Higher Level String Resonances in Four Dimensions

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This talk is based on

WZF, D. Lüst, O. Schlotterer, S. Stieberger and T.R. Taylor, arXiv:1007.5254

WZF and T.R. Taylor, arXiv:1110.1087

WZF, G. Shiu, P. Soler and F. Ye, arXiv:1401.5880, arXiv:1401.5890

L. Anchordoqui, I. Antoniadis, D.C. Dai, WZF, H. Goldberg, X. Huang, D. Lüst, D. Stojkovic and T.R. Taylor, to appear

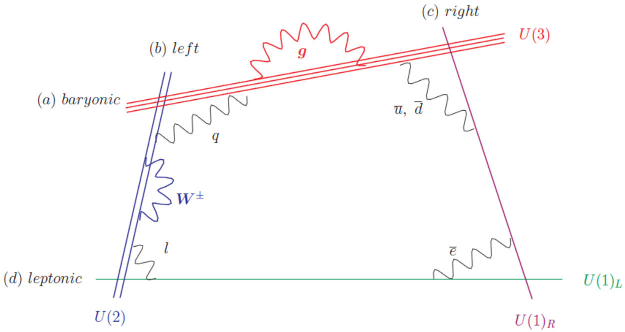
Overview

Motivation

- Landscape problem. The predictive power of string theory is lost.
- What string predictions can we make?
- As was pointed out in [D. Lüst, S. Stieberger and T.R. Taylor, '08; D. Lüst, O. Schlotterer, S. Stieberger and T.R. Taylor, '09], the scattering amplitudes of only gauge bosons and gauge bosons with up to two matter fermions are universal. These amplitudes are common to any compactification which allows CFT description, and to any number of supersymmetries (even $\mathcal{N} = 0$).
- Finding common signals of string models becomes possible by changing the value of just one parameter: the string mass scale M_s ~TeV [I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, '98] also see Antoniadis, Hashi and Kumar's talks.
- Assumptions: large extra dimensions; low mass string scale; weak coupling.

Local Model

We consider local model of intersecting type IIA/IIB D-branes on orientifolds embedded in a "Swiss cheese" type Calabi-Yau manifold.



Intersection pattern of four stacks of D6-branes giving rise to the MSSM

Decay of 2nd massive level string states

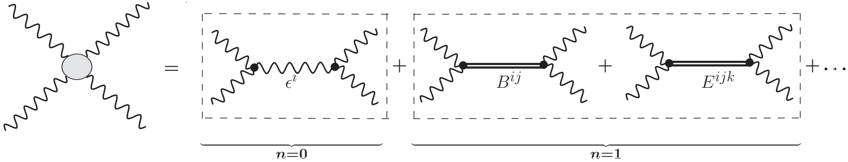
Low mass string resonances and the phenomenology have been studied a lot
[Anchordoqui,Antoniadis,Feng,Huang,Lüst,Nawata,Schlotterer,Stieberger,Taylor,Vlcek; Dong,Han,Huang,Shiu; Hashi,Kitazawa;...]

Here we would like to perform a comprehensive study of the 2nd massive level string states. There are three types of (universal) string excitations:

Bosonic sector – color octet $G^{(2)}$, color singlet $C^{(2)}$; fermionic sector $Q^{(2)}$. For these three types of massive string excitations, there are the following decay channels:

	2 massless string states	1 first level string state plus 1 massless string state	involve 1 or 2 color singlet(s)	involve 1 first level color singlet excitation
$G^{(2)}$	$gg, q\bar{q}$	$G^{(1)}g, Q^{(1)}\bar{q}, \bar{Q}^{(1)}q$	$gA_a, G^{(1)}A_a$	$C^{(1)}g$
$C^{(2)}$	$gg, q\bar{q}$	$G^{(1)}g, Q^{(1)}\bar{q}, \bar{Q}^{(1)}q$	A_aA_a	$C^{(1)}A_a$
$Q^{(2)}$	gq	$G^{(1)}q, Q^{(1)}g$	$qA_a, Q^{(1)}A_a$	$C^{(1)}q$

Factorization of the four-gluon amplitude



[WZF, D. Lüst, O. Schlotterer, S. Stieberger and T.R. Taylor, ’10]

In 4D

In 4D, we can use the helicity formalism and the maximally helicity violating (MHV) amplitude for four massless gluons reads

$$\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) = 4g^2 \langle 12 \rangle^4 \Big[\frac{V_t}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_2}T^{a_1}T^{a_4}T^{a_3}) + \frac{V_u}{\langle 13\rangle\langle 34\rangle\langle 42\rangle\langle 21\rangle} \mathrm{Tr}(T^{a_2}T^{a_1}T^{a_3}T^{a_4} + T^{a_1}T^{a_2}T^{a_4}T^{a_3}) + \frac{V_s}{\langle 14\rangle\langle 42\rangle\langle 23\rangle\langle 31\rangle} \mathrm{Tr}(T^{a_1}T^{a_3}T^{a_2}T^{a_4} + T^{a_3}T^{a_1}T^{a_4}T^{a_2}) \Big] ,$$

where the form factor function reads

$$V_t = V(s,t,u) = \frac{\Gamma(1 - s/M^2)\Gamma(1 - u/M^2)}{\Gamma(1 + t/M^2)} ,$$

$M^2 = 1/\alpha'$ sets the string scale, and u, t, s are Mandelstam variables, $u = -\frac{s}{2}(1 + \cos\theta)$ and $t = -\frac{s}{2}(1 - \cos\theta)$.

Form factor expansion

The form factor can be expanded in terms of s-channel resonances. Near the nth level pole ($s \to nM^2$) we have

$$V_t(n) = V(s,t,u) \approx \frac{1}{s - nM^2} \times \frac{M^{2-2n}}{(n-1)!} \prod_{J=0}^{n-1} (u + M^2 J) \,.$$

As was explained in [Anchordoqui, Goldberg and Taylor, '08], recasting the expansion one can re-express the amplitudes as the sum of a series Wigner d-matrices.

The Wigner d-matrix $d^{(j)}_{m',m}(\theta)$ denotes a state of total angular momentum j, with m' (m) showing the helicity difference of the initial (final) state particles.

Thus following this procedure, one can obtain two 3pt amplitudes of nth level massive (intermediate) string states decaying into different final states with specific spin combinations.

Two simple examples

The first level resonance is given in [Anchordoqui, Goldberg and Taylor, '08]

$$\mathcal{M}(g_1^-, g_2^+, g_3^+, g_4^-) \xrightarrow{n=1} 4g^2 \mathrm{Tr}(\{T^{a_1}, T^{a_2}\}\{T^{a_3}, T^{a_4}\}) \frac{M^2}{s - M^2} d^{(2)}_{2,2} ,$$
$$\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) \xrightarrow{n=1} 4g^2 \mathrm{Tr}(\{T^{a_1}, T^{a_2}\}\{T^{a_3}, T^{a_4}\}) \frac{M^2}{s - M^2} d^{(0)}_{0,0} ,$$

where

$$\mathrm{Tr}(\{T_a, T_b\}\{T_c, T_d\}) \to 8 \sum_e d_{abe} d_{cde} .$$

These were confirmed in [WZF, Lüst, Schlotterer, Stieberger and Taylor, '10].

Higher Levels Resonances

For the second massive level resonances

$$\mathcal{M}(g_1^-, g_2^+, g_3^+, g_4^-) \xrightarrow{n=2} 4g^2 \text{Tr}([T^{a_1}, T^{a_2}][T^{a_3}, T^{a_4}]) \frac{M^2}{s - 2M^2} \frac{2}{3} (d_{2.2}^{(3)} + 2d_{2,2}^{(2)}) ,$$

$$\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) \xrightarrow{n=2} 4g^2 \text{Tr}([T^{a_1}, T^{a_2}][T^{a_3}, T^{a_4}]) \frac{2M^2}{s - 2M^2} d_{0,0}^{(1)} ,$$

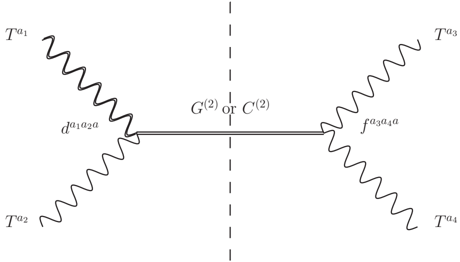
where

$$\text{Tr}([T_a, T_b][T_c, T_d]) = -\tfrac{1}{2} \sum_e f_{abe} f_{cde} ,$$

where f is the gauge group structure.

A general analysis of higher level string resonances is given in [WZF and T.R. Taylor, '11].

Factorization of 4pt amplitudes with 1 massive leg



Helicity Formalism

The helicity formalism can dramatically simplify the final results of scattering amplitudes. For massless spin-$\frac{1}{2}$ spinors, we define

$$|i\rangle = |k_i\rangle = u_+(k_i) = v_-(k_i) = \begin{pmatrix} 0 \\ k_i^{*\dot{a}} \end{pmatrix},$$

$$|i] = |k_i] = u_-(k_i) = v_+(k_i) = \begin{pmatrix} k_{i,a} \\ 0 \end{pmatrix},$$

$$[i| = [k_i| = \bar{u}_+(k_i) = \bar{v}_-(k_i) = (k_i^a, 0)\,,$$

$$\langle i| = \langle k_i| = \bar{u}_-(k_i) = \bar{v}_+(k_i) = \left(0, k^*_{i,\dot{a}}\right).$$

These commutative spinors are defined by

$$P^{\dot{a}a} = p_\mu \bar{\sigma}^{\mu\dot{a}a} = -p^{*\dot{a}}p^a\,, \qquad P_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}} = -p_a p^*_{\dot{a}}\,.$$

Then we can define the spinor products

$$\langle pq\rangle = p^*_{\dot{a}} q^{*\dot{a}}\,, \qquad [pq] = p^a q_a\,.$$

Massless gauge boson

The helicity wave functions (polarization vectors) of a massless spin one gauge boson can be written as

$$\epsilon_\mu^+(k,r) = \frac{\langle r|\gamma_\mu|k]}{\sqrt{2}\langle rk\rangle} = \frac{r^*_{\dot{a}}\bar{\sigma}_\mu^{\dot{a}a}k_a}{\sqrt{2}\langle rk\rangle} ,$$

$$\epsilon_\mu^-(k,r) = -\frac{[r|\gamma_\mu|k\rangle}{\sqrt{2}[rk]} = -\frac{r^a\sigma_{\mu a\dot{a}}k^{*\dot{a}}}{\sqrt{2}[rk]} ,$$

where k is the momentum of the gauge boson and r is the reference momentum which can be chosen to be any light-like momentum except k. The final results of the helicity amplitudes are independent of the choice of reference momentum r.

Massive boson

A spin-J particle contains $2J+1$ spin degrees of freedom associated to the eigenstates of J_z. The choice of the quantization axis $\vec{z}$ can be handled in an elegant way by decomposing the momentum k into two arbitrary light-like reference momenta p and q:

$$k^\mu = p^\mu + q^\mu, \qquad k^2 = -m^2 = 2pq, \qquad p^2 = q^2 = 0.$$

Then the spin quantization axis is chosen to be the direction of q in the rest frame. The $2J+1$ spin wave functions depend on p and q, however this dependence will drop out in the squared amplitudes summed over all spin directions and in unpolarized cross sections.

Massive spin one boson

The massive spin-1 field ξ^μ wave functions are given in [D. Spehler and S.F. Novaes, '91]

$$\xi_+^\mu(k) = \frac{1}{\sqrt{2}m} p^*_{\dot{a}} \bar{\sigma}^{\mu\dot{a}a} q_a \,,$$

$$\xi_0^\mu(k) = \frac{1}{2m} \bar{\sigma}^{\mu\dot{a}a} (p^*_{\dot{a}} p_a - q^*_{\dot{a}} q_a) \,,$$

$$\xi_-^\mu(k) = -\frac{1}{\sqrt{2}m} q^*_{\dot{a}} \bar{\sigma}^{\mu\dot{a}a} p_a \,.$$

Massive spin two boson

The wave function (polarization tensor) of massive spin two boson $\alpha^{\mu\nu}$ satisfies the following relations (symmetric, transverse, traceless), which read

$$\alpha^{\mu\nu}(k,\lambda) = \alpha^{\nu\mu}(k,\lambda)\,, \qquad k_\mu \alpha^{\mu\nu}(k,\lambda) = 0\,, \qquad g_{\mu\nu}\alpha^{\mu\nu}(k,\lambda) = 0\,,$$

Helicity wave functions of the massive spin-2 boson were constructed in [S.F. Novaes and D. Spehler, '92]:

$$\alpha^{\mu\nu}(k,+2) = \frac{1}{2m^2}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b} p^*_{\dot{a}} q_a p^*_{\dot{b}} q_b\,,$$

$$\alpha^{\mu\nu}(k,+1) = \frac{1}{4m^2}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\left[(p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)p^*_{\dot{b}}q_b + p^*_{\dot{a}}q_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)\right]\,,$$

$$\alpha^{\mu\nu}(k,\ 0\ ) = \frac{1}{2m^2\sqrt{6}}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\left[(p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b) - p^*_{\dot{a}}q_a q^*_{\dot{b}}p_b - q^*_{\dot{a}}p_a p^*_{\dot{b}}q_b\right]\,,$$

$$\alpha^{\mu\nu}(k,-1) = -\frac{1}{4m^2}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\left[(p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)q^*_{\dot{b}}p_b + q^*_{\dot{a}}p_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)\right]\,,$$

$$\alpha^{\mu\nu}(k,-2) = \frac{1}{2m^2}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b} q^*_{\dot{a}} p_a q^*_{\dot{b}} p_b\,.$$

Helicity wave functions for higher spin massive boson

A massive spin-n boson also satisfies symmetric, transverse, traceless conditions, and it has $2n+1$ d.o.f..

There are two methods to construct helicity wave functions for higher spin massive bosons:

(1) Consider spin-n boson as spin-$(n-1)$ and spin-1 coupling. Check any old angular momentum textbooks for the CG coefficients to determine the helicity wave functions for the spin-n boson.

(2) Make use of the J_- operator:
$J_-\Phi(n,m) = \sqrt{(j+m)(j-m+1)}\Phi(n,m-1)$.

Helicity wave functions for higher spin massive boson

The helicity wave function of an arbitrary j_z state of Φ_n can be written in a general form

$$\Phi_n^{\mu_1\mu_2\cdots\mu_n}(n,m) = \Big[\sum_\alpha \frac{2^{n-m-2\alpha}\cdot n!}{\alpha!(m+\alpha)!(n-2\alpha-m)!}(2m^2)^n\Big]^{-\frac{1}{2}} \times$$
$$\sum_\alpha \Big\{ \prod_i (p^*\bar{\sigma}^{(\underline{\mu_i}}q)^{m+\alpha} \prod_j (-q^*\bar{\sigma}^{\underline{\mu_j}}p)^\alpha \prod_k [\bar{\sigma}^{\underline{\mu_k})}(p^*p-q^*q)]^{n-m-2\alpha} \Big\},$$

where $m \geq 0$, the sum over α is over such values that the factorials are non-negative, and we symmetrize all the spacetime indices μ_i, μ_j, μ_k. These wave functions satisfy physical state conditions (symmetric, transverse and traceless). The helicity wave functions of $\Phi_n^{\mu_1\mu_2\cdots\mu_n}(n,-m)$ can be easily obtained by

$$\Phi_n^{\mu_1\mu_2\cdots\mu_n}(n,-m) = \Phi_n^{\mu_1\mu_2\cdots\mu_n}(n,m)^\dagger\,.$$

Massive spin three boson

$$\Phi_3^{\mu\nu\rho}(k,+3) = \frac{1}{(\sqrt{2}m)^3}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}p^*_{\dot{a}}q_a p^*_{\dot{b}}q_b p^*_{\dot{c}}q_c\,,$$

$$\Phi_3^{\mu\nu\rho}(k,+2) = \frac{\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}}{\sqrt{6}(\sqrt{2}m)^3}\Big[p^*_{\dot{a}}q_a p^*_{\dot{b}}q_b(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c) + p^*_{\dot{a}}q_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)p^*_{\dot{c}}q_c + (p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)p^*_{\dot{b}}q_b p^*_{\dot{c}}q_c\Big]\,,$$

$$\begin{aligned}\Phi_3^{\mu\nu\rho}(k,+1) = \frac{\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}}{\sqrt{15}(\sqrt{2}m)^3}\Big[&p^*_{\dot{a}}q_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c) + (p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)p^*_{\dot{b}}q_b(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c)\\ &+ (p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)p^*_{\dot{c}}q_c - p^*_{\dot{a}}q_a p^*_{\dot{b}}q_b q^*_{\dot{c}}p_c - p^*_{\dot{a}}q_a q^*_{\dot{b}}p_b p^*_{\dot{c}}q_c - q^*_{\dot{a}}p_a p^*_{\dot{b}}q_b p^*_{\dot{c}}q_c\Big]\,,\end{aligned}$$

$$\begin{aligned}\Phi_3^{\mu\nu\rho}(k,\ 0\ ) = \frac{\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}}{2\sqrt{5}(\sqrt{2}m)^3}\Big[&(p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c) - p^*_{\dot{a}}q_a q^*_{\dot{b}}p_b(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c)\\ &- q^*_{\dot{a}}p_a p^*_{\dot{b}}q_b(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c) - p^*_{\dot{a}}q_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)q^*_{\dot{c}}p_c - q^*_{\dot{a}}p_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)p^*_{\dot{c}}q_c\\ &- (p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)p^*_{\dot{b}}q_b q^*_{\dot{c}}p_c - (p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)q^*_{\dot{b}}p_b p^*_{\dot{c}}q_c\Big]\,,\end{aligned}$$

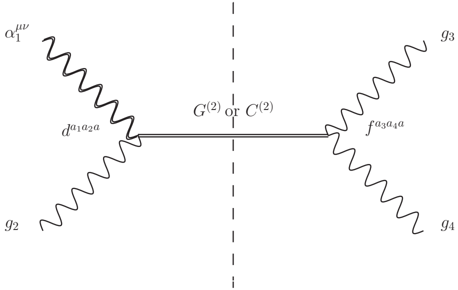
$$\begin{aligned}\Phi_3^{\mu\nu\rho}(k,-1) = -\frac{\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}}{\sqrt{15}(\sqrt{2}m)^3}\Big[&(p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)q^*_{\dot{c}}p_c + (p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)q^*_{\dot{b}}p_b(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c)\\ &+ q^*_{\dot{a}}p_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c) - p^*_{\dot{a}}q_a q^*_{\dot{b}}p_b q^*_{\dot{c}}p_c - q^*_{\dot{a}}p_a p^*_{\dot{b}}q_b q^*_{\dot{c}}p_c - q^*_{\dot{a}}p_a q^*_{\dot{b}}p_b p^*_{\dot{c}}q_c\Big]\,,\end{aligned}$$

$$\Phi_3^{\mu\nu\rho}(k,-2) = \frac{\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}}{\sqrt{6}(\sqrt{2}m)^3}\Big[(p^*_{\dot{a}}p_a - q^*_{\dot{a}}q_a)q^*_{\dot{b}}p_b q^*_{\dot{c}}p_c + q^*_{\dot{a}}p_a(p^*_{\dot{b}}p_b - q^*_{\dot{b}}q_b)q^*_{\dot{c}}p_c + q^*_{\dot{a}}p_a q^*_{\dot{b}}p_b(p^*_{\dot{c}}p_c - q^*_{\dot{c}}q_c)\Big]\,,$$

$$\Phi_3^{\mu\nu\rho}(k,-3) = -\frac{1}{(\sqrt{2}m)^3}\bar{\sigma}^{\mu\dot{a}a}\bar{\sigma}^{\nu\dot{b}b}\bar{\sigma}^{\rho\dot{c}c}q^*_{\dot{a}}p_a q^*_{\dot{b}}p_b q^*_{\dot{c}}p_c\,.$$

An example

$\mathcal{A}(\alpha_1, g_2, g_3, g_4)$



The full helicity amplitudes

The full helicity amplitudes read

$$\mathcal{A}[\alpha_1, \epsilon_2, \epsilon_3, \epsilon_4] = 8\, g^2 \big(V_t\, t^{a_1 a_2 a_3 a_4} + V_s\, t^{a_2 a_3 a_1 a_4} + V_u\, t^{a_3 a_1 a_2 a_4} \big) \sqrt{2\alpha'}\ \mathscr{A}[\alpha_1, \epsilon_2, \epsilon_3, \epsilon_4],$$

where

$$\mathscr{A}\,[\alpha(+2), +, +, -] = \frac{1}{2\sqrt{2}} \frac{\langle p4\rangle^4}{\langle 23\rangle\langle 34\rangle\langle 42\rangle}\,,$$

$$\mathscr{A}\,[\alpha(+1), +, +, -] = \frac{1}{\sqrt{2}}\ \frac{\langle p4\rangle^3\langle 4q\rangle}{\langle 23\rangle\langle 34\rangle\langle 42\rangle}\,,$$

$$\mathscr{A}\,[\alpha(\ 0\ ), +, +, -] = \frac{\sqrt{3}}{2}\ \frac{\langle p4\rangle^2\langle 4q\rangle^2}{\langle 23\rangle\langle 34\rangle\langle 42\rangle}\,,$$

$$\mathscr{A}\,[\alpha(-1), +, +, -] = \frac{1}{\sqrt{2}}\ \frac{\langle q4\rangle^3\langle 4p\rangle}{\langle 23\rangle\langle 34\rangle\langle 42\rangle}\,,$$

$$\mathscr{A}\,[\alpha(-2), +, +, -] = \frac{1}{2\sqrt{2}} \frac{\langle q4\rangle^4}{\langle 23\rangle\langle 34\rangle\langle 42\rangle}\,.$$

[WZF, D. Lüst, O. Schlotterer, S. Stieberger and T.R. Taylor, '10]

The results

$$\mathcal{A}[\alpha(+2),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\frac{16}{\sqrt{3}}d^3_{-3,-2}(\theta)\, f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{+2+a_1a_2} = F^{a,J=3}_{-2-a_1a_2} = 8gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(+1),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(\frac{16}{3}d^3_{-2,-2}(\theta) - \frac{16}{3}d^2_{-2,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{+1+a_1a_2} = F^{a,J=3}_{-1-a_1a_2} = \frac{8}{\sqrt{3}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{+1+a_1a_2} = F^{a,J=2}_{-1-a_1a_2} = 4\sqrt{\frac{2}{3}}gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(0),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(8\sqrt{\frac{2}{15}}d^3_{-1,-2}(\theta) - \frac{8}{\sqrt{3}}d^2_{-1,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{0+a_1a_2} = F^{a,J=3}_{0-a_1a_2} = 4\sqrt{\frac{2}{5}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{0+a_1a_2} = F^{a,J=2}_{0-a_1a_2} = 2\sqrt{2}gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(-1),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(4\sqrt{\frac{2}{15}}d^3_{0,-2}(\theta) - 4\sqrt{\frac{2}{3}}d^2_{0,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{-1+a_1a_2} = F^{a,J=3}_{+1-a_1a_2} = 2\sqrt{\frac{2}{5}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{-1+a_1a_2} = F^{a,J=2}_{+1-a_1a_2} = 2gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(-2),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(\frac{4}{3\sqrt{5}}d^3_{+1,-2}(\theta) - \frac{4\sqrt{2}}{3}d^2_{+1,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{-2+a_1a_2} = F^{a,J=3}_{+2-a_1a_2} = \frac{2}{\sqrt{15}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{-2+a_1a_2} = F^{a,J=2}_{+2-a_1a_2} = \frac{2}{\sqrt{3}}gM_s d^{a_1a_2a}\,.$$

The first massive level

This analysis for the first massive level was completely worked out in [WZF, Lüst, Schlotterer, Stieberger, Taylor, '11] [WZF, Lüst, Schlotterer, '12]. Here for illustration, we only focus on the universal part of the NS sector.

$$|n=1\rangle = \Big(\chi_{1\mu}\psi^{\mu}_{-\frac{3}{2}} + \chi_{2\mu\nu}\alpha^{\mu}_{-1}\psi^{\nu}_{-\frac{1}{2}} + \chi_{3\mu\nu\rho}\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}}\Big)|0;k\rangle\,.$$

Using the old covariant approach, the physical state conditions are:

$$(L_0-\frac{1}{2})|n=1\rangle = 0\,, \qquad L_1|n=1\rangle = 0\,, \qquad G_{\frac{3}{2}}|n=1\rangle = G_{\frac{1}{2}}|n=1\rangle = 0\,,$$

where the super Virasoro generators read

$$\begin{aligned} L_m &= \frac{1}{2}\sum_n :\alpha^{\lambda}_{m-n}\alpha_{n\lambda}: +\frac{1}{4}\sum_r (2r-m):\psi^{\lambda}_{m-r}\psi_{r\lambda}: +a\delta_{m,0}\,, \\ G_r &= \sum_n \alpha^{\lambda}_n \psi_{(r-n)\lambda}\,. \end{aligned}$$

Physical state conditions

We can get the following equations

$$\begin{aligned} \chi_{1\mu} k^{\mu} + \chi_{2\mu\nu} \eta^{\mu\nu} &= 0\,, \\ \chi_{1\mu} + \chi_{2\mu\nu} k^{\nu} &= 0\,, \\ \chi_{2\mu\nu} - \chi_{2\nu\mu} + 6\chi_{3\mu\nu\rho} k^{\rho} &= 0\,. \end{aligned}$$

- Solving above equations, one can get the 1st massive level physical fields. One should discard all the spurious states.
- These fields are all universal (their tree level amplitudes are independent of the internal geometry of the Calabi-Yau spaces).
- At 1st massive level, we could have spin-0,1,2 bosons in the NS sector. The possible solution for the the higher rank tensors can be a combination of spin-0,1,2 fields, and k_μ, $\eta_{\mu\nu}$ and $\varepsilon_{\mu\nu\rho\sigma}$.

Physical states

We could obtain the following solutions:
A spin-2 field

$$V_{\alpha^a}^{(-1)} = [T^a]_{\alpha_2}^{\alpha_1} g \alpha_{\mu\nu} i\partial X^\mu \psi^\nu \mathrm{e}^{-\phi} \mathrm{e}^{ikX} ,$$

with $\alpha_{\mu\nu} k^\nu = \alpha_{\mu\nu}\eta^{\mu\nu} = 0$.
One complex scalar field,

$$V_{\Phi_\pm^a}^{(-1)} = [T^a]_{\alpha_2}^{\alpha_1} \frac{g}{2} \Big\{ \left[(\eta_{\mu\nu} + 2\alpha' k_\mu k_\nu) i\partial X^\mu \psi^\nu + 2\alpha' k_\nu \partial \psi^\nu \right] \pm \frac{i}{6} 2\alpha' \varepsilon_{\mu\nu\rho\sigma} \psi^\mu \psi^\nu \psi^\rho k^\sigma \Big\} \mathrm{e}^{-\phi} \mathrm{e}^{ikX} ,$$

which only couple to (anti)self-dual gauge field configurations, i.e., to gluons in (++) or (−−) helicity configurations.
All these confirm the $n = 1$ resonance states predicted from factorization.

Two simple examples

The first level resonance is given in [L.A. Anchordoqui, H. Goldberg and T.R. Taylor, '08]

$$\mathcal{M}(g_1^-, g_2^+, g_3^+, g_4^-) \xrightarrow{n=1} 4g^2 \mathrm{Tr}(\{T^{a_1}, T^{a_2}\}\{T^{a_3}, T^{a_4}\}) \frac{M^2}{s - M^2} d^{(2)}_{2,2}\,,$$

$$\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) \xrightarrow{n=1} 4g^2 \mathrm{Tr}(\{T^{a_1}, T^{a_2}\}\{T^{a_3}, T^{a_4}\}) \frac{M^2}{s - M^2} d^{(0)}_{0,0}\,,$$

where

$$\mathrm{Tr}(\{T_a, T_b\}\{T_c, T_d\}) \to 8 \sum_e d_{abe} d_{cde}\,.$$

These were confirmed in [WZF, D. Lüst, O. Schlotterer, S. Stieberger and T.R. Taylor, '10].

The second massive level

The general open string state for the second massive level can be written as,

$$|n=2\rangle = \Big(\zeta_{1\mu}\psi^{\mu}_{-\frac{5}{2}} + \zeta_{2\mu\nu}\alpha^{\mu}_{-1}\psi^{\nu}_{-\frac{3}{2}} + \zeta'_{2\mu\nu}\alpha^{\mu}_{-2}\psi^{\nu}_{-\frac{1}{2}} + \zeta_{3\mu\nu\rho}\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\psi^{\rho}_{-\frac{1}{2}} + \zeta'_{3\mu\nu\rho}\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{3}{2}} + \zeta_{4\mu\nu\rho\sigma}\alpha^{\mu}_{-1}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}}\psi^{\sigma}_{-\frac{1}{2}} + \zeta_{5\mu\nu\rho\sigma\gamma}\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}}\psi^{\sigma}_{-\frac{1}{2}}\psi^{\gamma}_{-\frac{1}{2}}\Big)|0;k\rangle\,.$$

The physical state conditions are:

1. $(L_0 - \frac{1}{2})|n=2\rangle = 0$,
2. $L_2|n=2\rangle = L_1|n=2\rangle = 0$,
3. $G_{\frac{5}{2}}|n=2\rangle = G_{\frac{3}{2}}|n=2\rangle = G_{\frac{1}{2}}|n=2\rangle = 0$,

Physical State Conditions

$$\frac{3}{2}\zeta_{1\mu} + 2\sqrt{2\alpha'}\zeta'_{2\nu\mu}k^{\nu} + \zeta_{3\nu\rho\mu}\eta^{\nu\rho} + \frac{1}{2}(\zeta'_{3\nu\mu\rho}\eta^{\nu\rho} + \zeta'_{3\mu\nu\rho}\eta^{\nu\rho}) = 0$$

$$\zeta_{1\mu} + \sqrt{2\alpha'}\zeta'_{2\mu\nu}k^{\nu} = 0, \qquad 2\zeta_{1\mu} + \sqrt{2\alpha'}\zeta_{2\nu\mu}k^{\nu} = 0$$

$$(\zeta_{2\mu\nu} + \sqrt{2\alpha'}\zeta_{3\mu\nu\rho}k^{\rho})\alpha^{\mu}_{-1}\alpha^{\nu}_{-1} = 0, \qquad (\zeta'_{3\mu\nu\rho} + \sqrt{2\alpha'}\zeta_{4\sigma\mu\nu\rho}k^{\sigma})\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}} = 0$$

$$\zeta_{2\mu\nu} + 2\zeta'_{2\mu\nu} + \sqrt{2\alpha'}(\zeta_{3\rho\mu\nu} + \zeta_{3\mu\rho\nu})k^{\rho} = 0$$

$$\sqrt{2\alpha'}\zeta_{1\mu}k^{\mu} + (\zeta_{2\mu\nu} + 2\zeta'_{2\mu\nu})\eta^{\mu\nu} = 0$$

$$\zeta_{1\mu} + \sqrt{2\alpha'}\zeta_{2\mu\nu}k^{\nu} + (\zeta_{3\mu\nu\rho} + \zeta_{3\nu\mu\rho})\eta^{\nu\rho} = 0$$

$$\zeta_{4\mu\nu\rho\sigma}(\eta^{\mu\nu}\psi^{\rho}_{-\frac{1}{2}}\psi^{\sigma}_{-\frac{1}{2}} - \eta^{\mu\rho}\psi^{\nu}_{-\frac{1}{2}}\psi^{\sigma}_{-\frac{1}{2}} + \eta^{\mu\sigma}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}}) + (2\zeta'_{2\mu\nu} + \sqrt{2\alpha'}\zeta'_{3\mu\nu\rho}k^{\rho})\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}} = 0$$

$$(\zeta_{4\mu\nu\rho\sigma} + 5\sqrt{2\alpha'}\zeta_{5\mu\nu\rho\sigma\gamma}k^{\gamma})\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}}\psi^{\sigma}_{-\frac{1}{2}} = 0$$

$$\left[\zeta_{2\mu\nu} - 2\zeta'_{2\nu\mu} + \sqrt{2\alpha'}(\zeta'_{3\rho\mu\nu}k^{\rho} - \zeta'_{3\mu\rho\nu}k^{\rho})\right]\psi^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{3}{2}} = 0$$

$$\sqrt{2\alpha'}\zeta_{4\mu\nu\rho\sigma}\alpha^{\mu}_{-1}(k^{\nu}\psi^{\rho}_{-\frac{1}{2}}\psi^{\sigma}_{-\frac{1}{2}} - \psi^{\nu}_{-\frac{1}{2}}k^{\rho}\psi^{\sigma}_{-\frac{1}{2}} + \psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}}k^{\sigma})$$
$$+\left[(\zeta_{3\mu\nu\rho} + \zeta_{3\nu\mu\rho}) + \zeta'_{3\nu\rho\mu}\right]\alpha^{\mu}_{-1}\psi^{\nu}_{-\frac{1}{2}}\psi^{\rho}_{-\frac{1}{2}} = 0$$

The universal bosonic string states

For full solutions, c.f., [WZF, Taylor, '11]. In the paper, we focus on the decay of the spin-3,2 universal bosonic states:

$$V^{(-1)}_{\sigma^a} = [T^a]^{\alpha_1}_{\alpha_2} \frac{g}{2\sqrt{\alpha'}} \sigma_{\mu\nu\rho} i\partial X^\mu i\partial X^\nu \psi^\rho \mathrm{e}^{-\phi} \mathrm{e}^{ikX} ,$$

$$V^{(-1)}_{\pi^a} = [T^a]^{\alpha_1}_{\alpha_2} \frac{g}{4\sqrt{3}} k^\lambda \varepsilon_{\lambda(\mu|\rho\gamma|} \pi^\gamma{}_{\nu)} (i\partial X^\mu i\partial X^\nu \psi^\rho - 4\alpha' \partial\psi^\mu \psi^\nu \psi^\rho) \mathrm{e}^{-\phi} \mathrm{e}^{ikX} ,$$

where $\sigma_{\mu\nu\rho}$ and $\pi_{\mu\nu}$ are symmetric, transverse and traceless.

CFT computation

n-point tree level string amplitudes are obtained by calculating the n-point correlation functions of associate vertex operators inserted on the boundary of the disk worldsheet, which read

$$\mathcal{A}^{(n)} = \sum V_{\text{CKG}}^{-1} \int (\prod_{i=1}^{n} \mathrm{d}z_i) \langle V(z_1) V(z_2) V(z_3) V(z_4) \cdots V(z_n) \rangle$$

$$= \sum \langle c_1 V(z_1) c_2 V(z_2) c_3 V(z_3) \int (\prod_{i=4}^{n} \mathrm{d}z_i) V(z_4) \cdots V(z_n) \rangle \, .$$

$\sigma^{(2)} \to \alpha^{(1)} + g^{(0)}$

To obtain the decay rate of the $\sigma \to \alpha + g$, we need to compute the 3-point amplitude

$$\mathcal{A}(\sigma_1, \alpha_2, \epsilon_3)$$
$$= \mathrm{Tr}(T^{a_1}\{T^{a_2}, T^{a_3}\}) \frac{2g}{\sqrt{\alpha'}} \sigma_{\mu\nu\rho} \Big\{ (2\alpha')^2 \big[k_3^\mu k_3^\nu \epsilon_3^\rho \alpha_{\gamma\zeta} k_3^\gamma k_3^\zeta - k_3^\mu k_3^\nu k_3^\rho \alpha_{\gamma\zeta} \epsilon_3^\gamma k_3^\zeta$$
$$- k_3^\mu k_3^\nu \alpha^{\rho\gamma} k_{3\gamma} (\epsilon_3 \cdot k_2) \big] + (2\alpha') \big[3 k_3^\mu k_3^\nu \alpha^{\rho\gamma} \epsilon_{3\gamma} - 4 k_3^\mu \epsilon_3^\nu \alpha^{\rho\gamma} k_{3\gamma}$$
$$+ 2 k_3^\mu \alpha^{\nu\rho} (\epsilon_3 \cdot k_2) \big] + 2 \alpha^{\mu\nu} \epsilon_3^\rho \Big\} .$$

Choice of reference momentums

Plugging in helicity wave functions, then sum over the squared amplitudes one could obtain the decay rate of the channel. However, the calculation would be very tedious: $7 \times 5 \times 2$ helicity amplitudes.

First we observe $\Gamma(\sigma_1 \rightarrow \alpha_2 + \epsilon_3^+) = \Gamma(\sigma_1 \rightarrow \alpha_2 + \epsilon_3^-)$ which would reduce the total number of the helicity amplitudes to compute by half.

Then we need to choose the reference momentums very clearly to simplify the computation – if we align the spin axes of all the scattering fields to one direction, we only need to compute very few helicity amplitudes, the others should vanish automatically because of the angular momentum conservation.

Choice of reference momentums

The most clever choose of reference momentums read

$$p_1^\mu = -r^\mu \,, \quad q_1^\mu = -2k_3^\mu \,, \quad p_2^\mu = r^\mu \,, \quad q_2^\mu = k_3^\mu \,.$$

It can be easily verified that

$$k_1 + k_2 + k_3 = p_1 + q_1 + p_2 + q_2 + k_3 = 0 \,,$$
$$(p_1 + q_1)^2 = 2(p_2 + q_2)^2 \,.$$

And we fix the reference momentum r as $r \cdot k_3 = -1/(2\alpha')$.

Results

With these choices, we compute the helicity amplitude of $\mathcal{A}(\sigma_1, \alpha_2, \epsilon_3^+)$. Only five survive, which read

$$\mathscr{A}\left[\sigma_1(-3), \alpha_2(+2), \epsilon_3^+\right] = \frac{8g}{\sqrt{\alpha'}} d_{a_1 a_2 a_3} ,$$

$$\mathscr{A}\left[\sigma_1(-2), \alpha_2(+1), \epsilon_3^+\right] = \frac{8g}{\sqrt{3\alpha'}} d_{a_1 a_2 a_3} ,$$

$$\mathscr{A}\left[\sigma_1(-1), \alpha_2(\ 0\ ), \epsilon_3^+\right] = \frac{4\sqrt{2}g}{\sqrt{5\alpha'}} d_{a_1 a_2 a_3} ,$$

$$\mathscr{A}\left[\sigma_1(\ 0\ ), \alpha_2(-1), \epsilon_3^+\right] = \frac{4g}{\sqrt{10\alpha'}} d_{a_1 a_2 a_3} ,$$

$$\mathscr{A}\left[\sigma_1(+1), \alpha_2(-2), \epsilon_3^+\right] = \frac{2g}{\sqrt{15\alpha'}} d_{a_1 a_2 a_3} ,$$

which match exactly as the results we obtained from factorization.

The results from factorization

$$\mathcal{A}[\alpha(+2),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\frac{16}{\sqrt{3}}d^3_{-3,-2}(\theta)\, f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{+2+a_1a_2} = F^{a,J=3}_{-2-a_1a_2} = 8gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(+1),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(\frac{16}{3}d^3_{-2,-2}(\theta) - \frac{16}{3}d^2_{-2,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{+1+a_1a_2} = F^{a,J=3}_{-1-a_1a_2} = \frac{8}{\sqrt{3}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{+1+a_1a_2} = F^{a,J=2}_{-1-a_1a_2} = 4\sqrt{\frac{2}{3}}gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(0),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(8\sqrt{\frac{2}{15}}d^3_{-1,-2}(\theta) - \frac{8}{\sqrt{3}}d^2_{-1,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{0+a_1a_2} = F^{a,J=3}_{0-a_1a_2} = 4\sqrt{\frac{2}{5}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{0+a_1a_2} = F^{a,J=2}_{0-a_1a_2} = 2\sqrt{2}gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(-1),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(4\sqrt{\frac{2}{15}}d^3_{0,-2}(\theta) - 4\sqrt{\frac{2}{3}}d^2_{0,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

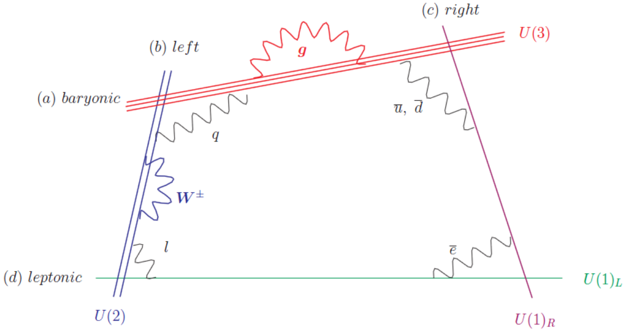
$$F^{a,J=3}_{-1+a_1a_2} = F^{a,J=3}_{+1-a_1a_2} = 2\sqrt{\frac{2}{5}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{-1+a_1a_2} = F^{a,J=2}_{+1-a_1a_2} = 2gM_s d^{a_1a_2a}\,.$$

$$\mathcal{A}[\alpha(-2),+,+,-] = \frac{g^2 M_s^2}{s-2M_s^2}\left(\frac{4}{3\sqrt{5}}d^3_{+1,-2}(\theta) - \frac{4\sqrt{2}}{3}d^2_{+1,-2}(\theta)\right) f^{a_1a_2a}d^{a_3a_4a}\,,$$

$$F^{a,J=3}_{-2+a_1a_2} = F^{a,J=3}_{+2-a_1a_2} = \frac{2}{\sqrt{15}}gM_s d^{a_1a_2a}\,,\quad F^{a,J=2}_{-2+a_1a_2} = F^{a,J=2}_{+2-a_1a_2} = \frac{2}{\sqrt{3}}gM_s d^{a_1a_2a}\,.$$

Model-dependent problems

Massive string states decay to anomalous $U(1)$'s



Intersection pattern of four stacks of D6-branes giving rise to the MSSM

The mass mixing effect

For a recent discussion on the mass mixing effect of the anomalous $U(1)$'s, see [WZF, Shiu, Soler, Ye, '14][2] and also Kumar and Soler's talks. Effectively (assuming no kinetic mixing) the Lagrangian for all the $U(1)$'s from an n-stack model can be written as

$$\mathcal{L} = -\frac{1}{4}\sum_a F_a^2 - \frac{1}{2}A_a M_{ab}^2 A_b + \sum_a \bar{\psi}_a (i\partial\!\!\!/ + g_a q_a A\!\!\!/_a)\psi_a \,,$$

where ψ_a denotes the matter fields charged under $U(1)_a$ $(a, b, \cdots$ label the stack of D-branes). The $U(1)$ mass-squared matrix is of the following form

$$M_{ab}^2 = g_a g_b K_{ai} G_{ij} K_{jb}^T \,,$$

where the integer-entry matrix K contains all the information of local model constructions.

Final Remarks

1. It's very interesting to study the universal amplitudes involving higher level massive string states.
2. The universal string states and their decay widths at mass level $n = 2$ are identified.
3. If the nature really has a low mass string scale, it could be tested at the LHC or future colliders.

Thank You!