Random Supergravity description of a heavy supersymmetric sector.

Work in collaboration with P. Ortiz

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Outline

- Effective Supergravity description of flux compactifications:

$$K = K(\phi, \bar{\phi}), \qquad W = N^a \Pi_a(\phi) + W_{np}$$

What are the stability properties of
dS vacua for a generic choice of fluxes?.

Random Supergravity approach

- Generic supergravity theory with a large number of fields, $\mathrm{N} \gg 1$.
- Couplings are treated as random variables (*Denef 04, 05*).

$$N^a \longrightarrow (W, D_I W, D_{IJ} W, D_{IJK} W)$$

- The Hessian is modelled using Random Matrix Theory
- The probability of a dS critical point being stable is exponentially suppressed: $\mathbb{P}_{min} \sim \mathrm{e}^{-\mathrm{N}^p}$ (*Marsh 12, Bachlechner 12*)

Introduction

Decoupling improves stability

The effective scalar potential in type-IIB

- Model independent minimum in the large volume limit (LVS).
  Conlon 05
  - Example: Compactification on $\mathbb{P}^4_{[1,1,1,6,9]}$: $h^{1,2} = 276$, $h^{1,1} = 2$.
- Numerical analysis of specific compactifications,
  e.g. *Sumitomo 12, 13*

$$\mathbb{T}^6 \text{ orientifold}: \qquad \mathbb{P}_{min} \sim 0.981, \qquad h^{2,1} = 5, \quad h^{1,1} = 1.$$

- Can we study the supersymmetric sector in isolation?
  - Do BF allowed tachyons lead to instabilities in the final dS vacuum?.
  - Is it sufficient/necessary to stabilise at SUSY minima?.

Covi 09, Parameswaran 10, Louis 12, Danielsson 13, Blaback 13

Supersymmetric sectors
Metaestability conditions

Necessary conditions for metaestability $\langle Z \mathcal{H} Z \rangle \geq 0$.

sGoldstino direction

$$\langle Z_{\pm X} \mathcal{H} \, Z_{\pm X} \rangle \geq 0 \quad \Longrightarrow \quad S[X] \geq -\frac{2}{3}\frac{1}{1+\gamma}$$

(Gomez-Reino 06-07, Covi 08-09)

Supersymmetric directions $\gamma > 0$

$$\langle Z_{\pm\lambda} \mathcal{H} \, Z_{\pm\lambda} \rangle \geq 0 \qquad (\geq \frac{9}{4}\gamma \text{ in AdS. } \textit{Breitenlohner 82})$$

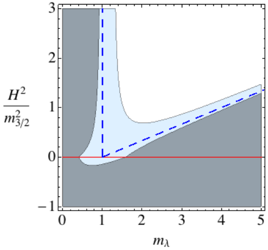
$$\mu^2_{\pm\lambda} = (m_\lambda \pm 1)(m_\lambda \pm (3\gamma+1)) \pm \sqrt{3(\gamma+1)}\,\frac{\partial m_\lambda}{\partial X} + 3(\gamma+1)\, B[X,\lambda] \geq 0,$$

$$\mathcal{M} Z_{\pm\lambda} = \pm m_\lambda \, Z_{\pm\lambda}, \qquad \gamma = \frac{H^2}{m^2_{3/2}}, \qquad S[X] = -R_{X\bar{X}X\bar{X}} \qquad B[X,\lambda] = -R_{X\bar{X}\lambda\bar{\lambda}}$$

Metaestability conditions

Perturbative stability of de Sitter vacua

$$\mu^2_{\pm\lambda} = (m_\lambda \pm 1)\big(m_\lambda \pm (3\gamma+1)\big) \pm \sqrt{3(\gamma+1)}\,\frac{\partial m_\lambda}{\partial X} + 3(\gamma+1)\,B[X,\lambda] \geq 0,$$



- In the SUSY limit $\gamma = -1$, these conditions become sufficient.
- Requiring the supersymmetric sector to be stabilised at a SUSY minimum, $m_\lambda > 2$, (*Denef 05*), is not sufficient in general.
- There are physically relevant situations where it is not necessary (LVS).
- Supersymmetric AdS maxima, $m_\lambda < 1$, appear to have particularly good stability properties for $\gamma \to \infty$.

Supersymmetric truncations
Definition and Properties

(Binetruy 04, Achucarro 08-09, Gallego 08-09, Brizi 09, KS 12)

Consistent supersymmetric truncations

- The field content is split in two sectors $\phi^I = (H^\alpha, L^i)$
- the heavy sector is frozen at $H^\alpha = H_0^\alpha$.
  - The solutions of the reduced theory are exact solutions of the full theory.
  - Supersymmetry is exactly preserved by the truncation.

- The Kähler function $G = K + \log|W|^2$ satisfies

$$G_\alpha(H, \bar{H}, L, \bar{L})|_{H_0} = 0 \qquad \text{for all } L^i.$$

  - The reduced Kähler manifold is totally geodesic
  - All the matrices involved in the Hessian are block diagonal:

$$\mathcal{H} = \mathcal{H}^{(h)} \otimes \mathcal{H}^{(l)} \qquad \mathcal{M} = \mathcal{M}^{(h)} \otimes \mathcal{M}^{(l)}$$

Supersymmetric truncations
Explicit examples

Type-IIB flux compactifications to leading order in α' and g_s

- The Kähler function is separable

$$G = G^{(h)}(H, \bar{H}) + G^{(l)}(L, \bar{L}), \qquad G^{(h)}_{\alpha}|_{H_0} = 0$$

$$\begin{aligned} G^{(h)} &= K_d(s, \bar{s}) + K_{c.s}(z^{\alpha}, \bar{z}^{\alpha}) + \log|W_{flux}(s, z^{\alpha})| \\ G^{(l)} &= K_k(t^i, \bar{t}^i). \end{aligned}$$

LVS of Type-IIB flux compactifications

- The Kähler function is small deformation of the tree-level one

$$G = G^{(h)}(H, \bar{H}) + G^{(l)}(L, \bar{L}) + \epsilon\, G^{mix}(H, \bar{H}, L, \bar{L}),$$

- with $\epsilon \sim 1/\mathcal{V}^p$ (Gallego 11)

Random couplings

- The couplings are described by a single distribution *(Denef 04, Marsh 12)*

$$\nabla_\alpha G_\beta \in \Omega(0,\sigma) \quad \text{if} \quad \alpha < \beta, \quad \text{and} \quad \nabla_\alpha G_\alpha \in \Omega(0,\sqrt{2}\sigma).$$

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Random Matrix Theory

- $\mathcal{M}_h$ can be modelled as an element of the $CI-$ensemble (*Altland 97*).
- The typical spectral density to leading order $\mathcal{O}(1/\mathrm{N})$ is

$$\rho(m) = \frac{4\,\mathrm{N}_h}{\pi\sigma_\chi^2}\sqrt{\sigma_\chi^2 - m^2}$$

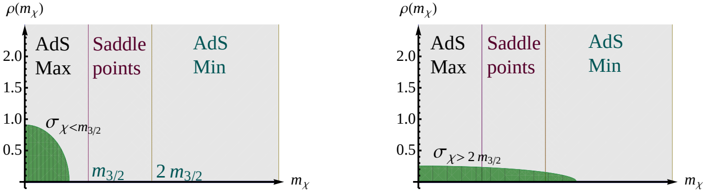
- Atypical spectra are possible at the cost of an exponentially suppressed probability $\mathbb{P} \sim \mathrm{e}^{-\mathrm{N}_h^p}$. (*Tracy 94*)

Supersymmetric AdS vacua

Mass spectrum of the fermions

Density function of fermion masses

$$\rho(m)dm = \frac{4\,\mathrm{N}_h}{\pi\sigma_\chi^2}\sqrt{\sigma_\chi^2 - m^2}dm.$$



- There is no finite value for σ_χ where $\mathbb{P}_{min} \sim \mathcal{O}(1)$.
- It is still possible to find configurations where all the fermion masses satisfy the constraint $m_\chi > 2m_{3/2}$,
- but their probability is exponentially suppressed, (*Bachlechner 12*)

$$P \sim \mathrm{e}^{-N^2}$$

de Sitter configurations

Separable Kähler functions

Type-IIB flux compactifications to leading order in α' and g_s

- Take a separable Kahler function

$$G(H, \bar{H}, L, \bar{L}) = G_h(H, \bar{H}) + G_l(L, \bar{L}), \qquad \text{with} \qquad \partial_\alpha G_h|_{H_0} = 0.$$

- The Hessian is diagonalised in the same basis as $\mathcal{M}$

$$\mathcal{H}_h = (\mathcal{M} + \mathbb{1})(\mathcal{M} + (3\gamma + 1)\mathbb{1}),$$

- The metaestability conditions are both necessary and sufficient

$$\mu^2_{\pm\lambda} = (m_\lambda \pm 1)(m_\lambda \pm (3\gamma + 1)) \geq 0$$

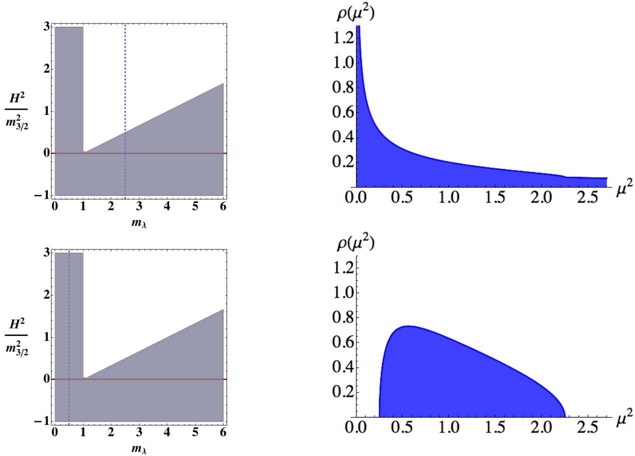
- We calculate the spectrum of masses of the scalar fields combinig this result with the spectral density of the fermions

$$\rho(\mu^2) d\mu^2 = \left[\rho(m_+^2) \left| \frac{dm_+^2}{d\mu^2} \right| + \rho(m_-^2) \left| \frac{dm_-^2}{d\mu^2} \right| \right] d\mu^2 \ .$$

de Sitter configurations

Separable Kähler functions.

$$\gamma = 0, \qquad \sigma_\chi = 2.5 \ \ (\text{Top}), \quad \sigma_\chi = 0.5 \ \ (\text{Bottom})$$



- The spectrum develops a gap for $\sigma_\chi > 1$: $\mu^2|_{min} = (\sigma_\chi - 1)^2$,
- and zero-modes are suppressed with probability e^{-N_h}.

de Sitter configurations

Quasi-separable Kähler functions: Large Volume Scenarios

- Take a quasi-separable Kähler function $\epsilon \ll 1$

$$G(H,\bar{H},L,\bar{L}) = G^{(h)}(H,\bar{H}) + G^{(l)}(L,\bar{L}) + \epsilon\, G_{int}(H,\bar{H},L,\bar{L}) \ ,$$

- The parameters $\mu^2_{\pm\lambda}$ coincide with the eigenvalues of $\mathcal{H}$ to first order in perturbation theory, $|\nabla_X \mathcal{M}|, \mathcal{R} \sim \mathcal{O}(\epsilon)$

$$\mathcal{H}_h = (\mathcal{M} + \mathbb{1})(\mathcal{M} + (3\gamma+1)\mathbb{1}) + \sqrt{3(\gamma+1)}\,\nabla_X \mathcal{M} - 3(\gamma+1)\mathcal{R}.$$

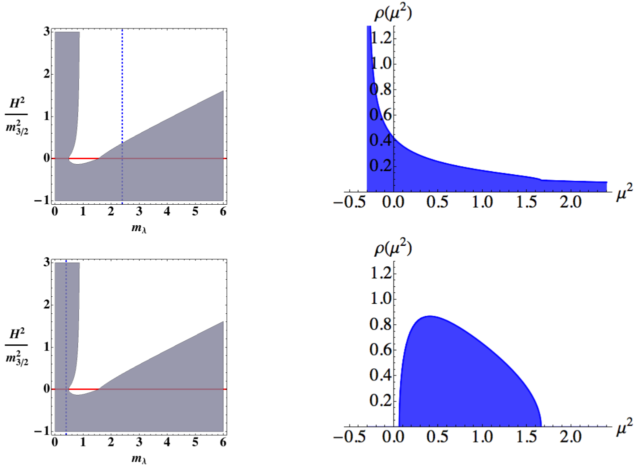
- The metaestability conditions are both necessary and sufficient

$$\mu^2_{\pm\lambda} = (m_\lambda \pm 1)(m_\lambda \pm (3\gamma+1)) \pm \sqrt{3(\gamma+1)}\,\frac{\partial m_\lambda}{\partial X} + 3(\gamma+1)\,B[X,\lambda] \geq 0$$

de Sitter configurations

Quasi-separable Kähler functions: Large Volume Scenarios

$\gamma = 0, \quad B[X,\lambda] = -0.1 \quad \sigma_\chi = 2.5$ (Top), $\sigma_\chi = 0.5$ (Bottom)

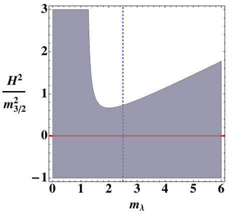


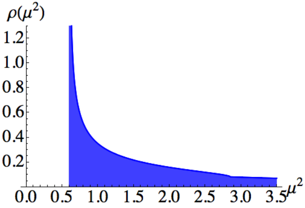
- The spectrum preserves the gap for $\sigma_\chi > 1$,
- and tachyons are suppressed with probability e^{-N_h}.

de Sitter configurations

Quasi-separable Kähler functions: Large Volume Scenarios

$$\gamma = 0, \qquad B[X,\lambda] = 0.2 \qquad \sigma_\chi = 2.5$$



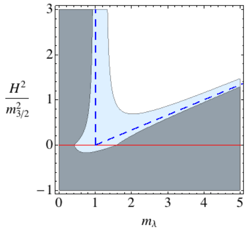


- Typical spectra are tachyon free-regardless of the choice of σ_χ, $\mathbb{P}_{min} \sim \mathcal{O}(1)$.
- In general stable configurations correspond to SUSY AdS maxima and saddle points in isolation $\gamma = -1$
  - Checked on the $\mathbb{P}^4_{[1,1,1,6,9]}$ model. *Blanco-Pillado 14 (to appear)*.
- The couplings between the complex structure and Kähler sectors turn all the BF allowed tachyons into stable directions of the potential.

Constraints on dS vacua and inflation

Stability of the inflationary trajectory

- These results can be extended to study dS vacua and inflationary models.
- They can be applied when the supersymmetric sector acts as a spectator both in supersymmetry breaking and inflation.



Stability	Suppressed fluctuations
$B[X,\lambda]_{max} \geq -\frac{m_1^2+3\gamma+1}{3(\gamma+1)}$	$B[X,\lambda]_{max} \geq -\frac{m_1^2+2\gamma+1}{3(\gamma+1)}$

Summary

Decoupling supersymmetric sectors

- We have derived a set of necessary conditions for the stability of de Sitter configurations (both vacua and inflation).
- These conditions become sufficient when studying c.s.m in LVS,
- where stabilising the heavy sector at a supersymmetric minimum is not necessary (neither sufficient) to guarantee the stability of the dS.

Summary

Decoupling supersymmetric sectors

- We have derived a set of necessary conditions for the stability of de Sitter configurations (both vacua and inflation).
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- where stabilising the heavy sector at a supersymmetric minimum is not necessary (neither sufficient) to guarantee the stability of the dS.

RandomSUGRA description of consistently truncated supersymmetric sectors

- We have analised the generic spectrum of masses for a heavy supersymmetric sector
- In particular we study lagrangians with the same structure of LVS.
- We have characterised the regions of parameter space with, $\mathbb{P}_{min} \sim \mathcal{O}(1)$.
- Configurations minimising the $m_{3/2}$ $(\mathrm{e}^K|W|^2)$ are a particularly robust type of minima, specially for $H \gg m_{3/2}$.