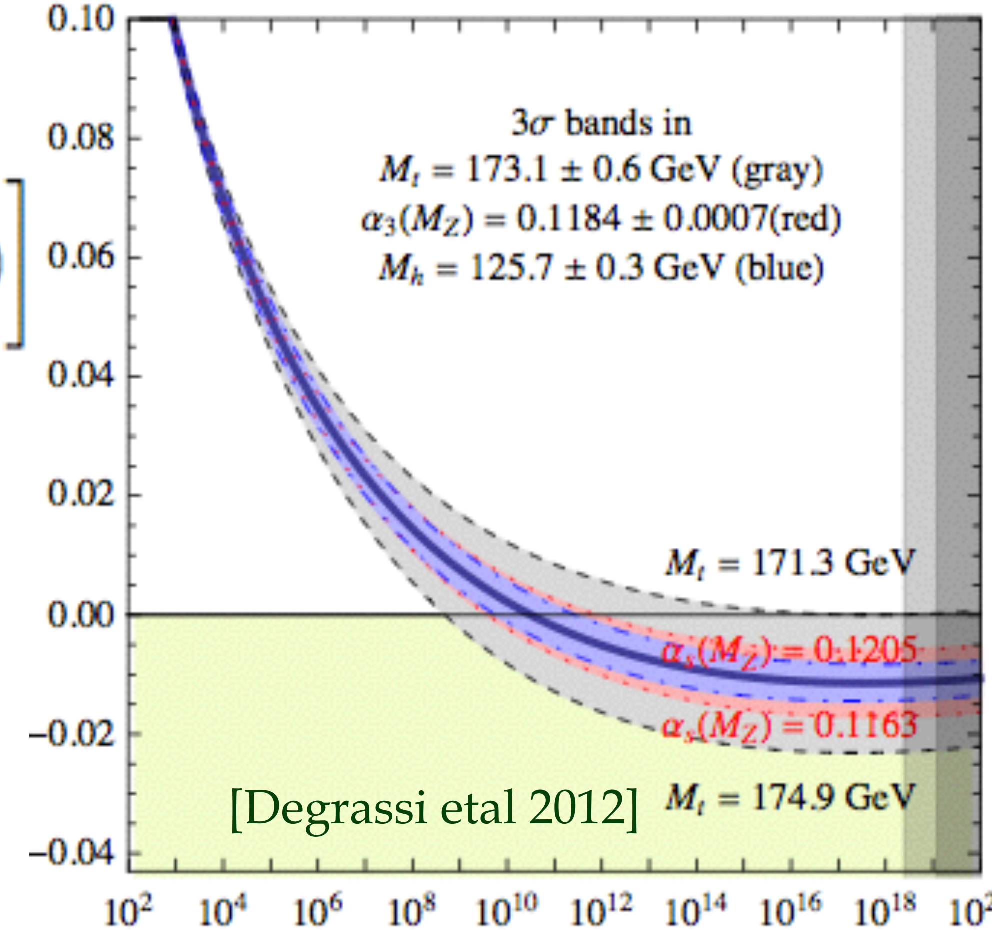
$$-m(m_Z)^2/\lambda(m_Z) \sim (246\ \text{GeV})^2$$

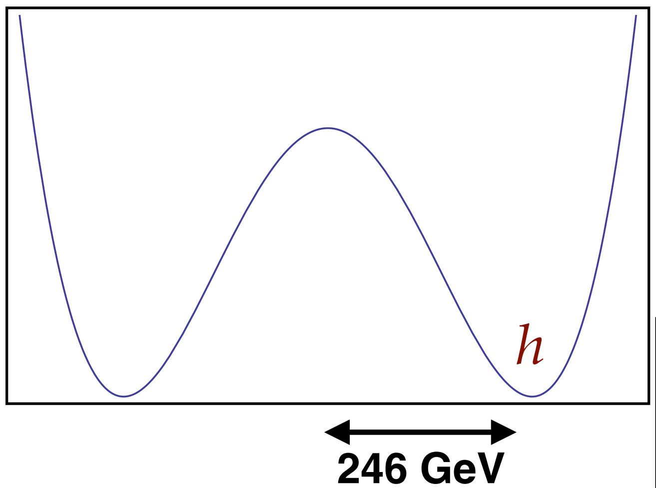
$$\lambda(m_Z) + \xi(m_Z)^2/2\alpha \sim 0.13$$

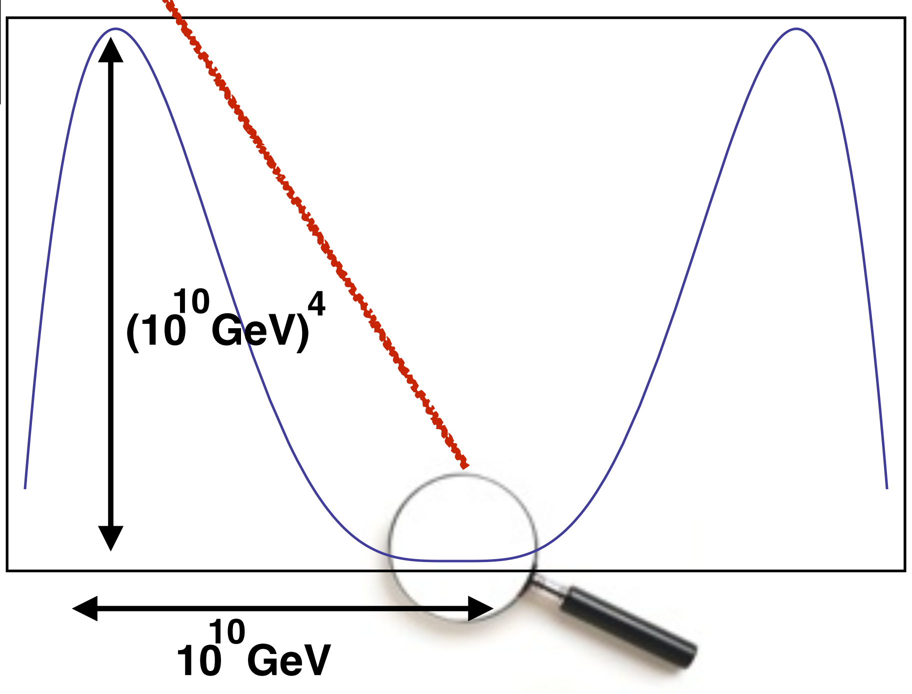
The Metastable Electroweak Vacuum

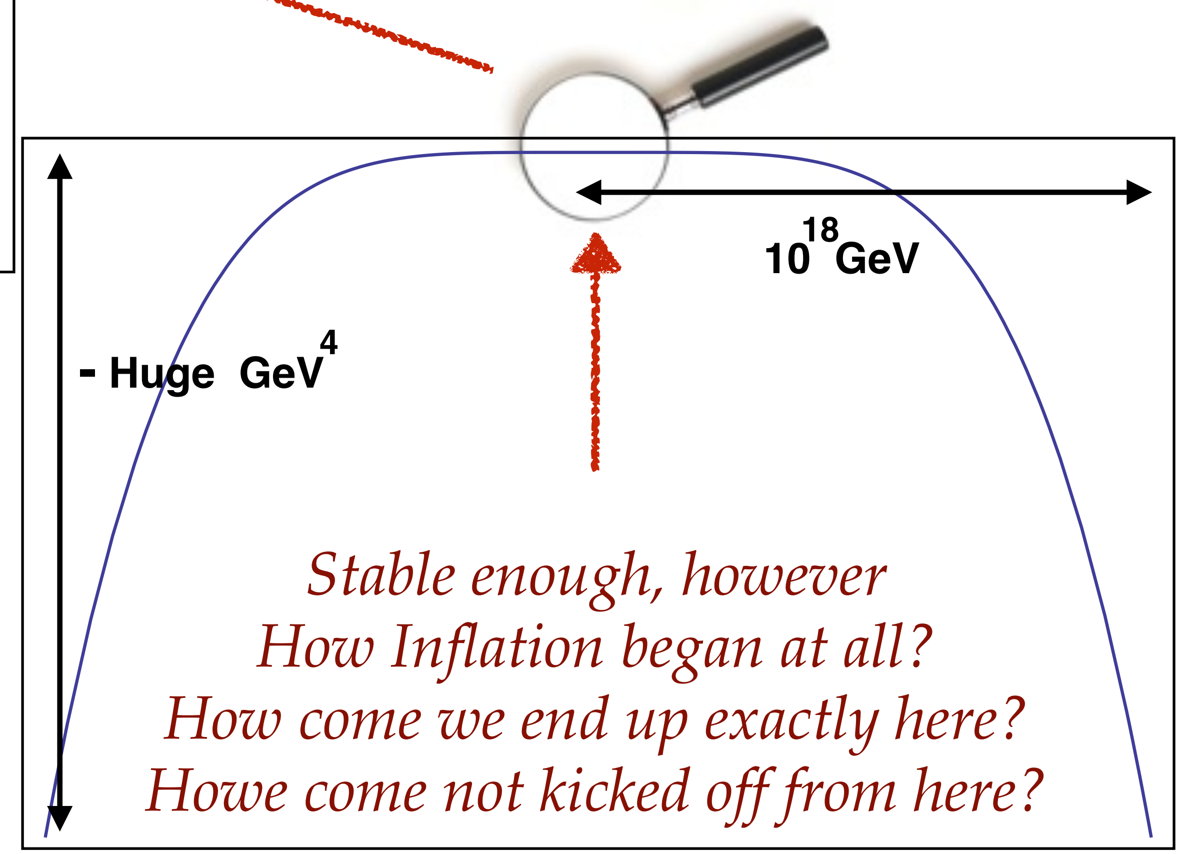
✔ The Standard Model extrapolated to HE:

$$\beta_\lambda = \frac{1}{(4\pi)^2}\left[24\lambda^2 - 6\,y_t^4 + \frac{9}{8}g^4 + \frac{3}{4}g^2 g'^2 + \frac{3}{8}g'^4 + \lambda(12y_t^2 - 9g^2 - 3g'^2)\right]$$









✔ Fine-tuning of the Higgs value!

[see Lebedev-Westphal 2013]

Higgs-like Field in the Jordan Frame

✔ The Higgs potential

$$V_E(\chi,\varphi) = V(\chi) + \frac{1}{2}e^{-2\tilde{\chi}}\left(m^2 - \xi(e^{-\tilde{\chi}}-1)m_{\rm Pl}^2/2\alpha\right)\varphi^2 + \frac{1}{4}e^{-2\tilde{\chi}}\left(\lambda + \xi^2/2\alpha\right)\varphi^4$$

✔ Boundedness from below implies that

$$\lambda(m_{\rm Pl}) + \xi(m_{\rm Pl})^2/2\alpha > 0$$

$$\boxed{\xi(m_{\rm Pl}) \gtrsim 50000}$$

$$\beta_\xi = \frac{6\xi+1}{(4\pi)^2}\left[2\lambda + y_t^2 - \frac{3}{4}g^2 - \frac{1}{4}g'^2\right]$$

Q. Is it too big a number?

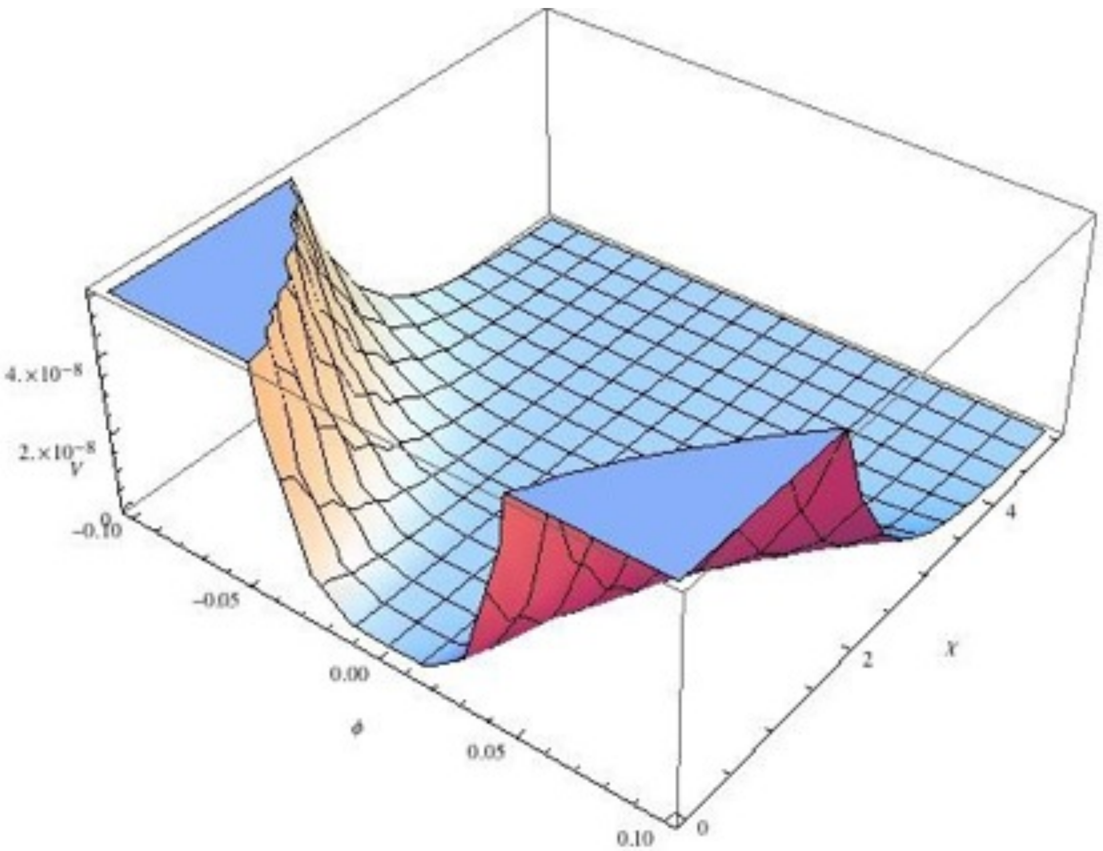
✔ Certainly consistent with cosmology $\xi\varphi_0^2 \lesssim m_{\rm Pl}^2$ $\quad \xi \lesssim 10^{32}$

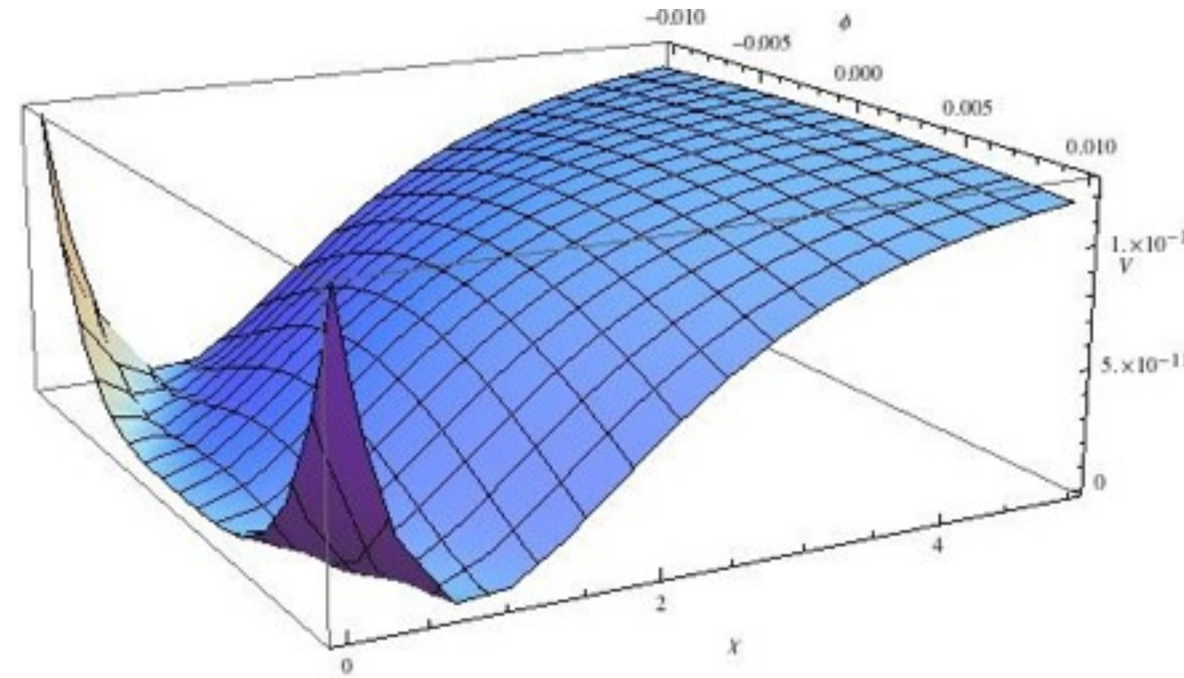
Q. Is that consistent with collider phenomenology?

CMS+Atlas exclude $|\xi| > 2.6\times 10^{15}$ at the 95% C.L.

[Atkins-Calmet PRL'13]

No Scalar Potential in the Jordan Frame: Revisited





✔ Higgs potential induces non-zero vev for Wely field

$$\chi_0 \approx \xi \varphi_0^2 m_{\rm Pl}^{-1} \sim 10^{-5} \xi \ {\rm eV}$$

✔ A non-zero vev and mass is induced by non-zero Weyl vev

$$\varphi_0^2 \approx \xi^{-1} m_{\rm Pl} \chi_0 \approx \xi^{-1} \xi_{\rm Higgs} v^2$$

$$m_\varphi^2 = (\xi^2/\alpha)\varphi_0^2 \approx \xi \xi_{\rm Higgs} v^2/\alpha$$

Starobinsky meets DBI in the Sky

[Kaviani-MT, to appear]

✔ The effect of the higher-order correction and non-minimal coupling on DBI inflation.

✔ An interesting brane-inflation model in string theory.
Not consistent with observation though: too much non-gaussianities.

✔ Scalar with non-minimal coupling and non-canonical kinetic term

$$S = \int \mathrm{d}^4x(-g)^{1/2}\Big(\frac{1}{2}m_{\rm Pl}^2 f(R,\varphi) + P(X,\varphi)\Big)$$

$$f(R,\varphi) = (1+\xi m_{\rm Pl}^{-2}\varphi^2)R + \alpha m_{\rm Pl}^{-2}R^2,$$

$$P(X,\varphi) = -f^{-1}(\varphi)\big[(1-2f(\varphi)X)^{1/2}-1\big] - V(\varphi)$$

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi,$$

Starobinsky meets DBI in the Sky

✔ A Weyl transformation from the Jordan to the Einstein frame:

$$g^E_{\mu\nu} = (1 + \xi m_{\rm Pl}^{-2}\varphi^2 + 2\alpha m_{\rm Pl}^{-2} R) g_{\mu\nu} \equiv e^{\tilde{\chi}} g_{\mu\nu},$$

✔ The Einstein frame action:

$$S = \int {\rm d}^4x (-g_E)^{1/2} \Big(\frac{1}{2} m_{\rm Pl}^2 R_E - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V_E(\chi) + P_E(X,\varphi) \Big)$$

$$V_E(\chi) = \frac{1}{8}\alpha^{-1} m_{\rm Pl}^4 \big(1 - e^{-\tilde{\chi}}\big)^2,$$

$$P_E(X,\varphi) = -e^{-2\tilde{\chi}} f^{-1}(\varphi) \big[(1 + f e^{\tilde{\chi}} g_E^{\mu\nu} \partial_\mu\varphi \partial_\nu\varphi)^{1/2} - 1 \big] - e^{-2\tilde{\chi}} V(\varphi)$$

Starobinsky meets DBI in the Sky

✔ The dynamics on the FRW geometry

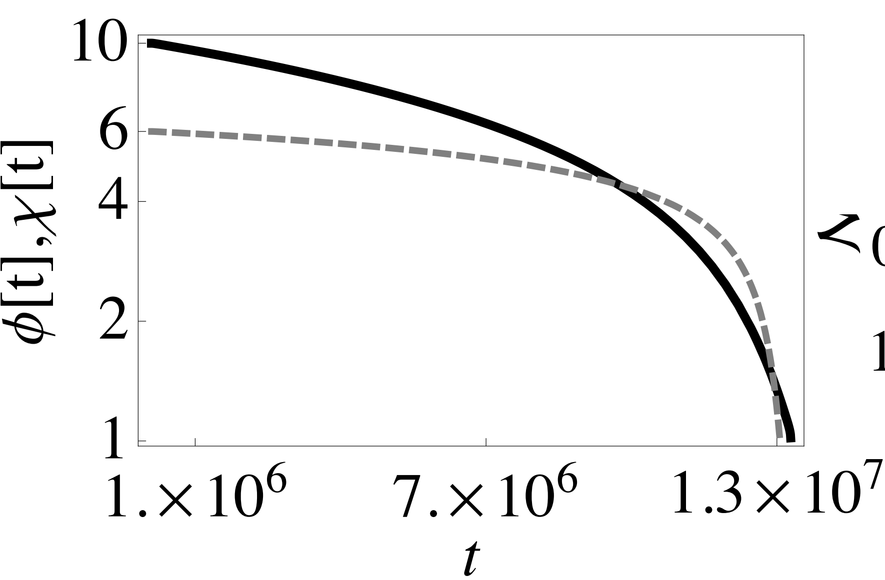
$$\ddot{\varphi} + 3H\gamma^{-2}\dot{\varphi} + e^{-\tilde{\chi}}\gamma^{-3}V_{\varphi}(\varphi) + \frac{1}{2}(1-3\gamma^{-2})\dot{\varphi}\dot{\chi} + \frac{1}{2}e^{-\tilde{\chi}}f^{-2}(\varphi)f_{\varphi}(\varphi)\big(1-3\gamma^{-2}+2\gamma^{-3}\big) = 0$$

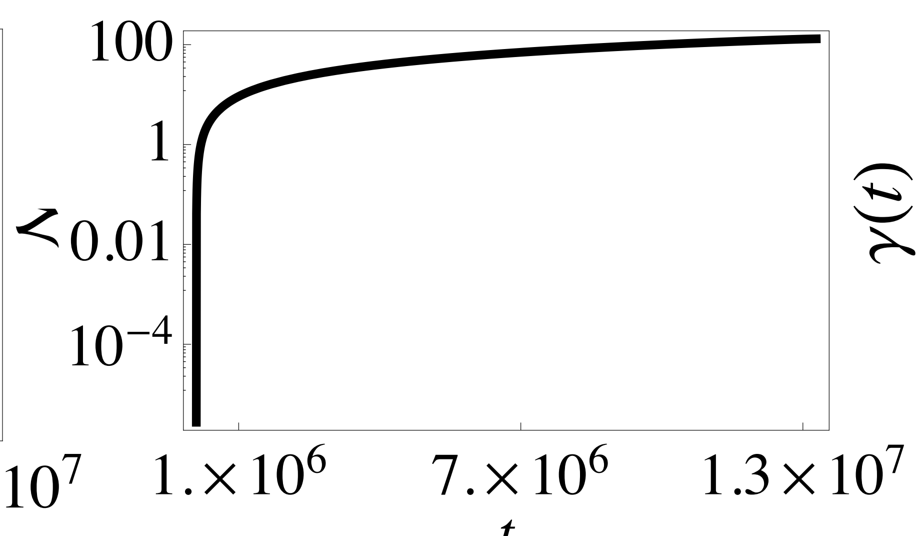
$$\ddot{\chi} + 3H\dot{\chi} + (8/3)^{1/2}\alpha^{-1}m_{\rm Pl}^{3}e^{-\tilde{\chi}}(1-e^{-\tilde{\chi}}) - 2e^{-2\tilde{\chi}}V(\varphi) + \frac{1}{2}e^{-2\tilde{\chi}}f^{-1}(\varphi)\big(4-\gamma-3\gamma^{-1}\big) = 0$$

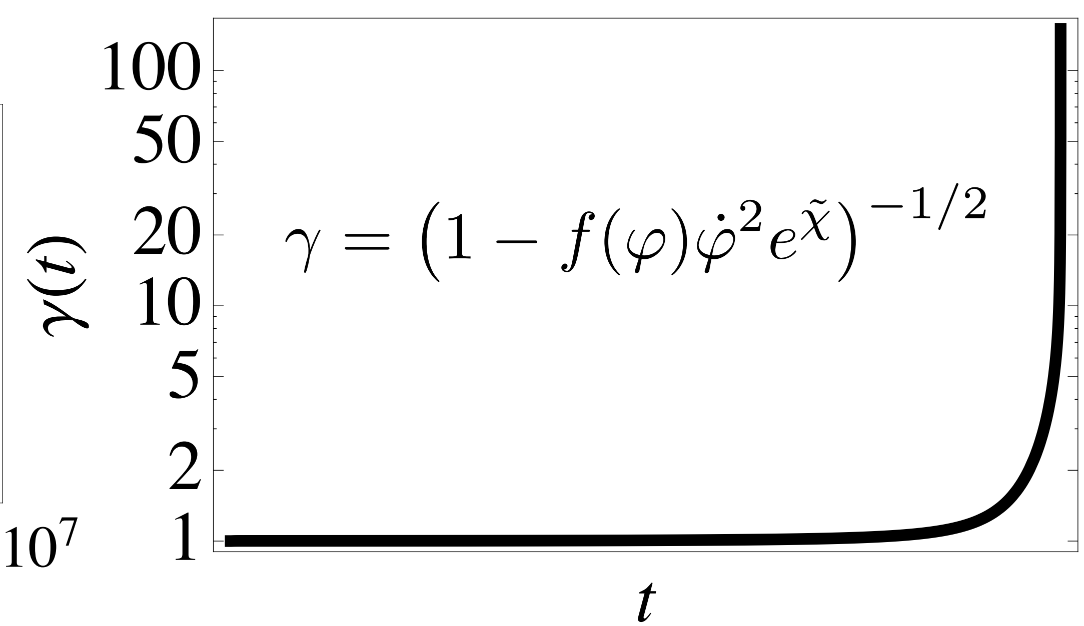
$$3H^2 m_{\rm Pl}^2 = \frac{1}{2}\dot{\chi}^2 + \alpha^{-1}m_{\rm Pl}^4(1-e^{-\tilde{\chi}})^2 + e^{-2\tilde{\chi}}V(\varphi) - e^{-2\tilde{\chi}}f^{-1}(1-\gamma)$$

$$-2\dot{H}m_{\rm Pl}^2 = \dot{\chi}^2 + e^{-2\tilde{\chi}}f^{-1}(\gamma-\gamma^{-1})$$

✔ A double-inflation model







Summary

A phenomenological study:

✔ The non-minimal coupling of scalar fields to an $f(R)$ theory of gravity changes the behavior of both the scalar and the gravity sectors.

✔ If the Standard Model is to be extrapolated to high scale, this coupling can stabilize the electroweak vacuum. Besides it explains why the Higgs ended up here.

✔ All (non-minimally coupled) scalars get stabilized even if they have no potential in the Jordan frame.

✔ This framework also changes the dynamics of scalars with non-canonical kinetic terms (DBI, K-essence).

Thank You