F-Theorey on Spin(7) Holonomy Manifolds

based on
1307.5858 with Bonetti, Grimm
1309.2287 with Bonetti, Grimm, Palti

Tom Pugh
OUTLINE

- Introduction
- Spin(7) Manifolds from Calabi-Yau Quotients
- F-theory on Spin(7) Manifolds
- Weak Coupling
- Conclusions
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**INTRODUCTION**

- The compactification of F-theory on Calabi-Yau fourfolds has been studied in great detail.
- However in 8 dimensions the largest special holonomy group is Spin(7) and not SU(4).
- We may therefore ask what features would be seen by compactifying on a Spin(7) holonomy manifold.
- To approach this problem we consider the compactification of M-theory on a Spin(7) holonomy manifold and then infer the F-theory dual.
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The decomposition of the chiral spinor of $Spin(8)$ under $SU(4)$ gives

$$8 \rightarrow 1 \oplus 1 \oplus 6$$

From the two covariantly constant nowhere vanishing spinors we can build

$$J_{m\bar{n}} = i\eta_1 \gamma_{m\bar{n}} \eta_2 \quad \Omega_{mnrs} = (\eta_1 + i\eta_2)\gamma_{mnrs}(\eta_1 + i\eta_2)$$

Alternatively, decomposing under $Spin(7)$ gives

$$8 \rightarrow 1 \oplus 7$$

From which we can build

$$\Phi_{mnrs} = \eta \gamma_{mnrs} \eta$$
CALABI-YAU QUOTIENTS

- We will consider Spin(7) holonomy manifolds constructed using the method described by Joyce.
- We quotient the Calabi-Yau by an anti-holomorphic isometric involution

\[ \sigma^2 = 1 \quad \sigma^*(g) = g \]

\[ \sigma^*(\Omega) = e^{2i\theta} \Omega \quad \sigma^*(J) = -J \]

- The Cayley calibration of the Spin(7) manifold produced in this way is then given by

\[ \Phi = \frac{1}{\mathcal{V}^2} \left( \frac{1}{||\Omega||} \text{Re}(e^{-i\theta} \Omega) + \frac{1}{8} J \wedge J \right) \]
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**Spin(7) Quotient of Elliptically Fibered Calabi-Yau Fourfolds**

- For the reduction to have an F-theory dual, the Calabi-Yau fourfold must be elliptically fibered

\[ P \equiv y^2 - x^3 - f(u^i) x z^4 - g(u^i) z^6 = 0 \]

- We then consider anti-holomorphic involutions which are compatible with this structure

\[ \sigma : u^i \rightarrow \sigma_{ah}(u^i) \quad P = 0 \rightarrow P = 0 \]
**Spin(7) Effective Action**

- By considering the action of $\sigma$ we can see how harmonic forms on the Calabi-Yau fit within the Spin(7) cohomology
  
  $$J = \nu^0 \omega_0 + \nu^\alpha \omega_\alpha + \nu^i \omega_i \quad \{\omega_{\alpha_+}\} = H^2_{21}(Z_8, \mathbb{R})$$

  $$\{\omega_0 \wedge J, \omega_{\alpha_-} \wedge J, \omega_i \wedge J\} \subset H^4_{1S}(Z_8, \mathbb{R}) \oplus H^4_{35A}(Z_8, \mathbb{R})$$

- The fields of the M-theory effective action can then be expanded in this basis

- $H^2_{21}(Z_8, \mathbb{R})$ perturbations of $C_3$

- $H^4_{1S}(Z_8, \mathbb{R}) \oplus H^4_{35A}(Z_8, \mathbb{R})$ perturbations of $g_{mn}$

- The effective action is then invariant under N=1 supersymmetry in 3d
**F-theory Lift**

- When lifting to F-theory the extra quotient which forms the Spin(7) manifold gives a quotient of the usual F-theory circle.

- The circle quotient projects out the 4d $N=1$ partners of the 3d $N=1$ fields.
- These are then restored in the 4d effective theory when the F-theory limit is taken.
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To see what happens in the weak coupling limit we can consider the Sen limit in the presence of the quotient.

This agrees with the result of following $\sigma$ through the T-duality.
The new anti-holomorphic quotient acts on the $x^3$ direction.

The effect of the quotient is then only seen in the F-theory picture at the fixed points.

When the F-theory limit is taken and the circle becomes large this quotient is then pushed away restoring 4d N=1 supersymmetry.
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CONCLUSIONS

- The compactification of M-theory on a Spin(7) holonomy manifold can be carried out in a way that remains compatible with M-theory/F-theory duality.

- For the Calabi-Yau quotients analyzed the quotient also acts on the F-theory circle and the localized effects are washed out upon taking the large circle limit.

- Several intrinsically Spin(7) effects remain to be studied.

- Can the SUSY and Lorentz breaking scales be separated?
Thank You