D7-BRANE CHAOTIC INFLATION

Lukas Witkowski

based on arXiv:1404.3711 with Arthur Hebecker and Sebastian C. Kraus
I. Renewed interest in large-field inflation in string theory.

II. One proposal: **Axion Monodromy Inflation**
   - axions (ALPs) exhibit shift symmetry
   - break shift-symmetry weakly and introduce monodromy

III. Original Axion Monodromy setup: break SUSY explicitly.

IV. Recently: Axion Monodromy models in spontaneously broken SUGRA

**Inflaton candidates:**
- massive Wilson line [Marchesano, Shiu, Uranga: 1404.3040]
- universal axion [Blumenhagen, Plauschinn: 1404.3542]
- D7 position modulus [Hebecker, Kraus, LW: 1404.3711]
I. Renewed interest in large-field inflation in string theory.

II. One proposal: **Axion Monodromy Inflation**
   - axions (ALPs) exhibit shift symmetry
   - break shift-symmetry weakly and introduce monodromy

III. Original Axion Monodromy setup: break SUSY explicitly.

IV. Recently: Axion Monodromy models in spontaneously broken SUGRA

Inflaton candidates:
   - massive Wilson line [Marchesano, Shiu, Uranga: 1404.3040]
   - universal axion [Blumenhagen, Plauschinn: 1404.3542]

This talk: **D7 position modulus** [Hebecker, Kraus, LW: 1404.3711]
IN SHORT

Ingredients:

• Inflaton: \( \text{Re}(c) \), the real part of the D7 position modulus \( c \).
• Shift-symmetric Kähler potential at Large Complex Structure.

\[
K \supset - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right)
\]

Mirror dual of shift symmetry of D6-brane Wilson line in IIA.
• In absence of fluxes, moduli space of \( c \) is a Riemann surface, which generically has one-cycles.
Ingredients continued:

• Fluxes will lead to an appearance of $c$ in the superpotential.

$$W = W_0 + \alpha c + \frac{\beta}{2} c^2$$

• The periodicity is broken and a monodromy arises.

• Inflation: initially $\text{Re}(c)$ displaced from minimum by large amount, then relaxes towards minimum circling the one-cycle multiple times.
IN SHORT

Virtues:

• Formulation in (spontaneously broken) SUGRA:

\[ K \supset -\ln \left( A + iB(c - \bar{c}) + \frac{D}{2} (c - \bar{c})^2 \right) \quad W = W_0 + \alpha c + \frac{\beta}{2} c^2 \]

• Can address moduli stabilisation quantitatively.

Downsides:

• Work at Large Complex Structure: ensure corrections \( e^{ic} \) small.

• Require weak violation of shift symmetry of \( \text{Re}(c) \). Need to tune \( \alpha, \beta \).
OUTLINE

1. Origin of shift-symmetric Kähler potential
2. Origin of monodromy: the superpotential
3. Moduli stabilisation
KÄHLER POTENTIAL

From F-theory:

- consider CY 4-fold at large volume:

\[ K_T \supset - \ln \left( \frac{\kappa_{ijkl}}{4!} (t^i - \bar{t}^i)(t^j - \bar{t}^j)(t^k - \bar{t}^k)(t^l - \bar{t}^l) + \ldots \right) \]

where \( t^i \) are 2-cycle moduli

\[ K_{CS} \supset - \ln \left( \frac{\kappa_{ijkl}}{4!} (z^i - \bar{z}^i)(z^j - \bar{z}^j)(z^k - \bar{z}^k)(z^l - \bar{z}^l) + \ldots \right) \]

where \( z^i \) are 4-fold complex structure moduli

- Arrive at shift symmetric \( K_{CS} \) at Large Complex Structure.
KÄHLER POTENTIAL

From F-theory:

• consider CY 4-fold at Large Complex Structure:

\[ K_{CS} \supset -\ln \left( \frac{K_{ijkl}}{4!}(z^i - \bar{z}^i)(z^j - \bar{z}^j)(z^k - \bar{z}^k)(z^l - \bar{z}^l) + \ldots \right) \]

• Go to weak coupling regime to recover IIB expression:

identify 4-fold complex structure moduli with axio-dilaton, 3-fold CS moduli and D7-brane moduli:

\[ z^i = \{ S, u^a, c^p \} \]

\[ K_{CS}^{g_s \rightarrow 0} = -\ln \left( \frac{\kappa^{(1)}_{abc}}{3!} (S - \bar{S})(u^a - \bar{u}^a)(u^b - \bar{u}^b)(u^c - \bar{u}^c) \right. \]

\[ + \frac{\kappa^{(2)}_{abpq}}{2!2!} (u^a - \bar{u}^a)(u^b - \bar{u}^b)(c^p - \bar{c}^p)(c^q - \bar{c}^q) + \ldots \right) \]

• In K3 x K3 one can verify that \( c^p \) appear only quadratically in \( \ln(\ldots) \).
KÄHLER POTENTIAL

From F-theory:

• consider CY 4-fold at Large Complex Structure:

\[ K_{CS} \supset -\ln \left( \frac{\kappa_{ijkl}}{4!} (z^i - \bar{z}^i)(z^j - \bar{z}^j)(z^k - \bar{z}^k)(z^l - \bar{z}^l) + \ldots \right) \]

• Go to weak coupling regime to recover IIB expression:

identify 4-fold complex structure moduli with axio-dilaton, 3-fold CS moduli and D7-brane moduli:

\[ z^i = \{ S, u^a, c^p \} \]

\[ K_{CS}^{g_s \to 0} = -\ln \left( \frac{\kappa_{abc}^{(1)}}{3!} (S - \bar{S})(u^a - \bar{u}^a)(u^b - \bar{u}^b)(u^c - \bar{u}^c) \right. \]

\[ + \frac{\kappa_{abpq}^{(2)}}{2!2!} (u^a - \bar{u}^a)(u^b - \bar{u}^b)(c^p - \bar{c}^p)(c^q - \bar{c}^q) + \ldots \]

• In K3 x K3 one can verify that \( c^p \) appear only quadratically in \( \ln(\ldots) \).
KÄHLER POTENTIAL

Moduli stabilisation - PART I:

\[ K_{GS}^{g_s \to 0} = - \ln \left( \frac{\kappa_{abc}^{(1)}}{3!} (S - \bar{S})(u^a - \bar{u}^a)(u^b - \bar{u}^b)(u^c - \bar{u}^c) \right) \]

\[ + \frac{\kappa_{abpq}^{(2)}}{2!2!} (u^a - \bar{u}^a)(u^b - \bar{u}^b)(c^p - \bar{c}^p)(c^q - \bar{c}^q) + \ldots \]

Stabilise \( S \), all \( u^a \) and all \( c^p \) except one \( c \) by fluxes:

- terms \((S - \bar{S})\) and \((u^a - \bar{u}^a)\) become model-dependent constants.
- terms \((c^p - \bar{c}^p)(c - \bar{c})\) become terms linear in \((c - \bar{c})\).
- also get a quadratic term \((c - \bar{c})^2\).

\[ K_{CS} = - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) \]
KÄHLER POTENTIAL

Moduli stabilisation - PART I:

\[ K_{CS}^{g_s \to 0} = - \ln \left( \frac{\kappa_{abc}^{(1)}}{3!} (S - \bar{S})(u^a - \bar{u}^a)(u^b - \bar{u}^b)(u^c - \bar{u}^c) \right. \]

\[ \left. + \frac{\kappa_{abpq}^{(2)}}{2!2!} (u^a - \bar{u}^a)(u^b - \bar{u}^b)(c^p - \bar{c}^p)(c^q - \bar{c}^q) + \ldots \right) \]

Stabilise \( S \), all \( u^a \) and all \( c^p \) except one \( c \) by fluxes:

- terms \((S - \bar{S})\) and \((u^a - \bar{u}^a)\) become model-dependent constants.
- terms \((c^p - \bar{c}^p)(c - \bar{c})\) become terms linear in \((c - \bar{c})\).
- also get a quadratic term \((c - \bar{c})^2\).

\[ K_{CS} = - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) \]
KÄHLER POTENTIAL

Arrive at

\[ K_{CS} = - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) \]

Summary:

• Kähler potential is shift-symmetric in \( \text{Re}(c) \).

• Work at Large Complex Structure:
  \[ A \sim (\text{Im } z)^4, \quad B \sim (\text{Im } z)^3, \quad D \sim (\text{Im } z)^2 \] are large.

• Need to ensure that \( \text{Im}(c) \) is stabilised at a large value such that non-perturbative corrections of the form \( e^{ic} \) are negligible.

• Alternatively, can reason via 3-fold mirror symmetry:
  Shift symmetry for D7-brane modulus in IIB as mirror dual of shift symmetry of D6-Wilson line in IIA (see A. Hebecker’s talk)
KÄHLER POTENTIAL

Arrive at

\[ K_{CS} = - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) \]

Summary:

- Kähler potential is shift-symmetric in \( \text{Re}(c) \).

- Work at Large Complex Structure:
  \[ A \sim (\text{Im } z)^4, \quad B \sim (\text{Im } z)^3, \quad D \sim (\text{Im } z)^2 \]
  are large.

- Need to ensure that \( \text{Im}(c) \) is stabilised at a large value such that non-perturbative corrections of the form \( e^{ic} \) are negligible.

- Alternatively, can reason via 3-fold mirror symmetry:
  Shift symmetry for D7-brane modulus in IIB as mirror dual of shift symmetry of D6-Wilson line in IIA (see A. Hebecker’s talk)
SUPERPOTENTIAL

From F-theory:

• The superpotential is given by

\[ W = N_J \Pi_J(z) \]

where \( N_J \) are flux numbers and \( \Pi_J \) is the period vector. \( \Pi_J \) is obtained by integrating the holomorphic 4-form \( \Omega_4 \) over 4-cycles:

\[ \Pi_J \sim (1, z^i, \kappa_{ijkl} z^k z^l, \kappa_{ijkl} z^j z^k z^l, \kappa_{ijkl} z^i z^j z^k z^l) \]

• In weak coupling limit identify \( z^i = \{S, u^a, c^p\} \) as before. Integrate out \( S, u^a \) and \( c^p \neq c \) as before.

\[ W = W_0 + \alpha c + \frac{\beta}{2} c^2 + \ldots \]
SUPERPOTENTIAL

From F-theory:

• The superpotential is given by

\[ W = N^J \Pi_J(z) \]

where \( N^J \) are flux numbers and \( \Pi_J \) is the period vector. \( \Pi_J \) is obtained by integrating the holomorphic 4-form \( \Omega_4 \) over 4-cycles:

\[ \Pi_J \sim (1, z^i, \kappa_{ijkl} z^k z^l, \kappa_{ijkl} z^j z^k z^l, \kappa_{ijkl} z^i z^j z^k z^l) \]

• In weak coupling limit identify \( z^i = \{ S, u^a, c^p \} \) as before. Integrate out \( S, u^a \) and \( c^p \neq c \) as before.

\[ W = W_0 + \alpha c + \frac{\beta}{2} c^2 + \ldots \]
SUPERPOTENTIAL

Summary:

\[ W = W_0 + \alpha c + \frac{\beta}{2} c^2 + \ldots \]

- Superpotential breaks the shift symmetry for \( \text{Re}(c) \) and gives rise to monodromy.
- Parameters \( \alpha, \beta \) depend on fluxes and can / need to be tuned.
- Superpotential quadratic in \( c \):
  in the end we will obtain a quadratic inflation potential.
MODULI STABILISATION

Also include Kähler moduli sector:

\[
K = -\ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) -2 \ln \left( V + \frac{\xi}{2} \right)
\]

\[
W = W_0 + \alpha c + \frac{\beta}{2}c^2 + \mathcal{A}e^{-\alpha_s T_s}
\]

- Fix Kähler moduli à la Large Volume Scenario (LVS).
- Also need to stabilise \( \text{Im}(c) \).

Examine scalar potential:

\[
V = e^K \left( K^{T_\gamma \bar{T}_\delta} D_{T_\gamma} W \overline{D_{T_\delta} W} - 3|W|^2 + K^{c\bar{c}} |D_c W|^2 \right)
\]
MODULI STABILISATION

Also include Kähler moduli sector:

\[ K = - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) - 2 \ln \left( V + \frac{\xi}{2} \right) \]

\[ W = W_0 + \alpha c + \frac{\beta}{2} c^2 + A e^{-a_s T_s} = \tilde{W}_0 + A e^{-a_s T_s} \]

- Fix Kähler moduli à la Large Volume Scenario (LVS).
- Also need to stabilise \( \text{Im}(c) \).

Examine scalar potential:

\[ V = e^K \left( K^{T\gamma \bar{T}\delta} D_{T\gamma} W \bar{D}_{T\delta} \bar{W} - 3|W|^2 + K^{\bar{c}c} |D_c W|^2 \right) \]

Volume scaling:

\[ \mathcal{O}(V^{-3}) \quad \mathcal{O}(V^{-2}) \]
MODULI STABILISATION

Also include Kähler moduli sector:

\[ K = - \ln \left( A + iB(c - \bar{c}) + \frac{D}{2} (c - \bar{c})^2 \right) - 2 \ln \left( \mathcal{V} + \frac{\xi}{2} \right) \]

\[ W = W_0 + \alpha c + \frac{\beta}{2} c^2 + \mathcal{A} e^{-a_s T_s} = \tilde{W}_0 + \mathcal{A} e^{-a_s T_s} \]

- Fix Kähler moduli à la Large Volume Scenario (LVS).
- Also need to stabilise \( \text{Im}(c) \).

Examine scalar potential:

\[ V = e^K \left( K^{T \gamma \bar{T} \delta} D_{T \gamma} \overline{W} \overline{D_{T \delta} W} - 3|W|^2 + K^{cc} |D_c W|^2 \right) \]

Minimum: \( \text{LVS minimum} \sim -\frac{\overline{W}_0^2}{\mathcal{V}^3} \)

Min. at \( D_c W = 0 \)
MODULI STABILISATION

Also include Kähler moduli sector:

\[
K = -\ln \left( A + iB (c - \bar{c}) + \frac{D}{2} (c - \bar{c})^2 \right) - 2 \ln \left( \nu + \frac{\xi}{2} \right)
\]

\[
W = W_0 + \alpha c + \frac{\beta}{2} c^2 + \mathcal{A} e^{-\alpha_s T_s} = \tilde{W}_0 + \mathcal{A} e^{-\alpha_s T_s}
\]

- Fix Kähler moduli à la Large Volume Scenario (LVS).
- Also need to stabilise \( \text{Im}(c) \).

Examine scalar potential:

\[
V = e^K \left( K^{T_\gamma T_\delta} D_{T_\gamma} \bar{W} D_{T_\delta} W - 3|W|^2 + K^{cc} |D_c W|^2 \right)
\]

Minimum: LVS minimum \( \sim -\frac{|W_0|^2}{\nu^3} \)

Min. at \( D_c W = 0 \)
Moduli spectrum:

\[ V = e^K \left( K^{T\gamma \bar{T}\delta} D_{T\gamma} W \bar{D}_{T\delta} \bar{W} - 3|W|^2 + K^{c\bar{c}} |D_c W|^2 \right) \]

\[ \mathcal{O}(\mathcal{V}^{-3}) \]

\[ \mathcal{O}(\mathcal{V}^{-2}) \]

\[ m \]

\[ m_{\text{Im}(c)} \sim \frac{|W_0|}{\mathcal{V} \text{Im}(z)^2} \]

\[ m_{\nu} \sim \frac{|W_0|}{\mathcal{V}^{3/2} \text{Im}(z)^2} \]

\[ m_{\phi} \sim \frac{|eta|}{\mathcal{V}} \]

Inflaton: \( \varphi = \sqrt{2K_{c\bar{c}}} \text{Re}(c) \)
Moduli Stabilisation

Moduli spectrum:

\[ V = e^K \left( K^{T_{\gamma} \bar{T}_{\delta}} D_{T_{\gamma}} \bar{W} D_{T_{\delta}} W - 3|W|^2 + K^{c \bar{c}} |D_c W|^2 \right) \]

\[ m_{\text{Im}(c)} \sim \frac{|W_0|}{\nu \text{Im}(z)^2} \]

Im(c) appears in Kähler potential: mass arises from \( K_c W_0 \subset D_c W \).

\[ m_\nu \sim \frac{|W_0|}{\nu^{3/2} \text{Im}(z)^2} \]

\[ m_\varphi \sim \frac{\beta}{\nu} \]

Inflaton: \( \varphi = \sqrt{2K_{c \bar{c}}} \text{Re}(c) \), mass arises from \( \partial_c W \subset D_c W \).
Moduli spectrum:

\[
V = e^K \left( K^{T_\gamma \bar{T}_\delta} D_{T_\gamma} W \overline{D_{T_\delta} W} - 3|W|^2 + K^{c\bar{c}} |D_c W|^2 \right)
\]

- \(m_{\text{Im}(c)} \sim \frac{|W_0|}{\nu \text{Im}(z)^2}\)
- \(m_{\nu} \sim \frac{|W_0|}{\nu^{3/2} \text{Im}(z)^2}\)
- \(m_\varphi \sim \frac{|\beta|}{\nu}\)

Inflaton potential:

\[
V_{\text{inf}} = \frac{1}{2} m_\varphi^2 \varphi^2 + \mathcal{O}(\{\alpha, \beta\}^4)
\]

\[
= \frac{1}{2} \frac{|\beta|^2}{\nu^2} \varphi^2 + \mathcal{O}(\{\alpha, \beta\}^4)
\]
Moduli spectrum: \[ V = e^K \left( K^{T \gamma T \delta} D_{T \gamma} W \overline{D_{T \delta} W} - 3|W|^2 + K^{c \bar{c}} |D_c W|^2 \right) \]

\[ V_{\text{min}} \sim -\frac{|W_0|^2}{\mathcal{V}^3} \]

Stability of \( \mathcal{V} \) ensured during inflation for small enough \( \beta \).

Inflaton potential: \[ V_{\text{inf}} = \frac{1}{2} m^2 \varphi^2 + \mathcal{O}(\{\alpha, \beta\}^4) \]

\[ = \frac{1}{2} \frac{|\beta|^2}{\mathcal{V}^2} \varphi^2 + \mathcal{O}(\{\alpha, \beta\}^4) \]
CONCLUSION

1. D7-brane position moduli exhibit all properties for successful Axion Monodromy Inflation in spontaneously broken SUGRA.

2. Kähler potential exhibits shift symmetry in regime of Large Complex Structure.


4. Moduli stabilisation can be addressed quantitatively:
   Kähler moduli can be stabilised according to Large Volume Scenario.

5. Here, obtain quadratic inflation:

\[ V_{\text{inflaton}} = \frac{1}{2} m_\phi^2 \phi^2 + \ldots = \frac{1}{2} \frac{|\beta|^2}{V^2} \phi^2 + \ldots \]
**DOWNSIDES & OPEN QUESTIONS**

**Downsides:**
1. Need to work at Large Complex Structure.
2. Need to tune $\alpha, \beta$ (to keep volume stable during inflation).

**Open questions:**
1. Consistency: is the required flux tuning possible?
2. Realise this model in explicit example geometries: How does the D7-brane modulus $c$ appear in $K$ and $W$?
3. Detailed phenomenology?
4. How do loop corrections affect the inflation potential?
1. D7-brane position moduli exhibit all properties for successful Axion Monodromy Inflation in spontaneously broken SUGRA.

2. Kähler potential exhibits shift symmetry in regime of Large Complex Structure:
   \[ K \supset -\ln \left( A + iB(c - \bar{c}) + \frac{D}{2}(c - \bar{c})^2 \right) \]

3. Shift symmetry broken weakly in superpotential by fluxes:
   \[ W = W_0 + \alpha c + \frac{\beta}{2}c^2 \]

4. Moduli stabilisation can be addressed quantitatively:
   Kähler moduli can be stabilised following the Large Volume Scenario.

**Mille grazie!**