Towards new non-geometric backgrounds

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this talk is based on ...

This talk is based on the paper T-duality revisited [arXiv:1310.4194], and on some work in progress [arXiv:1407.xxxx].

Moduli stabilization is one of the import tasks in string phenomenology.

Non-geometric fluxes can help with that, as they contribute to the superpotential

$$W = \int_{\mathcal{X}} \Omega_3 \wedge \Big(F_3 - i\,S\,H_3 + \mathcal{Q}\cdot(J + i\,B) \Big)\,.$$

Shelton, Taylor, Wecht - 2005

These fluxes are understood

- moderately-well in the effective theory (supergravity in *d=4*),
- but their string-theory origin is much less clear.

motivation

One example in string-theory is given by applying **T-duality** to tori with H-flux.

three-torus with H-flux	T-duality →	geometric background
	T-duality →	non-geometric background
	T-duality →	locally non-geometric background

Dasgupta, Rajesh, Sethi - 1999
Kachru, Schulz, Tripathy, Trivedi - 2002
Hellermann, McGreevy, Williams - 2002
Dabholkar, Hull - 2002
Hull - 2004
Bouwknegt, Hannabuss, Mathai - 2004
Shelton, Taylor, Wecht - 2005

Blumenhagen, EP - 2010
Lüst - 2010
...

motivation

One example in string-theory is given by applying T-duality to tori with H-flux.

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But, are there also other examples for non-geometric backgrounds?

Goal ::

- Construct new non-geometric backgrounds via T-dualities

$$H_{abc} \xleftrightarrow{T_c} f_{ab}{}^{c} \xleftrightarrow{T_b} Q_{a}{}^{bc} \xleftrightarrow{T_a} R^{abc}$$

Idea ::

- Consider the three-sphere.

outline

1. motivation
2. collective t-duality
3. three-torus
4. three-sphere
5. summary

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To study T-duality for three-spheres, a non-abelian version might be needed.

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de la Ossa, Quevedo - 1992
Giveon, Rocek - 1993
Alvarez, Alvarez-Gaume, [Barbon,] Lozano - 1993 & 1994

Consider the sigma-model action for the NS-NS sector of the closed string

$$\mathcal{S} = -\frac{1}{4\pi\alpha'}\int_{\partial\Sigma}\Big[G_{ij}\,dX^i\wedge\star dX^j + \alpha' R\,\phi\star 1\Big] - \frac{i}{2\pi\alpha'}\int_{\Sigma}\tfrac{1}{3!}\,H_{ijk}\,dX^i\wedge dX^j\wedge dX^k\,.$$

This action is invariant under global transformations $\delta_\epsilon X^i = \epsilon^\alpha\, k^i_\alpha(X)$ if

$$\mathcal{L}_{k_\alpha}G = 0\,, \qquad \iota_{k_\alpha}H = dv_\alpha\,, \qquad \mathcal{L}_{k_\alpha}\phi = 0\,.$$

In general, the isometry algebra is non-abelian $[k_\alpha, k_\beta]_{\mathrm{L}} = f_{\alpha\beta}{}^{\gamma}\,k_\gamma\,.$

Following Buscher's procedure, the gauged sigma-model action is found as

$$\begin{aligned}\widehat{\mathcal{S}} = &-\frac{1}{2\pi\alpha'}\int_{\partial\Sigma} \tfrac{1}{2}\,G_{ij}(dX^i + k^i_\alpha A^\alpha)\wedge\star(dX^j + k^j_\beta A^\beta)\\ &-\frac{i}{2\pi\alpha'}\int_{\Sigma} \tfrac{1}{3!}\,H_{ijk}\,dX^i\wedge dX^j\wedge dX^k\\ &-\frac{i}{2\pi\alpha'}\int_{\partial\Sigma}\Big[\,(v_\alpha + d\chi_\alpha)\wedge A^\alpha + \tfrac{1}{2}\big(\iota_{k_{[\underline{\alpha}}}v_{\underline{\beta}]} + f_{\alpha\beta}{}^\gamma\chi_\gamma\big)\,A^\alpha\wedge A^\beta\,\Big]\,.\end{aligned}$$

Hull, Spence - 1989 & 1991
Alvarez, Alvarez-Gaume, Barbon, Lozano - 1994

This gauging is subject to the following constraints

$$\mathcal{L}_{k_{[\underline{\alpha}}}v_{\underline{\beta}]} = f_{\alpha\beta}{}^\gamma v_\gamma\,, \qquad\qquad \iota_{k_{[\underline{\alpha}}}f_{\underline{\beta\gamma}]}{}^\delta v_\delta = \frac{1}{3}\,\iota_{k_\alpha}\iota_{k_\beta}\iota_{k_\gamma}H\,.$$

EP - 2014

t-duality :: recovering the original model

The original model is recovered via the equations of motion for χ_α

$$0 = dA^\alpha - \tfrac{1}{2} f_{\beta\gamma}{}^{\alpha} A^\beta \wedge A^\gamma \, .$$

The gauge action can then be rewritten in terms of $DX^i = dX^i + k^i_\alpha A^\alpha$ as

$$\begin{aligned}\widehat{\mathcal{S}} = &- \frac{1}{4\pi\alpha'} \int_{\partial\Sigma} \Big[G_{ij} \, DX^i \wedge \star DX^j + \alpha' R \, \phi \star 1 \Big] \\ &- \frac{i}{2\pi\alpha'} \int_{\Sigma} \tfrac{1}{3!} \, H_{ijk} \, DX^i \wedge DX^j \wedge DX^k \, .\end{aligned}$$

Ignoring technical details, one replaces $DX^i \to dY^i$ and obtains the ungauged action.

Note :: the following two slides are somewhat technical.

t-duality :: obtaining the dual model I

The dual model is obtained via the equations of motion for A^α

$$A^\alpha = -\Big(\big[\mathcal{G} - \mathcal{D}\,\mathcal{G}^{-1}\mathcal{D}\big]^{-1}\Big)^{\alpha\beta}\Big(\mathbb{1} + i\star\mathcal{D}\,\mathcal{G}^{-1}\Big)_\beta{}^\gamma\big(k + i\star\xi\big)_\gamma\,,$$

where

$$\mathcal{G}_{\alpha\beta} = k^i_\alpha G_{ij} k^j_\beta\,, \qquad \xi_\alpha = d\chi_\alpha + v_\alpha\,,$$

$$\mathcal{D}_{\alpha\beta} = \iota_{k_{[\underline{\alpha}}} v_{\underline{\beta}]} + f_{\alpha\beta}{}^\gamma \chi_\gamma\,, \qquad k_\alpha = k^i_\alpha G_{ij} dX^j\,.$$

The action of the dual sigma-model is found by integrating out A^α and reads

$$\check{\mathcal{S}} = -\frac{1}{4\pi\alpha'}\int_{\partial\Sigma}\Big[\check{G} + \alpha' R\,\phi\star 1\Big] - \frac{i}{2\pi\alpha'}\int_\Sigma \check{H}\,,$$

where, with $\mathcal{M} = \mathcal{G} - \mathcal{D}\mathcal{G}^{-1}\mathcal{D}$,

$$\check{G} = G + \begin{pmatrix} k \\ \xi \end{pmatrix}^T \begin{pmatrix} -\mathcal{M}^{-1} & -\mathcal{M}^{-1}\mathcal{D}\mathcal{G}^{-1} \\ +\mathcal{M}^{-1}\mathcal{D}\mathcal{G}^{-1} & +\mathcal{M}^{-1} \end{pmatrix} \wedge\star\begin{pmatrix} k \\ \xi \end{pmatrix},$$

$$\check{H} = H + \tfrac{1}{2}\,d\left[\begin{pmatrix} k \\ \xi \end{pmatrix}^T \begin{pmatrix} +\mathcal{M}^{-1}\mathcal{D}\mathcal{G}^{-1} & +\mathcal{M}^{-1} \\ -\mathcal{M}^{-1} & -\mathcal{M}^{-1}\mathcal{D}\mathcal{G}^{-1} \end{pmatrix} \wedge \begin{pmatrix} k \\ \xi \end{pmatrix}\right].$$

t-duality :: obtaining the dual model II

An enlarged target-space can be parametrized by coordinates X^i and χ_α.

The enlarged metric $\check{G}$ and field strength $\check{H}$ have null-eigenvectors (and isometries)

$$\iota_{\check{n}_\alpha}\check{G} = 0\,, \qquad \iota_{\check{n}_\alpha}\check{H} = 0\,, \qquad \check{n}_\alpha = k_\alpha + \mathcal{D}_{\alpha\beta}\,\partial_{\xi_\beta}\,.$$

The dual metric and field strength are obtained via a change of coordinates

$$\mathcal{T}^I{}_A = \begin{pmatrix} k & 0 \\ \mathcal{D} & \mathbb{1} \end{pmatrix}, \qquad \check{\mathsf{G}}_{AB} = (\mathcal{T}^T\check{G}\,\mathcal{T})_{AB} = \begin{pmatrix} 0 & 0 \\ 0 & \mathsf{G}_{\alpha\beta} \end{pmatrix},$$

$$\check{\mathsf{H}}_{ABC} = \check{H}_{IJK}\,\mathcal{T}^I{}_A\,\mathcal{T}^J{}_B\,\mathcal{T}^K{}_C\,,$$

$$\check{\mathsf{H}}_{iBC} = 0\,.$$

The T-duality transformation rules are obtained via Buscher's procedure of

1. gauging isometries in the sigma-model action,
2. integrating-out the gauge field,
3. performing a change of coordinates.

The possible gaugings are restricted by (recall that $\iota_{k_\alpha} H = dv_\alpha$)

$$\mathcal{L}_{k_{[\underline{\alpha}}} v_{\underline{\beta}]} = f_{\alpha\beta}{}^{\gamma} v_\gamma \,, \qquad\qquad \iota_{k_{[\underline{\alpha}}} f_{\underline{\beta\gamma}]}{}^{\delta} v_\delta = \frac{1}{3}\, \iota_{k_\alpha} \iota_{k_\beta} \iota_{k_\gamma} H \,.$$

outline

1. motivation
2. collective t-duality
3. **three-torus**
4. three-sphere
5. summary

Consider a three-torus with *H*-flux specified as follows

$$ds^2 = R_1^2 \left(dX^1\right)^2 + R_2^2 \left(dX^2\right)^2 + R_3^2 \left(dX^3\right)^2 , \qquad X^i \simeq X^i + \ell_{\rm s} ,$$

$$H = h\, dX^1 \wedge dX^2 \wedge dX^3 , \qquad h \in \ell_{\rm s}^{-1}\mathbb{Z} .$$

The Killing vectors (in the basis $\{\partial_1, \partial_2, \partial_3\}$) are abelian and can be chosen as

$$k_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad k_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad k_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Consider one T-duality along the Killing vector $k_1 = \partial_1$.

The constraints for gauging are trivially satisfied.

The dual background is a twisted torus specified by

$$\check{ds}^2 = \frac{1}{R_1^2}\left(d\chi + h\,X^2\,dX^3\right)^2 + R_2^2\left(dX^2\right)^2 + R_3^2\left(dX^3\right)^2\,,$$

$$\check{H} = 0\,.$$

torus :: one t-duality

Consider one T-duality along the Killing vector $k_1 =$

$$\mathcal{L}_{k_{[\underline{\alpha}}} v_{\underline{\beta}]} = f_{\alpha\beta}{}^{\gamma} v_{\gamma}$$

$$\iota_{k_{[\underline{\alpha}}} f_{\underline{\beta\gamma}]}{}^{\delta} v_{\delta} = \tfrac{1}{3}\, \iota_{k_\alpha} \iota_{k_\beta} \iota_{k_\gamma} H$$

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$$\check{H} = 0\,.$$

Consider two collective T-dualities along $k_1 = \partial_1$ and $k_2 = \partial_2$.

The constraints on gauging the sigma-model imply (for $\alpha \in \mathbb{R}$)

$$v_1 = h\,\alpha\,X^2\,dX^3 - h\,(1-\alpha)\,X^3\,dX^2\,,$$
$$v_2 = h\,(1+\alpha)\,X^3\,dX^1 + h\,\alpha\,X^1\,dX^3\,.$$

The dual model is the T-fold background (no ambiguities in the collective approach)

$$\check{\mathrm{ds}}^2 = \frac{1}{R_1^2 R_2^2 + \left[h\,X^3\right]^2}\left[R_1^2\,\left(d\tilde{\chi}_1\right)^2 + R_2^2\,\left(d\tilde{\chi}_2\right)^2\right] + R_3^2\,\left(dX^3\right)^2\,,$$

$$\check{\mathrm{H}} = -h\,\frac{R_1^2 R_2^2 - \left[h\,X^3\right]^2}{\left[R_1^2 R_2^2 + \left[h\,X^3\right]^2\right]^2}\,d\tilde{\chi}_1 \wedge d\tilde{\chi}_2 \wedge dX^3\,.$$

Finally, consider three collective T-dualities along $k_1 = \partial_1, \; k_2 = \partial_2$ and $k_3 = \partial_3$.

The constraints on gauging the sigma-model require the *H*-flux to be vanishing

$$\iota_{k_\alpha}\iota_{k_\beta}\iota_{k_\gamma}H = 0 \qquad \longrightarrow \qquad H = 0\,.$$

The dual model is, as expected, characterized by

$$\check{ds}^2 = \frac{1}{R_1^2}\left(d\chi_1\right)^2 + \frac{1}{R_2^2}\left(d\chi_2\right)^2 + \frac{1}{R_3^2}\left(d\chi_3\right)^2,$$

$$\check{H} = 0\,.$$

The formalism for T-duality introduced above works as expected.

three-torus with *H*-flux	1 T-duality →	twisted torus
	2 T-dualities →	T-fold
three-torus with *H=0*	3 T-dualities →	torus with $R \to 1/R$

outline

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Consider a three-sphere with *H*-flux, specified by

$$ds^2 = R^2 \left[\sin^2\eta \,(d\zeta_1)^2 + \cos^2\eta\,(d\zeta_2)^2 + (d\eta)^2 \right], \qquad \zeta_{1,2} = 0 \ldots 2\pi\,,$$

$$H = \frac{h}{2\pi^2} \sin\eta\cos\eta\, d\zeta_1 \wedge d\zeta_2 \wedge d\eta\,, \qquad \eta = 0 \ldots \frac{\pi}{2}\,.$$

This model is conformal if $h = 4\pi^2 R^2$.

The isometry algebra is $\mathfrak{so}(4) = \mathfrak{su}(2) \times \mathfrak{su}(2)$, and the Killing vectors satisfy (with $\alpha, \beta, \gamma \in \{1,2,3\}$)

$$[\mathsf{K}_\alpha, \mathsf{K}_\beta]_{\mathrm{L}} = \epsilon_{\alpha\beta}{}^{\gamma}\, \mathsf{K}_\gamma\,,$$

$$[\mathsf{K}_\alpha, \tilde{\mathsf{K}}_\beta]_{\mathrm{L}} = 0\,, \qquad |\mathsf{K}_\alpha|^2 = |\tilde{\mathsf{K}}_\alpha|^2 = \frac{R^2}{4}\,.$$

$$[\tilde{\mathsf{K}}_\alpha, \tilde{\mathsf{K}}_\beta]_{\mathrm{L}} = \epsilon_{\alpha\beta}{}^{\gamma}\, \tilde{\mathsf{K}}_\gamma\,,$$

sphere :: setting

Consider a three-sphere with H-flux, specified by

2π ,

$\frac{\pi}{2}$.

This mod

$$\mathsf{K}_1 = \frac{1}{2}\begin{pmatrix} +1 \\ -1 \\ 0 \end{pmatrix}, \qquad \tilde{\mathsf{K}}_1 = \frac{1}{2}\begin{pmatrix} +1 \\ +1 \\ 0 \end{pmatrix},$$

$$\mathsf{K}_2 = \frac{1}{2}\begin{pmatrix} -\sin(\zeta_1-\zeta_2)\cot\eta \\ -\sin(\zeta_1-\zeta_2)\tan\eta \\ \cos(\zeta_1-\zeta_2) \end{pmatrix}, \qquad \tilde{\mathsf{K}}_2 = \frac{1}{2}\begin{pmatrix} +\sin(\zeta_1+\zeta_2)\cot\eta \\ -\sin(\zeta_1+\zeta_2)\tan\eta \\ -\cos(\zeta_1+\zeta_2) \end{pmatrix},$$

$$\mathsf{K}_3 = \frac{1}{2}\begin{pmatrix} -\cos(\zeta_1-\zeta_2)\cot\eta \\ -\cos(\zeta_1-\zeta_2)\tan\eta \\ -\sin(\zeta_1-\zeta_2) \end{pmatrix}, \qquad \tilde{\mathsf{K}}_3 = \frac{1}{2}\begin{pmatrix} +\cos(\zeta_1+\zeta_2)\cot\eta \\ -\cos(\zeta_1+\zeta_2)\tan\eta \\ +\sin(\zeta_1+\zeta_2) \end{pmatrix}.$$

The isom

(with $\alpha, \beta, \gamma \in \{1,2,3\}$)

$$[\mathsf{K}_\alpha, \mathsf{K}_\beta]_{\mathrm{L}} = \epsilon_{\alpha\beta}{}^{\gamma}\,\mathsf{K}_\gamma\,,$$

$$[\mathsf{K}_\alpha, \tilde{\mathsf{K}}_\beta]_{\mathrm{L}} = 0\,,$$

$$[\tilde{\mathsf{K}}_\alpha, \tilde{\mathsf{K}}_\beta]_{\mathrm{L}} = \epsilon_{\alpha\beta}{}^{\gamma}\,\tilde{\mathsf{K}}_\gamma\,,$$

$$|\mathsf{K}_\alpha|^2 = |\tilde{\mathsf{K}}_\alpha|^2 = \frac{R^2}{4}\,.$$

Consider a three-sphere with *H*-flux, specified by

$$ds^2 = R^2\left[\sin^2\eta\,(d\zeta_1)^2 + \cos^2\eta\,(d\zeta_2)^2 + (d\eta)^2\right], \qquad \zeta_{1,2} = 0\ldots 2\pi\,,$$

$$H = \frac{h}{2\pi^2}\sin\eta\cos\eta\, d\zeta_1\wedge d\zeta_2\wedge d\eta\,, \qquad \eta = 0\ldots\frac{\pi}{2}\,.$$

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$$[\mathsf{K}_\alpha,\mathsf{K}_\beta]_{\mathrm{L}} = \epsilon_{\alpha\beta}{}^{\gamma}\,\mathsf{K}_\gamma\,,$$

$$[\mathsf{K}_\alpha,\tilde{\mathsf{K}}_\beta]_{\mathrm{L}} = 0\,, \qquad |\mathsf{K}_\alpha|^2 = |\tilde{\mathsf{K}}_\alpha|^2 = \frac{R^2}{4}\,.$$

$$[\tilde{\mathsf{K}}_\alpha,\tilde{\mathsf{K}}_\beta]_{\mathrm{L}} = \epsilon_{\alpha\beta}{}^{\gamma}\,\tilde{\mathsf{K}}_\gamma\,,$$

Consider one T-duality along K_1. In this case, all constraints are satisfied:

- constraints from gauging the sigma-model ✓
- the matrix $\mathcal{G}_{\alpha\beta} = k^i_\alpha \, G_{ij} \, k^j_\beta$ is invertible ✓

The dual model, obtained via the above formalism, is characterized by

$$\check{\mathsf{G}} = \frac{R^2}{4}\left[(d\tilde{\eta})^2 + \sin^2(\tilde{\eta})(d\tilde{\zeta})^2\right] + \frac{4}{R^2}\,\xi \wedge \star\xi\,,$$

$$\check{\mathsf{H}} = \sin\tilde{\eta}\; d\tilde{\zeta} \wedge d\tilde{\eta} \wedge \xi\,,$$

$$d\xi = -\frac{h}{16\pi^2}\,\sin\tilde{\eta}\, d\tilde{\eta} \wedge d\tilde{\zeta}\,.$$

This metric describes a circle fibered over a two-sphere.

Bouwknegt, Evslin, Mathai - 2003

For two collective T-dualities, consider the commuting Killing vectors K_1 and $\tilde{\mathsf{K}}_1$.

The constraints for this model are almost satisfied:

- constraints from gauging the sigma-model ✓
- the matrix $\mathcal{G}_{\alpha\beta} = k^i_\alpha \, G_{ij} \, k^j_\beta$ is invertible ✗ $\det \mathcal{G} = \frac{R^4}{16} \sin^2(2\eta)$

The dual model, via the above formalism, takes a form similar to the T-fold

$$\check{\mathsf{G}} = R^2 \, (d\eta)^2 + \frac{1}{R^2} \, \frac{(d\tilde{\chi}_1)^2}{\sin^2 \eta + \left[\frac{h}{4\pi^2 R^2}\right]^2 \cos^2 \eta} + \frac{1}{R^2} \, \frac{(d\tilde{\chi}_2)^2}{\cos^2 \eta + \left(\frac{h}{4\pi^2 R^2}\right)^2 \frac{\cos^4 \eta}{\sin^2 \eta}} \,,$$

$$\check{\mathsf{H}} = -8 \, h \, \pi^2 \big(h^2 - 16\pi^4 R^4\big) \, \frac{\sin \eta \cos \eta}{\big[16\pi^2 R^4 \sin^2 \eta + h^2 \cos^2 \eta\big]^2} \, d\eta \wedge d\tilde{\chi}_1 \wedge d\tilde{\chi}_2 \,.$$

But, when starting from a conformal model with $h = 4\pi^2 R^2$, the background becomes

$$\overline{\mathsf{G}} = R^2 \, (d\eta)^2 + \frac{1}{R^2} \left[(d\tilde{\chi}_1)^2 + \tan^2 \eta \, (d\tilde{\chi}_2)^2 \right],$$

$$\overline{\mathsf{H}} = 0 \, .$$

With dual dilaton $\overline{\phi} = -\log\bigl(R^2 \cos\eta\bigr) + \phi$, this is again a conformal model.

Despite being non-compact, this background appears to be geometric.

sphere :: three t-dualities

For a non-abelian T-duality along K_1, K_2 and K_3, the constraints imply *H*=0.

Determining the dual model is still work in progress …

sphere :: summary

In the formalism for T-duality introduced above, for a conformal model one finds:

three-sphere with H-flux —1 T-duality→ S^1 fibered over S^2

—2 T-dualities→ seemingly geometric

three-sphere with $H=0$ —3 T-dualities→ work in progress

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summary

T-duality can be performed conveniently via an enlarged target space formalism

→ reproduces known results.

For two collective T-duality transformations,

three-torus with H-flux (not conformal)	2 T-dualities →	non-geometric space
three-sphere with H-flux (conformal)	2 T-dualities →	geometric space

Thus, the origin of non-geometry remains unclear …