F-Theory on Spin(7) Holonomy Manifolds

based on
1307.5858 with Bonetti, Grimm
1309.2287 with Bonetti, Grimm, Palti

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Outline

- Introduction
- Spin(7) Manifolds from Calabi-Yau Quotients
- F-theory on Spin(7) Manifolds
- Weak Coupling
- Conclusions

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Introduction

- The compactification of F-theory on Calabi-Yau fourfolds has been studied in great detail.
- However in 8 dimensions the largest special holonomy group is Spin(7) and not SU(4).
- We may therefore ask what features would be seen by compactifying on a Spin(7) holonomy manifold.
- To approach this problem we consider the compactification of M-theory on a Spin(7) holonomy manifold and then infer the F-theory dual.

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Calabi-Yau Fourfolds and Spin(7) Holonomy Manifolds

- The decomposition of the chiral spinor of $Spin(8)$ under $SU(4)$ gives

$$\mathbf{8} \rightarrow \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{6}$$

- From the two covariantly constant nowhere vanishing spinors we can build

$$J_{m\bar{n}} = i\eta_1 \gamma_{m\bar{n}} \eta_2 \quad \Omega_{mnrs} = (\eta_1 + i\eta_2)\gamma_{mnrs}(\eta_1 + i\eta_2)$$

- Alternatively, decomposing under $Spin(7)$ gives

$$\mathbf{8} \rightarrow \mathbf{1} \oplus \mathbf{7}$$

- From which we can build

$$\Phi_{mnrs} = \eta \gamma_{mnrs} \eta$$

Calabi-Yau Quotients

- We will consider Spin(7) holonomy manifolds constructed using the method described by Joyce.
- We quotient the Calabi-Yau by an anti-holomorphic isometric involution

$$\sigma^2 = \mathbb{1} \qquad \sigma^*(g) = g$$

$$\sigma^*(\Omega) = e^{2i\theta}\bar{\Omega} \qquad \sigma^*(J) = -J$$

- The Cayley calibration of the Spin(7) manifold produced in this way is then given by

$$\Phi = \frac{1}{\mathcal{V}^2}\left(\frac{1}{||\Omega||}\mathrm{Re}(e^{-i\theta}\Omega) + \frac{1}{8}J \wedge J\right)$$

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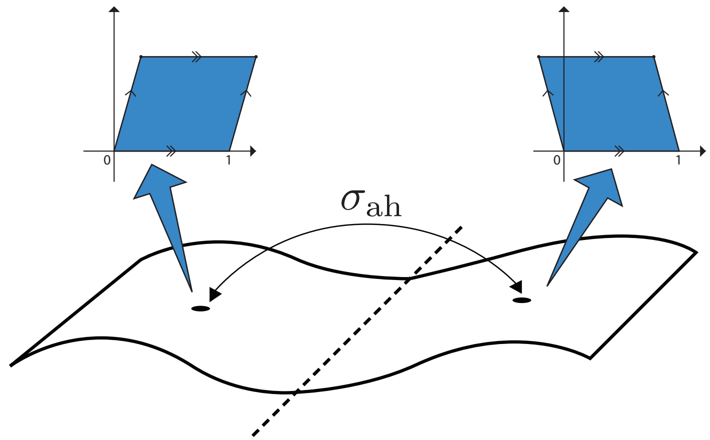
SPIN(7) QUOTIENT OF ELLIPTICALLY FIBERED CALABI-YAU FOURFOLDS

- For the reduction to have an F-theory dual, the Calabi-Yau fourfold must be elliptically fibered

$$P \equiv y^2 - x^3 - f(u^i)\, x\, z^4 - g(u^i)\, z^6 = 0$$

- We then consider anti-holomorphic involutions which are compatible with this structure

$$\sigma : \quad u^i \to \sigma_{ah}(u^i) \qquad P = 0 \to P = 0$$



Spin(7) Effective Action

- By considering the action of σ we can see how harmonic forms on the Calabi-Yau fit within the Spin(7) cohomology

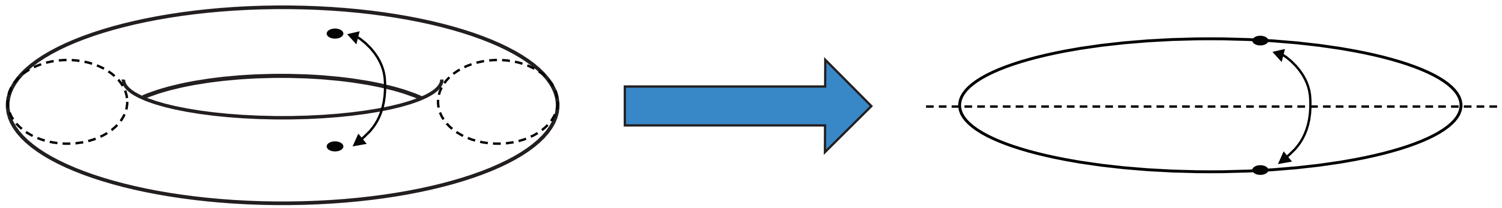
$$J = v^0\omega_0 + v^\alpha\omega_\alpha + v^i\omega_i \quad \{\omega_{\alpha_+}\} = H^2_{\mathbf{21}}(Z_8,\mathbb{R})$$

$$\{\omega_0\wedge J, \omega_{\alpha_-}\wedge J, \omega_i\wedge J\} \subset H^4_{\mathbf{1}\,\mathrm{S}}(Z_8,\mathbb{R}) \oplus H^4_{\mathbf{35}\,\mathrm{A}}(Z_8,\mathbb{R})$$

- The fields of the M-theory effective action can then be expanded in this basis
- $H^2_{\mathbf{21}}(Z_8,\mathbb{R})$ perturbations of C_3
- $H^4_{\mathbf{1}\,\mathrm{S}}(Z_8,\mathbb{R}) \oplus H^4_{\mathbf{35}\,\mathrm{A}}(Z_8,\mathbb{R})$ perturbations of g_{mn}
- The effective action is then invariant under N=1 supersymmetry in 3d

F-theory Lift

- When lifting to F-theory the extra quotient which forms the Spin(7) manifold gives a quotient of the usual F-theory circle



- The circle quotient projects out the 4d N=1 partners of the 3d N=1 fields
- These are then restored in the 4d effective theory when the F-theory limit is taken

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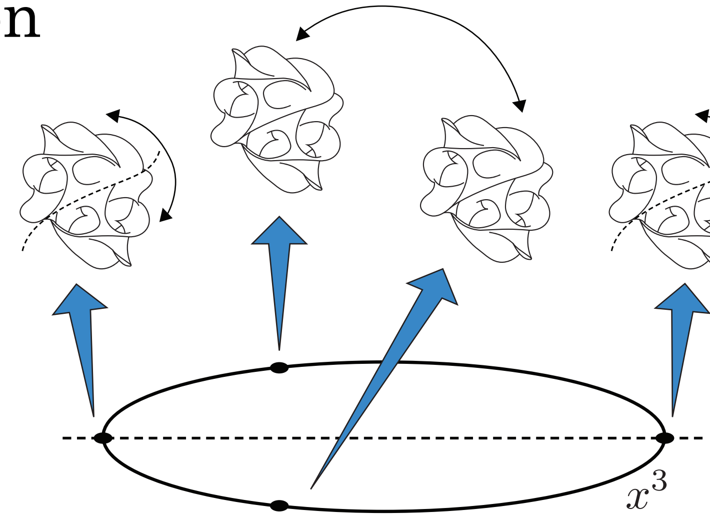
Quotients in the Weak Coupling Limit

- To see what happens in the weak coupling limit we can consider the Sen limit in the presence of the quotient.
- This agrees with the result of following σ through the T-duality.

Type IIB quotient		x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
$\mathcal{O}_1 = \Omega_p\,\sigma_{\rm h}\,(-1)^{F_L}$	O7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		
$\mathcal{O}_2 = R_3\,\sigma_{\rm ah}\,(-1)^{F_L}$	X5	$\times$	$\times$	$\times$		$\times$		$\times$		$\times$	
$\mathcal{O}_3 = \mathcal{O}_1\,\mathcal{O}_2$	O5	$\times$	$\times$	$\times$		$\times$		$\times$			$\times$

4D Effects of Spin(7) Quotients

- The new anti-holomorphic quotient acts on the x^3 direction



- The effect of the quotient is then only seen in the F-theory picture at the fixed points.
- When the F-theory limit is taken and the circle becomes large this quotient is then pushed away restoring 4d N=1 supersymmetry.

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Conclusions

- The compactification of M-theory on a Spin(7) holonomy manifold can be carried out in a way that remains compatible with M-theory/F-theory duality.
- For the Calabi-Yau quotients analyzed the quotient also acts on the F-theory circle and the localized effects are washed out upon taking the large circle limit.
- Several intrinsically Spin(7) effects remain to be studied.
- Can the SUSY and Lorentz breaking scales be separated?

Thank You