Large Number of Axions and Dark Radiation in String/M-theory

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Overview

- Axiverse cosmological history
- Dark radiation/ Moduli-induced axion problem
- Decay width coefficients
- Constraint on internal manifold
- G_2 compactified M-theory
- Conclusion and outlook

Axiverse cosmological history

- Moduli and axions arise from 11D metric and antisymmetric tensor fields
- The moduli space of X is parameterised by moduli s_i and axion fields t_i

$$z_i = t_i + is_i$$

- *The number of moduli/axions is expected to be large* ($N \sim 100 - 1000$)
- Moduli start oscillate when $H \sim m_X \rightarrow$ matter dominated universe
- Moduli decay $\Gamma \sim \frac{m_X^3}{M_{Pl}^2}$ can spoil the BBN prediction $\rightarrow m_X \gtrsim 10\text{TeV}$ (de Carlos, Casas, Quevedo, Roulet, 1993, Banks, Kaplan, Nelson, 1993)
- Shift symmetry of axion is broken by non-perturbative effects $\rightarrow$ axions have mass range $10^{-33}\text{eV} - 1\text{eV}$
- Sizeable coupling $\rightarrow$ moduli decay produces *too much energy density of axion*

Dark radiation problem

- Dark radiation is quantified by effective number of relativistic species, N_{eff}
- The axionic dark radiation from moduli decay can be calculated by

$$\Delta N_{eff} = \frac{43}{7}\frac{\Gamma_{axions}}{\Gamma_{visible}}\left(\frac{g^*}{g^*_{reheat}}\right)^{1/3}$$

- The effective number of relativistic species from observables is

$$N_{eff} = 3.55 \pm 0.60 \text{ (WMAP9)},\ N_{eff} = 3.30 \pm 0.27 \text{ (Planck)}$$

- $T_{\text{reheat}} \lesssim 100$ GeV $\rightarrow \frac{\Gamma_{axions}}{\Gamma_{visible}} \sim 0.1$ (Higaki, Nakayama, Takahashi, 2013)
- A model with only one axion → tight constraint on couplings (Cicoli, Conlon, Quevedo, 2012; Angus, Conlon, Haisch, Powell, 2013)
- $\Gamma_{axions} \propto N \rightarrow \Delta N_{eff} \propto N$
- Axiverse with $O(100)$ axions → very tight constraint on number of axions

Our goal: study a (more generic) solution with large number of axions

Analysis: decay width coefficients

- The interaction Lagrangian can be written as following

$$\mathcal{L} = \frac{1}{2}K_{ij}\partial^{\mu}s^{i}\partial_{\mu}s^{j} + \frac{1}{2}K_{ij}\partial^{\mu}t^{i}\partial_{\mu}t^{j} - \frac{1}{4}\left(\sum_{i=1}^{N}N_i z_i\right)F_{\mu\nu}F^{\mu\nu} + K_{\alpha\beta}D_{\mu}f^{\alpha}D^{\mu}f^{\beta} + K_{\alpha\beta}\tilde{f}^{\alpha}\not{D}\tilde{f}^{\beta}$$

- Moduli decay width is straightforward

$$\Gamma_{\text{axions}} = \sum_{j=1}^{N}\left(\sum_{i=1}^{N}C_{ij}U_{ik}\right)^2 \frac{m_{s_k}^3}{M_{PL}^2}$$

$$\Gamma_{\text{gauge particles}} = n_G\left(\sum_{i=1}^{N}B_i U_{ik}\right)^2 \frac{m_{s_k}^3}{M_{PL}^2}$$

$$\Gamma_{\text{fermions/sfermions}} = n_f\left(\sum_{i=1}^{N}D_i U_{ik}\right)^2 \frac{m_{s_k}^3}{M_{PL}^2}$$

- where $C_{ij} = \frac{1}{\sqrt{K_{ii}^D}}\frac{\partial \ln K_{ii}^D}{\partial s_j}, B_i = \frac{\alpha}{\sqrt{K_{ii}^D}}N_i, D_i = \frac{1}{\sqrt{K_{ii}^D}}\frac{\partial \ln K_{\alpha\alpha}^D}{\partial s_i}$

Analysis: decay width coefficients

- Assume that all couplings are roughly the same order, $C_{ij} \sim \langle C \rangle \sim B_j \sim \langle B \rangle \sim D_j \sim \langle D \rangle$

- As one would expect, increasing number of axions is equivalent to increasing dark radiation.

$$\langle \Delta N_{eff} \rangle \propto \frac{\Gamma_{axions}}{\Gamma_{visible}} \propto \frac{N \langle C \rangle^2}{n_G \langle B \rangle^2 + n_f \langle D \rangle^2} \propto N$$

where the unity in mean value of $(\sum_{i=1}^{N} U_{ik})^2$ is used.

- Very bad situation for axiverse, $\Delta N_{eff} \sim N \sim 100 - 1000$

Analysis: decay width coefficients

- Simplest way out $\rightarrow$ force each modulus to interact with only its partner $\rightarrow$ Dark radiation is N independent

- if we impose condition on coefficients:

$$C_{ij} = \frac{1}{\sqrt{K_{ii}^D}} \frac{\partial \ln K_{ii}^D}{\partial s_j} \equiv C_i \delta_{ij}$$

- on average, dark radiation becomes

$$\langle \Delta N_{eff} \rangle \propto \frac{\langle C \rangle^2}{n_G \langle B \rangle^2 + n_f \langle D \rangle^2}$$

where the orthogonality of rotation matrix, $(\sum_{i=1}^{N} U_{ik}^2) = 1$, is introduced.

- The strong correlation is relaxed $\rightarrow$ which manifold exhibits this property?

Constraints on internal manifold: Examples

- Calabi-Yau

$$V_X = \sum_{i,j,k}^{N} d_{ijk} S_i S_j S_k, \quad K = -\gamma \ln V_X$$

generically $C_{ij} \neq C_i \delta_{ij}$ unless the coefficients is carefully chosen, for example $V_X = d\ S_1^3$ or $V_X = d\ S_1 S_2 S_3$

- G_2 manifold

$$V_X = \prod_{i=1}^{N} S_i^{a_i}, \quad K = -\gamma \sum_{i=1}^{N} a_i \ln S_i, \quad K_{ij} = \frac{\gamma a_i \delta_{ij}}{s_i^2}$$

Only non-zero component is

$$\partial_i K_{ii} = \frac{-2\gamma a_i}{s_i} \to C_{ij} = \frac{-2\langle s_i \rangle^2}{\sqrt{\gamma a_i}} \delta_{ij}$$

Condition on moduli mass matrix

- Can we do better?
- We found important relation between moduli mass matrix and eigenvalue of Kahler metric which exhibit $\frac{1}{N}$ behaviour of dark radiation.

$$\sqrt{K_i} \propto U_{ij}$$

where K_i is eigenvalues of Kahler metric and U_{ij} is rotation matrix of moduli fields.

- In this case, correlation becomes

$$\langle \Delta N_{eff} \rangle \propto \frac{N \langle C \rangle^2}{n_G N^2 \langle B \rangle^2 + n_f N^2 \langle D \rangle^2} \propto \frac{1}{N}$$

- How can we realise this in practice? $\rightarrow$ M-theory on G_2 manifold

G_2 compactified M theory

- M theory on manifold with G_2 holonomy - G_2 MSSM (Acharya, Bobkov, Kane, Shao, Kumar, 2008; Acharya, Kane, Kumar, 2012)
- The Kahler potential arising from G_2 manifold compactification is controlled by moduli fields and given by the function

$$K = -3 \ln \left(\prod_{i=1}^{N} s_i^{a_i} \right), \quad \sum_{i=1}^{N} a_i = \frac{7}{3}$$

- It has been shown that a setup with a hidden sector with two gauge groups where first group is non-abelian with 1 flavour of quarks and second group is pure glue non-abelian leads to dS vacua.
- In this realistic setup, superpotential is written as

$$W = A_1 \phi^a e^{ib_1 \sum_i^N N_i S_i} + A_2 e^{ib_2 \sum_i^N N_i S_i}$$

G_2 compactified M theory

- It is straightforward to work out rotation matrix from moduli stabilisation.

$$U_{kj} = \sqrt{\frac{a_{j+1}}{(\sum_{i=1}^{j} a_i)(\sum_{i=1}^{j+1} a_i)}}\sqrt{a_k}, \quad , \quad k \leq j$$

$$U_{kj} = -\sqrt{\frac{\sum_{i=1}^{j} a_i}{\sum_{i=1}^{j+1} a_i}} \quad , \quad k = j+1$$

$$U_{kN} = \sqrt{\frac{3a_k}{7}}$$

where $i = 1 \ldots N-1$ are degenerated lightest moduli and $i = N$ is heavy modulus.

- Notice that except $k = j+1$, $U_{kj} \propto \sqrt{a_k} \propto \sqrt{K_k}$. Therefore, we can suppress the element $U_{j+1,j}$ by setting

$$\sum_{i=1}^{j} a_i \ll a_{j+1}$$

We expect $\frac{1}{N}$ suppression on dark radiation under this condition.

G_2 compactified M theory

- In stead of scanning N parameters space satisfying condition, $\sum_{i=1}^{N-1} a_i \ll a_N$, we will give toy models which can be parametrised by single parameter.

- n-moduli dominated configuration

$$a = \{\underbrace{\epsilon\bar{a}, \ldots, \epsilon\bar{a}}_{N-n}, \underbrace{\frac{7}{3n} - \frac{\epsilon\bar{a}(N-n)}{n}, \ldots, \frac{7}{3n} - \frac{\epsilon\bar{a}(N-n)}{n}}_{n}\}$$

- geometric sequence configuration

$$a = \{a_0, a_0, a_0 r, a_0 r^2, \ldots, a_0 r^{N-2}\}$$

Results

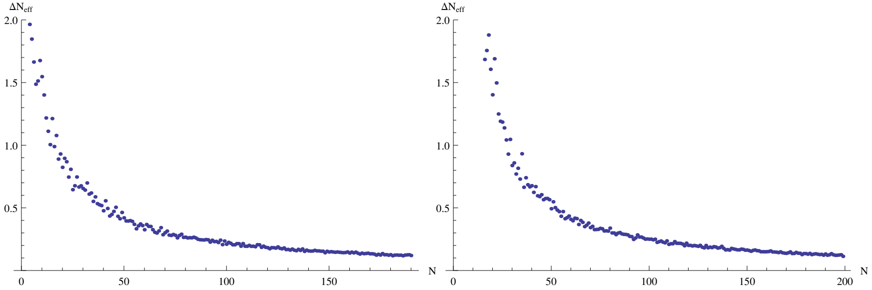


Figure: Result from geometric sequence and double moduli dominated configurations showing ΔN_{eff} as a function of N, where $r = 2$

Results

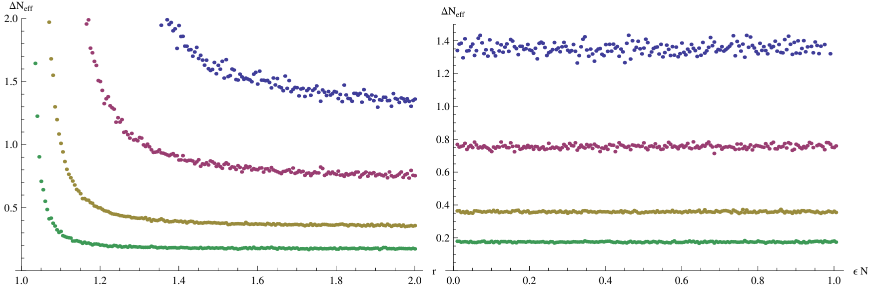


Figure: Result from geometric sequence and double moduli dominated configurations showing ΔN_{eff} as a function of ϵN, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

Conclusions

- Because of large number of axions, generic axiverse model have excessive amount of dark radiation comparing with observations.
- G_2 manifold relaxes strong correlation between number of moduli/axions and dark radiation.
- It is possible to suppress $(\frac{1}{N})$ dark radiation by proposing relation between moduli couplings and moduli mass matrix.
- Within this framework realistic scenario of early universe can be achieved for large number of moduli/axions.

Back up slides

G_2MSSM, Reheating temperature and dark matter relics

$$\Omega_{LSP}h^2 = 0.15\left(\frac{1}{D_{\text{total}}}\right)^{1/2}\left(\frac{10.75}{g_\star}\right)^{1/4}\left(\frac{m_{LSP}}{100\text{GeV}}\right)\left(\frac{\sigma_0}{\langle\sigma v\rangle}\right)\left(\frac{100\text{TeV}}{m_X}\right)^{3/2}$$

$$T_r = \left(\frac{40}{\pi g_\star}\right)^{1/4}\sqrt{\frac{D_{\text{total}}m_X^3}{M_{Pl}}}$$

We use $m_{LSP} = 100MeV$, $\langle\sigma v\rangle \sim \sigma_0 \sim 3\times 10^{-24}\ cm^3 s^{-1}$.

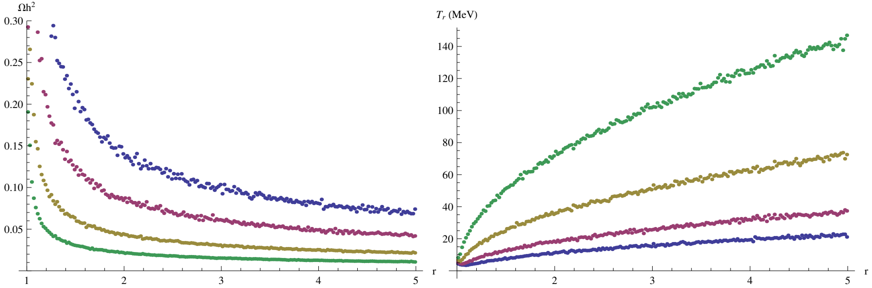


Figure: Geometric sequence configuration result showing $\Omega_{LSP}h^2$ and reheating temperature as a function of common ratio r, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

G_2MSSM, Reheating temperature and dark matter relics

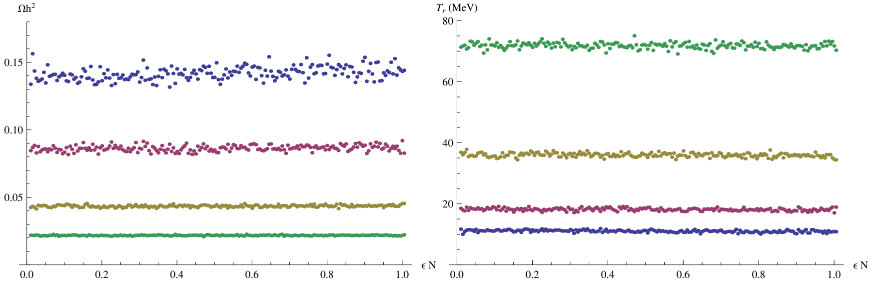


Figure: Double moduli dominated configuration result showing $\Omega_{LSP}h^2$ and reheating temperature as a function of common ratio r, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

N_{eff} calculation

- At the time of baryogenesis, the effective number of neutrinos is defined as

$$\rho_{rad} = \rho_{e^\pm} + \rho_\gamma + N_{eff}\,\rho_\nu$$

- The extra radiation part can be included inside the definition of N_{eff} as following.

$$\begin{aligned} \rho'_{rad} &= \rho_{e^\pm} + \rho_\gamma + N_{eff}\,\rho_\nu \left(1 + \frac{\rho_a}{\rho_{3\nu}}\right) \\ \frac{\Delta N_{eff}}{N_{eff}} &= \frac{\rho_a}{\rho_{3\nu}} \end{aligned}$$

- Since both neutrinos and axion are not in thermal equilibrium

$$\frac{\rho_a(\mathrm{BBN})}{\rho_{3\nu}(\mathrm{BBN})} = \frac{\rho_a(\nu\mathrm{dec})}{\rho_{3\nu}(\nu\mathrm{dec})}$$

where

$$\begin{aligned} \frac{\rho_{rad}}{\rho_{3\nu}} &= 1 + \frac{\rho_{e^\pm}}{\rho_{3\nu}} + \frac{\rho_\gamma}{\rho_{3\nu}} \\ &= \frac{43}{21} \end{aligned}$$

- Putting all relations together, we obtain

$$\Delta N_{eff} = \frac{43\rho_a(\nu\mathrm{dec})}{7\rho_{rad}(\nu\mathrm{dec})}$$

N_{eff} calculation

- To make prediction from moduli branching fraction, we need to relate this quantity to the time of reheating in moduli decay scenario. Firstly, because of its very weak coupling, axion has never been in thermal equilibrium. As a consequence, its energy density scale as $1/a^4$. We can write

$$\frac{\rho_a(\nu\text{dec})}{\rho_a(reheat)} = \frac{a^4(\text{reheat})}{a^4(\nu\text{dec})}$$

- Using the fact that comoving entropy is a conserved quantity $S \sim a^3 g_\star(T) T^3$ or $T \sim \frac{1}{a g_\star^{1/3}(T)}$, we get

$$\begin{aligned}
\rho_{rad} &\sim g_\star(T) T^4 \\
&\sim \frac{1}{a^4 g_\star^{1/3}(T)} \\
\frac{\rho_{rad}(\nu\text{dec})}{\rho_{rad}(\text{reheat})} &= \frac{a^4(\text{reheat}) g_\star^{1/3}(\text{reheat})}{a^4(\nu\text{dec}) g_\star^{1/3}(\nu\text{dec})}
\end{aligned}$$

- Substitute back, we obtain the effective number of neutrinos

$$\begin{aligned}
\Delta N_{eff} &= \frac{43}{7} \frac{\rho_a(\text{reheat})}{\rho_{rad}(\text{reheat})} \frac{g_\star^{1/3}(\nu\text{dec})}{g_\star^{1/3}(\text{reheat})} \\
&= \frac{43}{7} \frac{\text{Br}(X_i \to \text{axions})}{1 - \text{Br}(X_i \to \text{axions})} \frac{g_\star^{1/3}(\nu\text{dec})}{g_\star^{1/3}(\text{reheat})}
\end{aligned}$$

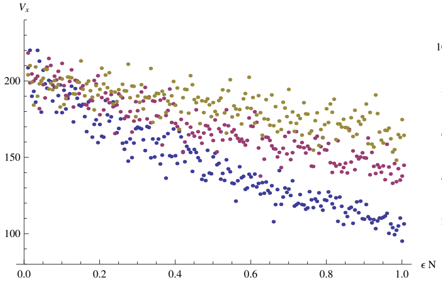
G_2MSSM, supergravity limits

- We can further test configurations a_i by GUT coupling constraint and supergravity limit.
- The volume function can be written by

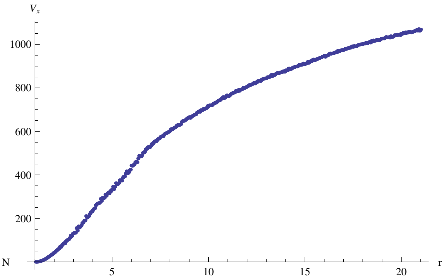
$$V_X = \sum_{i=1}^{N} \langle s_i \rangle^{a_i} = \sum_{i=1}^{N} \left(\frac{a_i}{N_i^{sm}} \frac{3P_{eff}Q}{14\pi(Q-P)} \right)^{a_i}$$

- where we match N_i^{sm} with gauge coupling at GUT scale $\sum_{i=1}^{N} N_i^{sm} \langle s_i \rangle = \frac{1}{\alpha} \approx 25$

G_2MSSM, supergravity limits



(a) Double moduli dominated



(b) Geometric sequence

Figure: Result from each configurations showing V_X as a function of parameter ϵN and common ratio r, where points in blue, red, yellow are $N = 20, 50, 100$ respectively