Vacua of β-supergravity

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arXiv:1306.4381, 1402.5972 and work in progress
by D. A. and André Betz

String phenomenology 2014,
08/07/2014, ICTP, Trieste, Italy

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NSNS sector in 10D and 4D supergravities

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4D gauged supergravity: also non-geometric fluxes $Q_a{}^{bc}$, R^{abc}

all fluxes are components of the embedding tensor

$\Leftrightarrow$ some "structure constants" in gauging algebra

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10D origin of Q and R? Naively not present in $\mathcal{L}_{\rm NSNS}$

+ argued to descend from 10D non-geometric backgrounds...

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$\hookrightarrow \tilde{\mathcal{L}}_\beta(f^a{}_{bc},\ Q_a{}^{bc},\ R^{abc})$, and standard geometry restored
$\hookrightarrow$ allows compactification, uplift of 4D gauged supergravities.

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Relation to conditions for supersymmetric vacua with SU(3)×SU(3) structure.

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Lagrangian $\tilde{\mathcal{L}}_\beta(f, Q, R)$

Reformulation of $\mathcal{L}_{\text{NSNS}}$ via a field redefinition

Build on earlier results: arXiv:1303.0251 by D. A.

arXiv:1106.4015 by D. A., M. Larfors, D. Lüst, P. Patalong

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$$\mathcal{H} = \begin{pmatrix} g - bg^{-1}b & -bg^{-1} \\ g^{-1}b & g^{-1} \end{pmatrix} = \mathcal{E}^T \, \mathbb{I} \, \mathcal{E} = \tilde{\mathcal{E}}^T \, \mathbb{I} \, \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{g} & \tilde{g}\beta \\ -\beta\tilde{g} & \tilde{g}^{-1} - \beta\tilde{g}\beta \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} e & 0 \\ e^{-T}b & e^{-T} \end{pmatrix}, \ \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{e} & \tilde{e}\beta \\ 0 & \tilde{e}^{-T} \end{pmatrix}, \ \mathbb{I} = \begin{pmatrix} \eta_d & 0 \\ 0 & \eta_d^{-1} \end{pmatrix}, \quad \begin{matrix} g = e^T \eta_d e \\ \tilde{g} = \tilde{e}^T \eta_d \tilde{e} \end{matrix}$$

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Idea: field redef. $(g_{mn}, b_{mn}, \phi) \leftrightarrow (\tilde{g}_{mn}, \beta^{mn}, \tilde{\phi})$, β antisym.

$$\tilde{g}^{-1} = (g+b)^{-1} g (g-b)^{-1}$$
$$\beta = -(g+b)^{-1} b (g-b)^{-1}$$
$$\Leftrightarrow (g+b)^{-1} = (\tilde{g}^{-1} + \beta) \ , \ \frac{e^{-2\tilde{\phi}}}{e^{-2\phi}} = \frac{\sqrt{|g|}}{\sqrt{|\tilde{g}|}}$$

$\Leftrightarrow$ reparametrization of gen. metric $\mathcal{H}$, i.e. new gen. vielbein

$$\mathcal{H} = \begin{pmatrix} g - bg^{-1}b & -bg^{-1} \\ g^{-1}b & g^{-1} \end{pmatrix} = \mathcal{E}^T \, \mathbb{I} \, \mathcal{E} = \tilde{\mathcal{E}}^T \, \mathbb{I} \, \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{g} & \tilde{g}\beta \\ -\beta\tilde{g} & \tilde{g}^{-1} - \beta\tilde{g}\beta \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} e & 0 \\ e^{-T}b & e^{-T} \end{pmatrix}, \ \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{e} & \tilde{e}\beta \\ 0 & \tilde{e}^{-T} \end{pmatrix}, \ \mathbb{I} = \begin{pmatrix} \eta_d & 0 \\ 0 & \eta_d^{-1} \end{pmatrix}, \quad \begin{matrix} g = e^T \eta_d e \\ \tilde{g} = \tilde{e}^T \eta_d \tilde{e} \end{matrix}$$

β w.r.t. Q, R: motivations from Gen. Complex Geom./sugra

hep-th/0609084, arXiv:0708.2392 by P. Grange, S. Schäfer-Nameki

arXiv:0807.4527 by M. Graña, R. Minasian, M. Petrini, D. Waldram

Apply the field redefinition to $\mathcal{L}_{\text{NSNS}}(g, b, \phi)$

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To see the fluxes, Q, better to use flat tangent space indices

$$Q_c{}^{ab} = \partial_c \beta^{ab} - 2\beta^{d[a} f^{b]}{}_{cd} \ , \quad R^{abc} = 3\beta^{d[a} \nabla_d \beta^{bc]}$$

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10D theory with non-geometric fluxes (β-supergravity),
uplift of 4D gauged supergravity ✓

More structure

David
ANDRIOT

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More structure

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Structures: natural with Generalized Geometry formalism

10D examples of non-geometries

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10D examples of non-geometries

Famous toroidal example: NSNS sector:

$$T^3 + H_{123} \overset{T_1}{\longleftrightarrow} \text{twisted torus } (f^1{}_{23}) \overset{T_2}{\longleftrightarrow} \text{non} - \text{geometric config.}$$

hep-th/0211182 by S. Kachru, M. B. Schulz, P. K. Tripathy, S. P. Trivedi

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arXiv:1402.5972 by D. A. and André Betz

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Other example: *NS*-branes (NSNS backgrounds)

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$$\mathrm{d}s^2 = \mathrm{d}s^2_6 + f_Q \left(\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\varphi^2\right) + f_Q^{-1} \left(1 + \frac{a^2}{f_Q^2}\right)^{-1} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \; b = \dots$$

$$f_Q(\rho) = \mathrm{cst} - 2q\pi \ln \rho \, , \; a = -\varphi \; \rho \partial_\rho f_Q$$

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arXiv:1004.2521, 1209.6056 by J. de Boer and M. Shigemori; arXiv:1109.4484 by E. A. Bergshoeff, T. Ortin and F. Riccioni; arXiv:1303.1413 by F. Hassler and D. Lüst; D. Geissbühler, D. Marqués, C. Núñez and V. Penas; T. Kimura and S. Sasaki; A. Chatzistavrakidis, F. F. Gautason, G. Moutsopoulos and M. Zagermann

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Comparison to the 4D T-duality chain $\Rightarrow$ Q-flux ?!

10D examples of non-geometries

Famous toroidal example: NSNS sector:

$$T^3 + H_{123} \xleftrightarrow{T_1} \text{twisted torus } (f^1{}_{23}) \xleftrightarrow{T_2} \text{non} - \text{geometric config.}$$

hep-th/0211182 by S. Kachru, M. B. Schulz, P. K. Tripathy, S. P. Trivedi
hep-th/0303173 by D. A. Lowe, H. Nastase, S. Ramgoolam

$$g = h_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{h_0} \end{pmatrix}, \; b = h_0 \begin{pmatrix} 0 & -Kz^3 & 0 \\ Kz^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; h_0(z^3) = \frac{1}{1 + (Kz^3)^2}$$

2 patches for $S^1_{z^3}$, proof that no diffeo. to glue g on overlaps
+ fields glue with T-duality element $\in$ O(2,2) $\Rightarrow$ non-geometry

arXiv:1402.5972 by D. A. and André Betz

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Comparison to the 4D T-duality chain $\Rightarrow$ Q-flux ?! $\rightarrow$ β-sugra

Non-geometry/geometry: global aspects

David
ANDRIOT

Introduction

β-supergravity

Lagrangian

More structure

(Non)-geometry

BI, *NS*-branes, ...

Conclusion

Non-geometry/geometry: global aspects

Field redefinition always possible off-shell/locally

$$\mathcal{L}_{\text{NSNS}}(g, b, \phi) = \tilde{\mathcal{L}}_{\beta}(\tilde{g}, \beta, \tilde{\phi}) + \partial(\dots)$$

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Geometry restored: T^3, Q-flux ✓.

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arXiv:1402.5972 by D. A. and André Betz

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**β-supergravity has 10D non-geometric fluxes,
it restores geometry $\Rightarrow$ alternative description, compactif. ✓**

Introduction
β-supergravity
BI, NS-branes, ...
BI
NS-branes
$\mathcal{D}$
SUSY
Conclusion

Bianchi identities and NS-branes

Bianchi identities for the fluxes

Bianchi identities and *NS*-branes

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4D, constant fluxes: gaugings of gauged supergravity

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hep-th/0508133 by J. Shelton, W. Taylor, B. Wecht

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Automatically satisfied with 10D expressions of f, Q, R

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More involved expressions for Bianchi identities

arXiv:1205.1522 by R. Blumenhagen, A. Deser, E. Plauschinn and F. Rennecke

NS-branes as sources

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On brane solutions $\Rightarrow$ Poisson equations

KK-monopole: $\Delta_3 f_K = c_K\ \delta^{(3)}(r_3)$

Q-brane: $\Delta_2 f_Q = c_Q\ \delta^{(2)}(r_2)$

Generalized derivative $\mathcal{D}$

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Γ^A: Clifford algebra: $\{\Gamma^A,\Gamma^B\} = 2\eta^{AB}$, use the representation

$\Gamma^A:\ \Gamma^a = 2\tilde{e}^a\ ,\ \Gamma_a = 2\iota_a$, spinor Ψ: polyform

SUSY vacua with SU(3)×SU(3) structure

For standard supergravity, $\mathcal{D}A = 2e^{\phi}(\mathrm{d} - H\wedge)(e^{-\phi}A)$ appears for RR (spinors): $F \sim \mathcal{D}C$

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For standard supergravity, $\mathcal{D}A = 2e^{\phi}(\mathrm{d} - H\wedge)(e^{-\phi}A)$ appears for RR (spinors): $F \sim \mathcal{D}C$, and in conditions for supersymmetric vacua with SU(3)×SU(3) structure:

$$\text{Killing spinor eq.} \Leftrightarrow \begin{cases} \mathcal{D}\Phi_1 = 4\varepsilon\ e^{-A}\mu\ \mathrm{Re}(\Phi_2) \\ \mathcal{D}\Phi_2 = 6\varepsilon\ e^{-A}\ \mathrm{i}\,\mathrm{Im}(\overline{\mu}\Phi_1) + \mathrm{RR} \end{cases}$$

where $e^A = \text{constant}$, $|\mu|^2 \sim -\Lambda$, and

$$\text{IIA} : \Phi_1 = \Phi_+ \ , \ \Phi_2 = \Phi_- \ , \ \varepsilon = +1 \ ,$$
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hep-th/0505212, hep-th/0609124 by M. Graña, R. Minasian, M. Petrini and A. Tomasiello

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SUSY vacua with SU(3)×SU(3) structure

For standard supergravity, $\mathcal{D}A = 2e^{\phi}(\mathrm{d} - H\wedge)(e^{-\phi}A)$ appears for RR (spinors): $F \sim \mathcal{D}C$, and in conditions for supersymmetric vacua with SU(3)×SU(3) structure:

$$\text{Killing spinor eq.} \Leftrightarrow \begin{cases} \mathcal{D}\Phi_1 = 4\varepsilon\ e^{-A}\mu\ \mathrm{Re}(\Phi_2) \\ \mathcal{D}\Phi_2 = 6\varepsilon\ e^{-A}\ \mathrm{i}\,\mathrm{Im}(\overline{\mu}\Phi_1) + \mathrm{RR} \end{cases}$$

where $e^A = \text{constant}$, $|\mu|^2 \sim -\Lambda$, and

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In β-supergravity: we derive exactly the same result, with $\mathcal{D}\Phi = 2e^{\tilde{\phi}}\left(\mathrm{d} - \check{\nabla}^a \cdot \iota_a + \mathcal{T}\vee + R\vee\right)\left(e^{-\tilde{\phi}}\Phi\right)$.

Conclusion

4D non-geometric fluxes, 10D non-geometry

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Source corrections with NS-branes, Poisson equations.
Reproduced from $\mathcal{D}^2 = 0$, Dirac op. $Spin(d,d) \times \mathbb{R}^+$ cov. der.
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Generalized Geometry/Double Field Theory: formalisms that reproduce standard and β-supergravity; structures: natural.
$Spin(d,d) \times \mathbb{R}^+$ cov. derivative, Lagrangian, e.o.m., SUSY.

Extension beyond the NSNS sector: RR non-geometric fluxes? exotic D-branes? Exceptional geometry/field theory could help

Additional Bianchi identities $\Rightarrow$ R-brane?

$\hookrightarrow$ Get new pheno. interesting backgrounds, de Sitter vacua.

Generalized Geometry formalism

$O(d,d) \times \mathbb{R}^+$ structure and examples

An $O(d,d) \times \mathbb{R}^+$ covariant formalism that reformulates standard type IIA/B supergravity; we follow and build on

arXiv:1107.1733 by A. Coimbra, C. Strickland-Constable, D. Waldram

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Some $O(d,d) \times \mathbb{R}^+$ covariant objects

David
ANDRIOT

Introduction

β-supergravity

BI, NS-branes, ...

Conclusion

Some $O(d,d) \times \mathbb{R}^+$ covariant objects : generalized derivative

$$D_A V^B = \partial_A V^B + \hat{\Omega}_A{}^B{}_C V^C \,, \quad \hat{\Omega}_A{}^B{}_C = \Omega_A{}^B{}_C - \Lambda_A \delta^B_C \,, \; \Lambda_a \equiv \lambda_a \,, \; \Lambda^a \equiv \xi^a$$

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Determine $\hat{\Omega}$: consider gen. torsion $T^A{}_{BC}$ (L_V)

$$T_{ABC} = -3\hat{\Omega}_{[ABC]} + \hat{\Omega}_D{}^D{}_B \eta_{AC} - e^{4d}\langle e^{-2d}\ \mathring{\mathcal{E}}_A, L_{\mathring{\mathcal{E}}_B}(e^{-2d}\ \mathring{\mathcal{E}}_C)\rangle$$

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Natural appearance of $\beta^{ab}\partial_b$, ω_Q, $\check{\nabla}$, $2(\beta^{ac}\partial_c\tilde{\phi} - \mathcal{T}^a) = \xi^a$

$Spin(d,d) \times \mathbb{R}^+$ cov. derivative and Dirac operator

see also

arXiv:1107.0008 by O. Hohm, S. K. Kwak and B. Zwiebach

arXiv:1304.1472 by D. Geissbühler, D. Marqués, C. Núñez, V. Penas

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$O(d,d) \times \mathbb{R}^+$ spinor Ψ

$$D_A \Psi = \left(\partial_A + \frac{1}{4} \Omega_A{}^D{}_C \Gamma^{BC} \eta_{DB} - \frac{1}{2} \Lambda_A \right) \Psi$$

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We compute:

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Can be understood from RR F, C, and SUSY.

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Can be understood from RR F, C, and SUSY.

4D derivative: $\mathcal{D}_\sharp = \frac{1}{2}\mathcal{D}_2$

$O(d-1,1) \times O(1,d-1)$ structure, $\tilde{\mathcal{L}}_\beta$ and e.o.m.

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Derivative $D_A W^B$ for $\tilde{\mathcal{E}}(\beta)$

$$\begin{cases} D_a w^b = \nabla_a w^b - \eta_{ad}\check{\nabla}^d w^b + \frac{1}{6}\eta_{ad}\eta_{cf}R^{dbf}w^c - \frac{1}{9}(\delta^b_a\Lambda_c - \eta_{ac}\eta^{be}\Lambda_e)w^c \\ D_a w^{\overline{b}} = \nabla_a w^{\overline{b}} - \eta_{ad}\check{\nabla}^d w^{\overline{b}} - \frac{1}{2}\eta_{ad}\overline{\eta_{cf}}R^{d\overline{bf}}w^{\overline{c}} \\ D_{\overline{a}} w^b = \nabla_{\overline{a}} w^b + \overline{\eta_{ad}}\check{\nabla}^{\overline{d}} w^b - \frac{1}{2}\overline{\eta_{ad}}\eta_{cf}R^{\overline{d}bf}w^c \\ D_{\overline{a}} w^{\overline{b}} = \nabla_{\overline{a}} w^{\overline{b}} + \overline{\eta_{ad}}\check{\nabla}^{\overline{d}} w^{\overline{b}} + \frac{1}{6}\overline{\eta_{ad}\eta_{cf}}R^{\overline{dbf}}w^{\overline{c}} - \frac{1}{9}(\delta^{\overline{b}}_{\overline{a}}\Lambda_{\overline{c}} - \overline{\eta_{ac}}\overline{\eta^{be}}\Lambda_{\overline{e}})w^{\overline{c}} \end{cases}$$

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Another interest: supersymmetry, Killing spinor equations...