SU(3) structures and heterotic domain wall solutions

Magdalena Larfors

Uppsala University

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X. de la Ossa, ML, E. Svanes (work in progress),
J. Gray, ML, D. Lüst (1205.6208)

Motivation and summary

Why string compactifications?

Physics: string models with good phenomenology (particle physics and cosmology)

- Calabi–Yau manifolds: good 4D physics models, but with unstable moduli.
- Background fluxes can stabilise moduli.
- Fluxes deform geometry $\implies$ SU(3) structure instead of SU(3) holonomy.

Math: probe (non-complex, non-Kähler) compact geometry.

This talk: Heterotic compactifications on SU(3) structure manifolds

- Properties of heterotic 4D $\mathcal{N} = 1/2$ domain wall solutions.
- Flow of SU(3) structures and moduli spaces.

Heterotic string compactifications

Heterotic string theory

Low energy: heterotic supergravity (10D)

- Bosonic fields:
  Metric G, B-field B, dilaton ϕ, gauge field A
- Fermionic fields:
  Gravitino Ψ_M, dilatino λ, gaugino χ
- Bosonic action:

$$S = \frac{1}{2\alpha'} \int \mathrm{d}^{10} x \, e^{-2\phi} \sqrt{|G|} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + \alpha'(\dots) \right)$$

  where $H = \mathrm{d}B$.
- Work at lowest order in α': no gauge fields, Bianchi identity $dH = 0$

Heterotic string compactifications

Supersymmetric vacua

No fermionic fields; vanishing SUSY variations

$$\delta\psi_M = \left(\nabla_M + \frac{1}{8}H_M\right)\epsilon \ , \ \delta\lambda = \left(\not{\nabla}\hat{\phi} + \frac{1}{12}\not{H}\right)\epsilon$$

where $\not{\nabla} = \Gamma^M \nabla_M$, $\not{H} = \Gamma^{MNP} H_{MNP}$, $H_M = \Gamma^{NP} H_{MNP}$

Compactifications

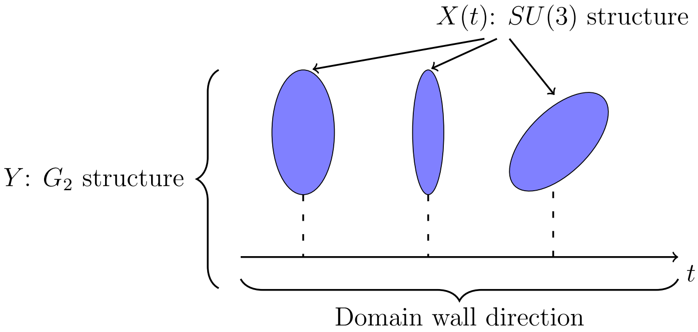
- $\mathcal{M}_{10} = \mathcal{M}_E \times X$
- SUSY $\iff$ nowhere vanishing spinor on X: $\epsilon = \rho_E \otimes \eta$
  $\iff$ X has reduced structure group

Hitchin:02, Gualtieri:04, Grana et al:05, ...

Heretotic $\mathcal{N} = \frac{1}{2}$ domain wall solutions

Lukas *et al*:10, 11, 12; Gray, Larfors, Lüst:12

4D domain wall vacuum: $\mathcal{M}_{10} = \mathcal{M}_4 \times_W X(t) \equiv \mathcal{M}_3 \times Y$
$\mathcal{M}_4 = \mathcal{M}_3 \times \mathbb{R}$, $\mathcal{M}_3$ max. symmetric



7D flux $\widehat{H}$ allowed by symmetry: $\widehat{H}_{\alpha\beta\gamma} = f\,\epsilon_{\alpha\beta\gamma}$, $\widehat{H}_{tmn}$, $\widehat{H}_{mnp}$

G_2 perspective

G_2 structure: Fernandez–Gray:82, Chiossi–Salamon:02

SUSY $\iff$ G_2 structure determined by 3-form φ ($\Phi = *_7\varphi$)

$$\mathrm{d}_7\varphi = \tau_0\,\psi + 3\,\tau_1 \wedge \varphi + *_7\tau_3\ ,$$
$$\mathrm{d}_7\psi = 4\,\tau_1 \wedge \psi + *_7\tau_2\ .$$

with torsion

$$\tau_0 = -\tfrac{15}{14}\,f\ \ ,\quad \tau_1 = \tfrac{1}{2}\,d_7\phi\ ,$$
$$\tau_2 = 0\quad ,\quad \tau_3 = -\widehat{H} + \tfrac{1}{14}\,f\,\varphi - \tfrac{1}{2}\,d_7\phi \lrcorner \Phi$$

This is an integrable G_2 structure.

- Bianchi identity $\mathrm{d}_7\widehat{H} = 0$ constrains the G_2 structure further.
- To zeroth order in α', can show
  SUSY + BI $\Longrightarrow$ Einstein equation + dilaton EOM + flux EOM

SU(3) perspective

$SU(3)$ structure: Gray–Hervalla:80, Chiossi–Salamon:02

SU(3) structure determined by $(3,0)$-form Ψ and real $(1,1)$ form ω such that

$$\omega \wedge \Psi = 0 \ , \qquad \omega \wedge \omega \wedge \omega \sim \Psi \wedge \bar{\Psi}$$

with torsion

$$\mathrm{d}\omega = -\frac{12}{||\Psi||^2}\ \mathrm{Im}(W_0\,\overline{\Psi}) + W_1^{\omega} \wedge \omega + W_3 \ ,$$

$$\mathrm{d}\Psi = W_0\ \omega \wedge \omega + W_2 \wedge \omega + \overline{W}_1^{\Psi} \wedge \Psi \ .$$

Embed in G_2 using 1-form $N = N_t\,\mathrm{d}t$:

$$\varphi = N \wedge \omega + \mathrm{Re}\Psi \ .$$

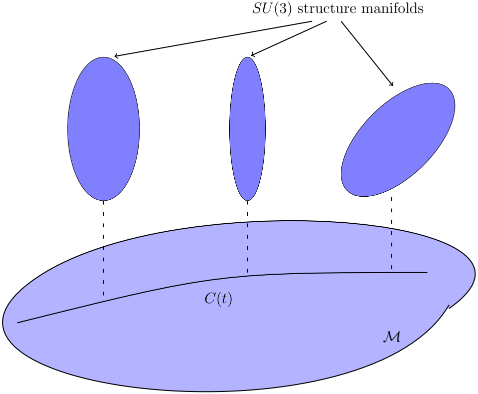
SUSY and BI

SU(3) torsion: X non-complex and conformally balanced .

SU(3) flow: $\partial_t \omega$ fixed in terms of N_t, ϕ and SU(3) torsion

$\partial_t \Psi$ fixed up to primitive (2,1)+(1,2)-form γ.

Flow of SU(3) structures



t parametrizes a curve in the moduli space of SU(3) structures

Two options:

- Fix torsion classes of SU(3) structure.
- Flow between different types of SU(3) structure.

Example: Calabi–Yau flow

Flow that preserves CY

Assume that $W_i = 0$ for all $t \iff X$ is CY for all t.

Analysis of SUSY and BI gives

$$
\begin{aligned}
\mathrm{d}\partial_t\omega = 0 \quad &\iff \quad \mathrm{d}\mathrm{d}^\dagger(N_t\,\Psi) = 0 \quad \iff \quad \mathrm{d}N_t = 0\ , \\
\mathrm{d}\partial_t\Psi = 0 \quad &\iff \quad \mathrm{d}\gamma = 0\ , \\
\mathrm{d}_7\widehat{H} = 0 \quad &\iff \quad \mathrm{d}^\dagger\gamma = 0
\end{aligned}
$$

Conclusion:
Flow preserves CY $\iff$ N_t is constant and the primitive form γ is harmonic

Example: Calabi–Yau flow

Flow away from CY

Assume X has $W_i = 0$ for $t = 0$.

What does X flow to if N_t is non-constant and γ is not harmonic?

- Taylor expand all forms in the equations $\beta(t) = \beta_0 + \delta_1\beta\, t + \mathcal{O}(t^2)$
- Solve for W_i order by order

First order result:

$$\begin{aligned}
\delta_1 W_0 &= -\tfrac{i}{3}\, \mathrm{d}^{\dagger_0}\mathrm{d}\, N_0 \ , \\
\delta_1 W_1^{\omega} &= 0 \ , \\
\delta_1 \overline{W}_1^{\Psi} &= -N_0^{-1}\,(\partial_m N_0)\,\Delta_0^m + \tfrac{1}{2}\,\left(\lambda_0\, N_0^{-1} + \tfrac{7}{2}\, i\, \tau_0\right)\,\bar{\partial} N_0 \ , \\
\delta_1 W_2 &= -2\,\omega_0 \lrcorner \bar{\partial}(N_0\,\gamma_0)^{(2,1)} + i\,\left(\tfrac{1}{3}\,(\mathrm{d}^{\dagger_0}\mathrm{d}\, N_0)\,\omega_0 - \mathrm{d}(J(\mathrm{d}N_0))\right) \ , \\
\delta_1 W_3 &= \tfrac{1}{2}\,\left(\bar{\partial}\partial^{\dagger_0}(N_0\,\Psi) + \partial\bar{\partial}^{\dagger_0}(N_0\,\overline{\Psi})\right) \ .
\end{aligned}$$

where $\Delta_0^m = \frac{1}{8}\,\overline{\Psi}_0^{mpq}\,\left(2\,(\partial_p N_0)\,\omega_{0\,qn} - N_0\,\gamma_{0\,pqn}^{(2,1)}\right)\mathrm{d}x^n$

Example: Calabi–Yau flow

Flow away from CY: 1st order results

Remarks:

- No flow from a CY to a complex non-CY manifold
- Integrability of non-CY flow: under study

Conclusions

Conclusions

General 4D heterotic $\mathcal{N} = 1/2$ domain wall solutions

- Y Non-compact with integrable G_2 structure
- $X(t)$ Conformally balanced (non-complex) SU(3) structure
- Flow equations generalize Hitchin flow
- Flow can change the SU(3) structure

Work in progress and outlook

- Integrability of flow
- Study moduli space of SU(3) structure manifolds
- Higher order in α': gauge sector, BI
- Non-perturbative "uplift" to 4D AdS.

Lukas *et al*:11, 12, 13 (CY and Nearly-Kähler cosets)

Thank You

Example: Hitchin flow

Assume G_2 holonomy: $\tau_a = 0$, $a = 0, .., 4$.

$\Longrightarrow$ No flux and constant dilaton

$\Longrightarrow$ Half-flat SU(3) structure

$$\begin{aligned} \mathrm{d}(\omega \wedge \omega) &= 0 \ , \\ \mathrm{dRe}(\Psi) &= 0 \ , \\ \mathrm{dIm}(\Psi) &= \mathrm{Im}(W_0)\,\omega \wedge \omega + \mathrm{Im}(W_2) \wedge \omega \ . \end{aligned}$$

Hitchin flow:

$$\begin{aligned} \partial_t(\omega \wedge \omega) &= 2\mathrm{dIm}(\Psi) \\ \partial_t \mathrm{Re}(\Psi) &= \mathrm{d}\omega \ . \end{aligned}$$

The presence of flux/G_2 torsion allows to find generalisations of Hitchin flow.