Mordell-Weil Torsion and the Global Structure of Gauge Groups in F-theory

Oskar Till,

Institut für Theoretische Physik,

Ruprecht-Karls Universität Heidelberg

in collaboration with C. Mayrhofer,
D R. Morrison and T. Weigand.
arXiv: 1405.3656

Outline

- Introduction to the Mordell-Weil group
- Torsion sections
- The Shioda map of a torsion section
- Non-abelian matter representations
- Implications for gauge theories
- Outlook

Introduction to the Mordell-Weil group

Points on an elliptic curve $E = \mathbb{C}/\Lambda$, for $\Lambda = \langle 1, \tau \rangle$ are additive as complex numbers.

Points with rational coordinates on E, over the field K form an abelian group under this addition.

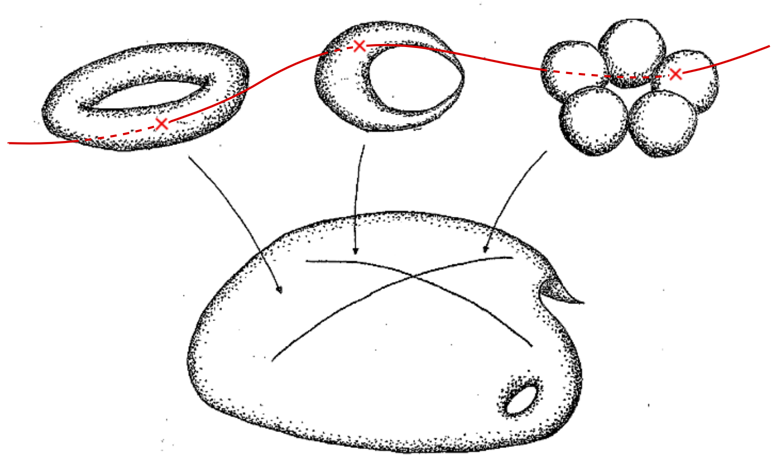
The Mordell-Weil theorem for elliptic curves states that this group, $E(K)$ is finitely generated, thus

$$E(K) = \mathbb{Z}^r \oplus \mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_i} \tag{1}$$

where the finite part is the torsion subgroup.

Mordell-Weil theorem for elliptic fibrations

For an elliptic fibration $Y \to \mathcal{B}$ the Mordell-Weil group is a group of sections. The group law is obtained by fiberwise addition of points over each point in $\mathcal{B}$.

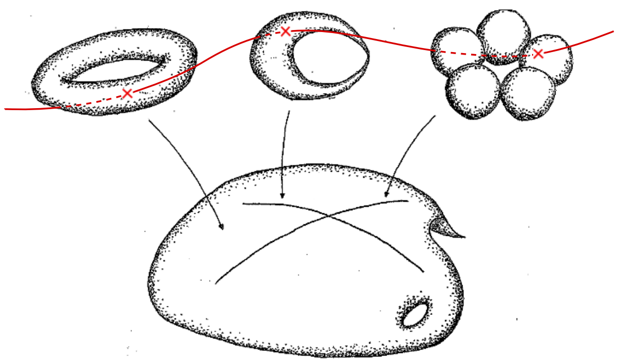


Mordell-Weil theorem for elliptic fibrations

Example: The possible torsion subgroups for an elliptic K3 surface are [Shimada '00]

$\mathbb{Z}_k\,(2 \le k \le 8),\quad \mathbb{Z}_2\oplus\mathbb{Z}_{2k}\,(1 \le k \le 3),\quad \mathbb{Z}_3\oplus\mathbb{Z}_3,\quad \mathbb{Z}_4\oplus\mathbb{Z}_4$

No classification exists for higher dimensional Calabi-Yau varieties.



Torsion sections

Torsion sections are $\mathbb{Z}_k$-valued in $E(K)$.

Among the 16 reflexive polygons, three admit torsion sections as the restriction of ambient toric divisors to the hypersurface. This way the Mordell-Weil groups $\mathbb{Z}_2$, $\mathbb{Z} \oplus \mathbb{Z}_2$ and $\mathbb{Z}_3$ are realized.

These sections are not torsion in homology, due to the singularities of the hypersurface.

However, modulo linear combinations of resolution divisors they are torsion

The Shioda map

The Shioda map is a map from the group of sections $E(K)$ to the group of divisors $NS(Y)$.

By construction, it gives a generator which does not lie in the Cartan of any non-abelian gauge group, and has one leg in the fiber.

Taking the Shioda map of a $\mathbb{Z}_k$-torsion section $\mathcal{T}$ gives a trivial divisor class on the fourfold Y

$$\mathcal{T} \mapsto T - Z - \bar{\mathcal{K}} + \frac{1}{k} \sum a_i F_i \quad a_i \in \mathbb{Z} \,. \tag{2}$$

The Shioda map II

Triviality on the fourfold means that this class is not a $U(1)$-generator. Using this we write

$$\Xi_k \equiv T - Z - \bar{\mathcal{K}} = -\frac{1}{k}\sum a_i F_i \quad a_i \in \mathbb{Z}\,. \tag{3}$$

The class Ξ_k has zero intersection with all resolution divisors $F_i \leftrightarrow$ the non-abelian gauge algebra is unchanged.

That is, the root Q and coroot $Q^\vee$ lattices are unaffected by the presence of a torsion section.

Non-abelian matter representations

At codimension two loci, where the fiber further degenerates, charged matter representations are found.

The charges of these states $\leftrightarrow$ elements in the weight lattice Λ.

The intersection pairing of $\Xi_k = -\frac{1}{k}\sum a_i F_i\,, \quad a_i \in \mathbb{Z}$ with the split curves over matter loci is integer.

That is, the divisor class Ξ_k is to be identified with an extra coweight.

Implications for gauge theories

Restricted matter spectrum. The only allowed representations are the ones integer charged under Ξ_k.

The root and coroot lattices Q and $Q^\vee$ are sublattices of the weight and coweight lattices Λ and $\Lambda^\vee$, respectively.

The center Z_G and the fundamental group of the gauge group G are given by

$$Z_G = \frac{\Lambda}{Q} \qquad \pi_1(G) = \frac{\Lambda^\vee}{Q^\vee} \tag{4}$$

Implications for gauge theories II

The presence of a torsion section refines the coweight lattice, compared to the case without torsion.

This enhances the fundamental group of G, or equivalently reduces the center of the gauge group.

Example: An A_2 fibration without torsion gives rise to the gauge group $SU(3)$.

If there is an $\mathbb{Z}_3$-section, the gauge group is instead $SU(3)/\mathbb{Z}_3$.

This constrains the matter spectrum to the representations invariant under the action of the center $\mathbb{Z}_3$.

Outlook

We have considered fibrations based on the reflexive polygons admitting torsion sections.

How to treat other torsion subgroups? Complete intersections? Non-toric methods?

Phenomenology of torsion sections. Realizing the MSSM with gauge group $SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$.