F-Theory duals of heterotic K3 orbifolds

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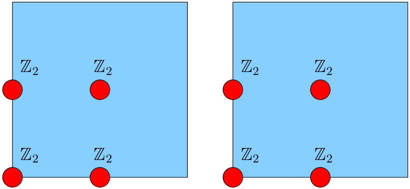
Based on Ludeling, Ruehle: [1405.2928]

$T^4/\mathbb{Z}_2$ Orbifold

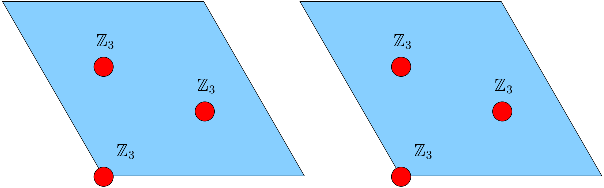


Details

- $\theta : (z_1, z_2) \mapsto (e^{2\pi i/2} z_1, e^{-2\pi i/2} z_2) = (-z_1, -z_2)$
- Singularities: $4 \cdot 4 = 16 \times \mathbb{Z}_2$
- Gauge group: $E_7 \times SU(2) \times (E_8)$
- Spectrum: $[(\mathbf{56}, \mathbf{2}) + 4(\mathbf{1}, \mathbf{1})]_U + [8(\mathbf{56}, \mathbf{1}) + 32(\mathbf{1}, \mathbf{2})]_T$
- Note:
  - $(\mathbf{56}, \mathbf{1})$, $(\mathbf{1}, \mathbf{2})$ pseudo-real $\Rightarrow$ half-hypers
  - Complex structures $\tau_{1,2}$, radii $b_{1,2}$ of tori unfixed

[cf. talk by Vaudrevange] (spectrums from [Honecker, Trapletti])

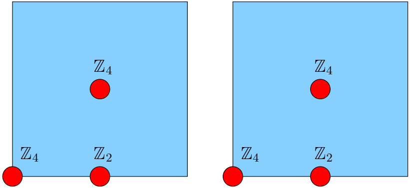
$T^4/\mathbb{Z}_3$ Orbifold



Details

- $\theta : (z_1, z_2) \mapsto (e^{2\pi i/3} z_1, e^{-2\pi i/3} z_2)$
- Singularities: $3 \cdot 3 = 9 \times \mathbb{Z}_3$
- Gauge group: $E_7 \times U(1) \times (E_8)$
- Spectrum: $[(\mathbf{56})_1 + (\mathbf{1})_2 + 2(\mathbf{1})_0]_U + [9(\mathbf{56}) + 63(\mathbf{1})]_T$
- Note:
  - States differ by U(1) charges, all full hypers
  - CS fixed by rotation to $\tau_{1,2} = e^{2\pi i/3}$, radii $b_{1,2}$ unfixed

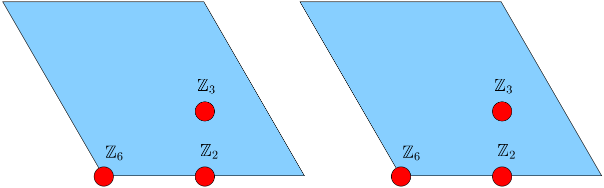
$T^4/\mathbb{Z}_4$ Orbifold



Details

- $\theta : (z_1, z_2) \mapsto (e^{2\pi i/4} z_1, e^{-2\pi i/4} z_2) = (iz_1, -iz_2)$
- Singularities: $2 \times \mathbb{Z}_4$, $1 \times \mathbb{Z}_2$ per T^2
- Gauge group: $E_7 \times U(1) \times (E_8)$
- Spectrum: $[(\mathbf{56})_1 + 2(\mathbf{1})_0]_U + [9(\mathbf{56}) + 64(\mathbf{1})]_T$
- Note:
  - States differ by U(1) charges, 5 $(\mathbf{56})_0$ are 10 half-hypers
  - CS fixed by rotation to $\tau_{1,2} = i$, radii $b_{1,2}$ unfixed

$T^4/\mathbb{Z}_6$ Orbifold



Details

- $\theta : (z_1, z_2) \mapsto (e^{2\pi i/6} z_1, e^{-2\pi i/6} z_2)$
- Singularities: $1 \times \mathbb{Z}_6$, $1 \times \mathbb{Z}_3$, $1 \times \mathbb{Z}_2$ per T^2
- Gauge group: $E_7 \times U(1) \times (E_8)$
- Spectrum: $[(\mathbf{56})_1 + 2(\mathbf{1})_0]_U + [9(\mathbf{56}) + 64(\mathbf{1})]_T$
- Note:
  - States differ by U(1) charges, 3 $(\mathbf{56})_0$ are 6 half-hypers
  - CS fixed by rotation to $\tau_{1,2} = e^{2\pi i/3}$, radii $b_{1,2}$ unfixed

Introduction to F-theory

Constraints for heterotic duality

- F-theory: Introduce extra torus whose CS encodes varying Type II axio-dilaton ⇒ CY 3-fold [Vafa]

Introduction to F-theory

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Introduction to F-theory

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- For duality need special fibration structure [Morrison, Vafa]
  - elliptic fibration over base B_2
  - K3 fibration over $\mathbb{P}^1$

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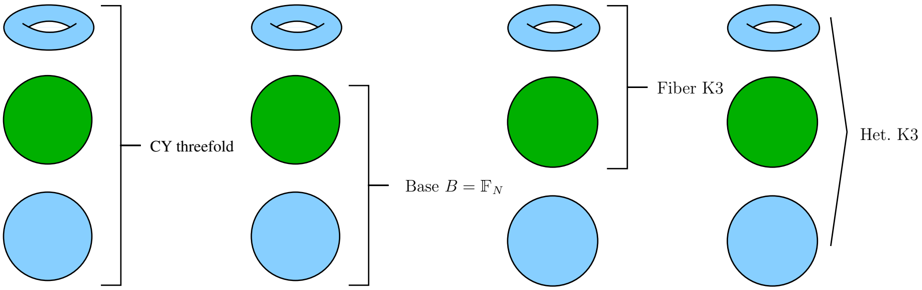

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- Take base Hirzebruch surface $\mathbb{F}_N$ ($\mathbb{P}^1$ fibration over $\mathbb{P}^1$)
- Gauge instantons embedded as $(12+N, 12-N)$ into $\mathrm{E}_8 \times \mathrm{E}_8$

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- Gauge instantons embedded as $(12+N, 12-N)$ into $\mathrm{E}_8 \times \mathrm{E}_8$
- Second E_8 unbroken in standard embedding $\Rightarrow N = 12$

Introduction to F-theory

Weierstrass description of elliptic fibration

- Equation: $y^2 = x^3 + fxz^4 + gz^6$ (f, g sections of base $\mathbb{F}_{12}$)
- Discriminant: $\Delta = 4f^3 + 27g^2$ [cf. talks Palti, Cvetic, Schafer-Nameki]
- j-function: $j(\tau) = f^3/\Delta \quad j(\mathrm{i}) = 1\,, \qquad j(e^{2\pi \mathrm{i}/3}) = 0$

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Scaling	s	t	u	v	x	y	z	f	g	Δ
λ	1	1	12	0	28	42	0	56	84	168
μ	0	0	1	1	4	6	0	8	12	24
ν	0	0	0	0	2	3	1	0	0	0

Expand f, g in u, v, s, t

$$f = c_{56}v^8 + c_{44}uv^7 + c_{32}u^2v^6 + c_{20}u^3v^5 + c_8u^4v^4$$

$$g = d_{84}v^{12} + d_{72}uv^{11} + d_{60}u^2v^{10} + d_{48}u^3v^9 + d_{36}u^4v^8 + d_{24}u^5v^7 + d_{12}u^6v^6 + d_0u^7v^5$$

$T^4/\mathbb{Z}_2$ Orbifold case – Ansatz

Heterotic side: $E_7 \times SU(2) \times E_8$

- Spectrum: $(\mathbf{56}, \mathbf{2}) + 8(\mathbf{56}, \mathbf{1}) + 32(\mathbf{1}, \mathbf{2}) + 4(\mathbf{1}, \mathbf{1})$

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F-theory side

- Restrict f, g and relate c_i, d_j s.t. III^*, I_2, II^* appear:
- $f = u^3 v^4(\ldots)\,,\ \ g = u^5 v^5(\ldots)\,,\ \ \Delta = u^9 v^{10}(u + p_{12} v)^2 \Delta_{\text{red}}$

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Problems

- Fractional instantons: $24/16 = 3/2$ per $\mathbb{Z}_2$ fixed point

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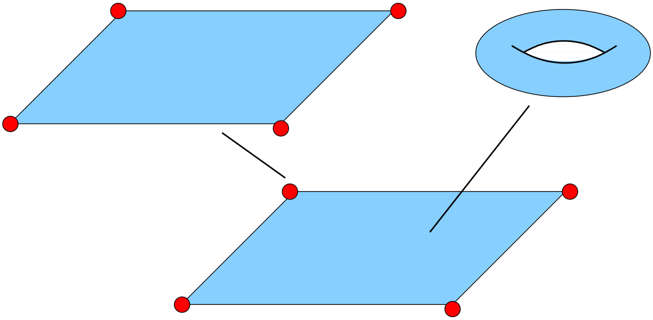
- $\{u = 0\} \cap \{u + p_{12} v = 0\} \Rightarrow$ expect $12 \times (\mathbf{56},\mathbf{2})$, not 1
- $\{\Delta_{\text{red}} = 0\} \cap \{u + p_{12} v = 0\} \Rightarrow$ expect multiples of 12 for hypers $(\mathbf{1},\mathbf{2})$, not 32

Duals for $T^4/\mathbb{Z}_2$ Orbifold

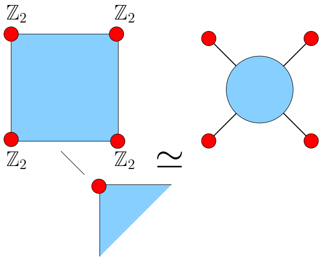


Alternative way of looking at the orbifold geometry

- Smooth fiber torus in the bulk of the base (away from singularities)
- Four fiber singularities over each base singularity

[Braun,Ebert,Hebecker,Valandro; Buchmüller,Louis,Schmidt,Valandro]

Duals for $T^4/\mathbb{Z}_2$ Orbifold

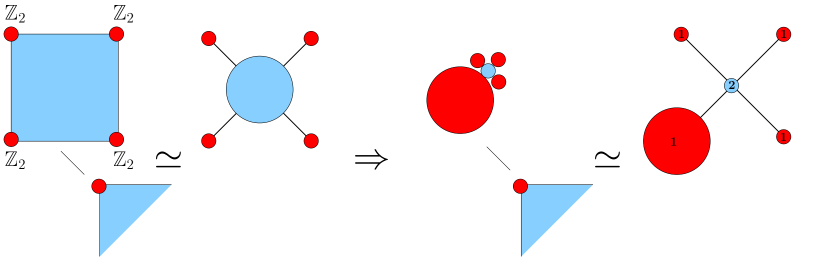


Idea

Find Weierstrass description of heterotic model w/ base $\mathbb{P}^1$ (s, t) and fiber T^2 (x, y, z)

1. Pick one section (i.e. one pillow corner)

Duals for $T^4/\mathbb{Z}_2$ Orbifold



Idea

Find Weierstrass description of heterotic model w/ base $\mathbb{P}^1$ (s,t) and fiber T^2 (x,y,z)

1. Pick one section (i.e. one pillow corner)
2. Blow up fiber singularity it hits (replace $\mathbb{Z}_2$ singularity w/ $\mathbb{P}^1$)
3. Blow down other finite fiber component (original fiber pillow)

Same method also used in [Braun,Ebert,Hebecker,Valandro]

Duals for $T^4/\mathbb{Z}_2$ Orbifold

Results for heterotic Weierstrass

- Get Weierstrass model with four D_4 singularities

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- Heterotic Weierstrass: $y^2 = x^3 + f_8^{\text{H}} x z^4 + g_{12}^{\text{H}} z^6$

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- Need vanishing $(f_8^{\mathrm{H}}, g_{12}^{\mathrm{H}}, \Delta_{24}^{\mathrm{H}}) = (2,3,6)$ @ 4 points
  $f_8^{\mathrm{H}} = \alpha p_4^2\,, \qquad g_{12}^{\mathrm{H}} = \beta p_4^3\,, \qquad \Delta_{24}^{\mathrm{H}} = (\alpha^3 + \beta^2) p_4^6$

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- $j(\tau) = (f_8^{\mathrm{H}})^3/\Delta_{24}^{\mathrm{H}} = \alpha^3/(\alpha^3 + \beta^2) \quad \Rightarrow \quad$ CS τ not fixed

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Results for F-theory Weierstrass

- $c_8 = \alpha p_4^2\,, \qquad d_{12} = \beta p_4^3$
- $d_{24} \leftrightarrow$ Instanton position $\Rightarrow d_{24} = \gamma p_4^6$

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- Extra I_2 locus $\Rightarrow c_{20} = \kappa p_4^5$
- Numerical coefficients $\alpha, \beta, \gamma, \kappa$ related amongst each other to lead to factorization

Duals for $T^4/\mathbb{Z}_2$ Orbifold

Spectrum

- E_8 at $v = 0$, E_7 at $u = 0$, I_2 at $(u + 6p_4^3 v) = 0$, I_1 at $(\dots) = 0$

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Spectrum

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- Quantization in multiples of 12 broken to multiples of 4

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- Nothing intersects $v = 0$, everything else intersects @ $u = p_4 = 0$
- Quantization in multiples of 12 broken to multiples of 4
- Get $(\mathbf{56})$'s by deforming I_2 away and matching w/ SE
  - 4 half-hypers at $u = p_4 = 0 \leftrightarrow (\mathbf{56}, \mathbf{2})$
  - 16 half-hypers at $u + 6p_4^3 v = (\ldots) = 0 \leftrightarrow (\mathbf{56}, \mathbf{1})$

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  - 4 half-hypers at $u=p_4=0 \leftrightarrow (\mathbf{56},\mathbf{2})$
  - 16 half-hypers at $u+6p_4^3 v=(\ldots)=0 \leftrightarrow (\mathbf{56},\mathbf{1})$
- Get $(\mathbf{1})$'s from parameters in the Weierstrass equation
  - In total 9: 5 from p_4 and $\alpha,\beta,\gamma,\kappa$
  - 5 are related $\Rightarrow$ four singlets

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- Nothing intersects $v=0$, everything else intersects @ $u=p_4=0$
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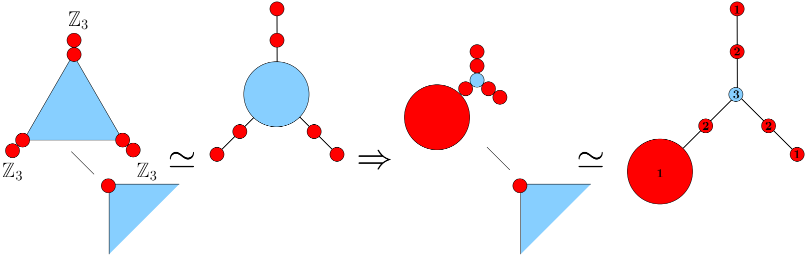
  - 16 half-hypers at $u+6p_4^3 v=(\ldots)=0 \leftrightarrow (\mathbf{56},\mathbf{1})$
- Get $(\mathbf{1})$'s from parameters in the Weierstrass equation
  - In total 9: 5 from p_4 and $\alpha,\beta,\gamma,\kappa$
  - 5 are related $\Rightarrow$ four singlets
- Get $(\mathbf{2})$'s from parameters that destroy I_2 locus $\rightarrow$ smooth SE
  - 69 overall in $c_{20}, c_8, d_{24}, d_{12}, d_0$
  - Subtract 1 scaling, 4 singlets preserving $I_2 \Rightarrow$ 64 half-hyper $(\mathbf{2})$

Duals for $T^4/\mathbb{Z}_3$ Orbifold



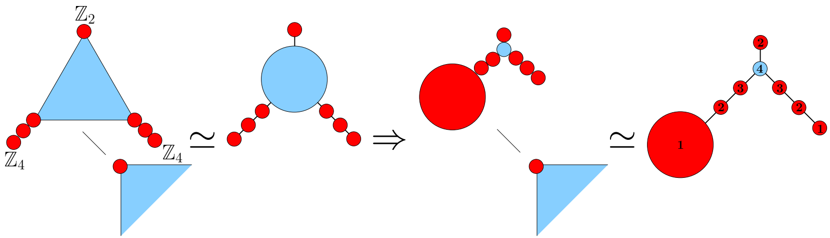
Results for heterotic Weierstrass

- Get Weierstrass model with three E_6 singularities
- Heterotic Weierstrass: $y^2 = x^3 + f_8^H x z^4 + g_{12}^H z^6$
- Need vanishing $(f_8^H, g_{12}^H, \Delta_{24}^H) = (\geq 3, 4, 8)$

$$f_8^H \equiv 0\,, \qquad g_{12}^H = \beta p_3^4\,, \qquad \Delta_{24}^H = (\beta^2) p_3^8$$

- $j(\tau) = (f_8^H)^3/\Delta_{24}^H = 0 \quad \Rightarrow \quad$ CS fixed to $\tau = e^{2\pi i/3}$

Duals for $T^4/\mathbb{Z}_4$ Orbifold



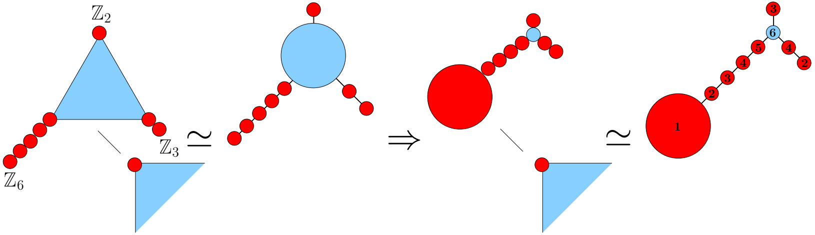
Results for heterotic Weierstrass

- Get Weierstrass model with two E_7, one D_4 singularities
- Heterotic Weierstrass: $y^2 = x^3 + f_8^{\mathrm{H}} x z^4 + g_{12}^{\mathrm{H}} z^6$
- At E_7 need vanishing $(f_8^{\mathrm{H}}, g_{12}^{\mathrm{H}}, \Delta_{24}^{\mathrm{H}}) = (3, \geq 5, 9)$

  At D_4 need vanishing $(f_8^{\mathrm{H}}, g_{12}^{\mathrm{H}}, \Delta_{24}^{\mathrm{H}}) = (2, \geq 3, 6)$

  Combined vanishing $(8, \geq 13, 24) \Rightarrow f_8^{\mathrm{H}} = \alpha p_1^3 q_1^3 r_1^2,\ g_{12}^{\mathrm{H}} \equiv 0$
- $j(\tau) = (f_8^{\mathrm{H}})^3/\Delta_{24}^{\mathrm{H}} = (f_8^{\mathrm{H}})^3/(f_8^{\mathrm{H}})^3 = 1 \ \Rightarrow$ CS fixed to $\tau = \mathrm{i}$

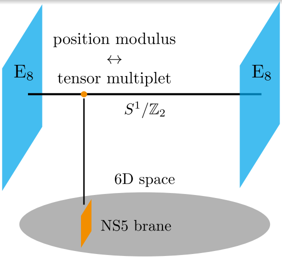
Duals for $T^6/\mathbb{Z}_6$ Orbifold



Results for heterotic Weierstrass

- Get Weierstrass model with one E_8, E_6, D_4 singularities
- Heterotic Weierstrass: $y^2 = x^3 + f_8^H x z^4 + g_{12}^H z^6$
- At E_8 $(f_8^H, g_{12}^H, \Delta_{24}^H) = (\geq 4, 5, 10)$
  At E_6 $(f_8^H, g_{12}^H, \Delta_{24}^H) = (3, 4, 8)$
  At D_4 $(f_8^H, g_{12}^H, \Delta_{24}^H) = (\geq 2, 3, 6)$
  Combined vanishing $(\geq 9, 12, 24) \Rightarrow f_8^H \equiv 0,\ g_{12}^H = \beta p_1^5 q_1^4 r_1^3$
- $j(\tau) = (f_8^H)^3/\Delta_{24}^H = 0 \quad \Rightarrow \quad$ CS fixed to $\tau = e^{2\pi i/3}$

General gauge sector with different instanton embedding



Construction

- Embedding instantons as $(12+N, 12-N) \leftrightarrow \mathbb{F}_N$
- Transition from $\mathbb{F}_N$ to $\mathbb{F}_{N\pm1}$ $\leftrightarrow$ blowup/blowdown in base
- Base Blowup introduces extra tensor multiplet whose scalar component encodes NS5 brane position in $S^1/\mathbb{Z}_2$ in the Hořava–Witten theory [Hořava, Witten]
- NS5 brane leaves one E_8 brane, travels through bulk, recombines with other E_8 brane [Seiberg, Witten; Morrison, Vafa]

Conclusion

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Approaches to duality

- Problem with usual approach in singular case:
  Fractional instantons, unbroken GG, everything @ FPs

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  - Fixes complex structure of tori as needed by orbifold
  - $\mathbb{Z}_N$ orbifolds have central node w/ multiplicity N

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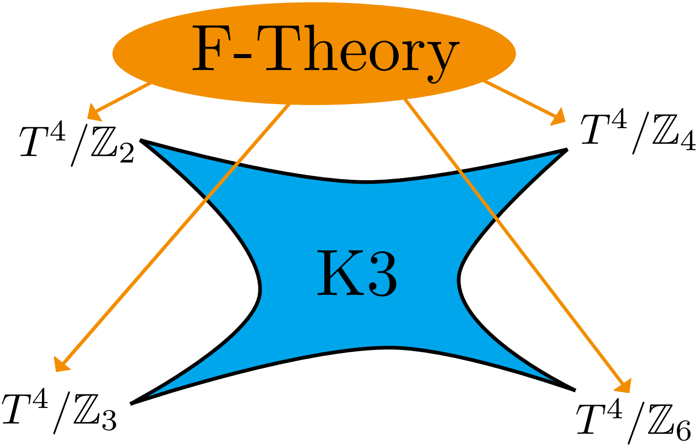
Outlook

- Apply to 4D models
- Use description w/o section
- Compare to M-Theory w/ frozen singularities



Thank you for your attention!