

Landau-Ginzburg Orbifolds and their Symmetries

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Motivation

The Landau-Ginzburg Orbifold

'The internal manifold is just a point'

- Quasi exact solvable, flow to exact CFT's in the IR
'The lagrangian description of minimal model CFTs'
- Describe consistent heterotic compactifications
- Instantons are switched off
- Duals of Calabi-Yau compactifications

Interesting playgrounds to study

① The heterotic moduli space [Gepner,Aspinwall,Melnikov,Plesser...]

② Model building [Font,Ibanez,Quevedo,Schellekens,Blumenhagen,Weigand...]

③ Mirror symmetry

[Greene,Plesser,Adams,Basu,Lerche,Warner,Candelas,Kreutzer,Sethi,Schimmrigk,Skarke,Blumenhagen,Wisskirchen,Flohr, Rahn..]

④ Engineer hybrid CFT's [Bertolini, Melnikov, Plesser '13]

⑤ Selection rules in heterotic compactifications [Gepner,Chun, Mas, Lauer,

Nilles,Aspinwall,Plesser,Diestler,Kachru...]

Outline

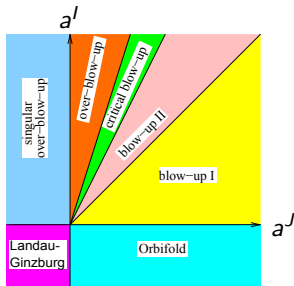
- Motivation
- The Landau Ginzburg Spectrum
- Classification of A_1^9 Gepner models
- Matching LGO symmetries with geometric Dual
- Summary and conclusions

Phases of Gauged Linear Sigma Models

Consider **two dimensional field theory** that exhibits

- (2,2) with chiral multiplets $\Phi^i = \phi^i + \theta_\alpha \gamma^{i,\alpha} + \theta^2 F^i$
- Exhibits a $U(1)_L \times U(1)_R$ R-symmetry and additional gauged U(1)'s [Witten'93]
- The U(1) charges **constraint** by **2D anomalies**
 → **CY condition** for target space manifold [See Hans Jockers talk]

$$\mathcal{L} = \int d\theta^4 K(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) + \int d\theta^2 \Psi^I \cdot \hat{\mathcal{W}}(\Phi)_I + \text{FI-terms}(a^K)$$



[Blaszczyk, Groot Nibbelink, Rühle'11]

The Landau-Ginzburg Orbifold Phase

- with $a^j < 0$, $|\psi^j| \neq 0$
 → U(1)'s broken to \mathbb{Z}_N Symmetry
- F- & D-Term equation:

target space shrinks to a point

Landau-Ginzburg Generalities

Defining properties and constraints

- The quasi homogenous Superpotential: $\hat{\mathcal{W}}(\kappa^{Q_0} \Phi^i) = \kappa \hat{\mathcal{W}}(\Phi^i)$, $0 < Q_0 \leq 1$
- **Flows to CFT** in IR with right central charge:

$$c = 3 \sum_i (1 - 2Q_0^i) \stackrel{!}{=} 9$$

- Possesses a $\mathbb{Z}_{N_0} \times \dots \times \mathbb{Z}_{N_n}$ symmetry

Defining a discrete quotient

Assign charges to fields $\phi \vec{Q}_i \in \mathbb{Z}_{N_i}$ to mod them out

- Q_0^i World sheet R-charge
- $\sum_i Q_a^i = 0 \quad \forall a > 0 \rightarrow$ Anomaly free \mathbb{Z}_{N_a} discrete Symmetry
- Add 10 Majorana Weyl Fermions λ^I & E_8 current algebra.
- Combined GSO and Orbifold projection: $g = e^{-i(\pi Q_0 + \lambda)}$, $e^{2\pi i Q^i}$
We get **twisted sectors**

LGO-Spectrum

Twisted Modding

$$\phi^i(\sigma + 2\pi) = e^{-i\pi k Q^i} \phi^i(\sigma)$$

→ twisted Vacuum (Energie, q_i, q_R charges) $|g^k, \vec{Q}_i\rangle$ [Kachru, Witten'93]

- 4D theory has additional $U(1)$'s, with charges q_i of world sheet fields with $q_L = \sum_i q_i$
- Govern **gauge representation**: $E_6 \supset SO(10) \times U(1)_L$:

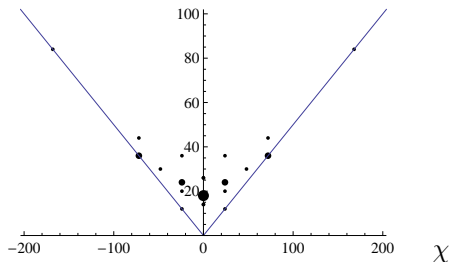
$$78 \rightarrow 45_0 \oplus 16_{-3/2} \oplus \overline{16}_{3/2} \oplus 1_0$$

$$27 \rightarrow 16_{-1/2} \oplus 10_1 \oplus 1_2$$

- The $U(1)_R$ charge specifies the space-time Lorentz representation

$(A_1)^9$ Fermat Classification

$$h^{(1,1)} + h^{(2,1)}$$

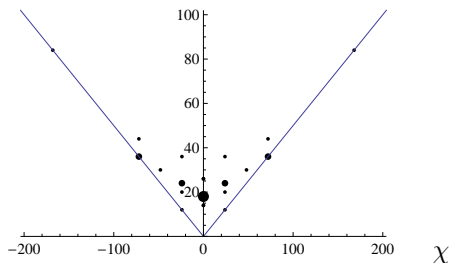


The classification

We **systematically** constructed all Abelian orbifolds of the $(A_1)^9$ Gepner model and computed the whole massless matter spectrum

$(A_1)^9$ Fermat Classification

$$h^{(1,1)} + h^{(2,1)}$$



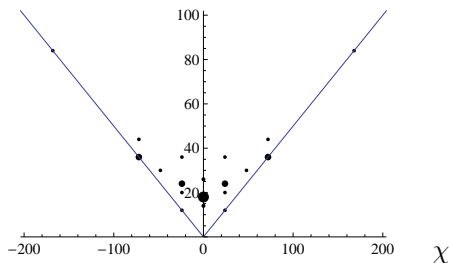
Mirror Symmetry

The set of models is closed under **Mirror Symmetry** [Greene, Plesser]

Acquire the Mirror Model by orbifold by the maximal subgroup!

$(A_1)^9$ Fermat Classification

$$h^{(1,1)} + h^{(2,1)}$$

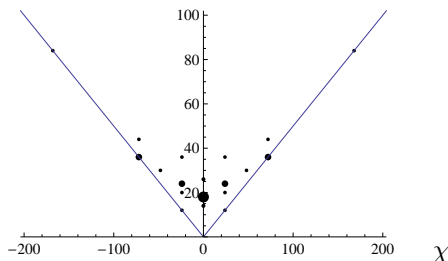


Most (geometric) models have Orbifold phase: [\[Fischer,Ratz,Torrado,Vaudrevange'12\]](#)

\mathcal{N}	$h^{(1,1)}$	$h^{(2,1)}$	Add Gauge Group	Add Singlets	Geometric Phase
1	36	0	$U(1)^6 \times SU(3)$	270	T^6/\mathbb{Z}_3
1	12	0	$U(1)^8$	252	$T^6/(\mathbb{Z}_3 \times \mathbb{Z}_{3,\text{free}})$
1	9	9	$U(1)^6 \times SU(3)$	270	?
2	21	21	$U(1)^6 \times SU(3)$	194	$T^2 \times K3$
4	9	9	$E_6 \times SU(3)$	258	T^6

$(A_1)^9$ Fermat Classification

$$h^{(1,1)} + h^{(2,1)}$$



Observations

- The additional vector multiplets \hookrightarrow Gauged isometries of self dual point

- Some Numerology:**

$$\# \text{Singlets} - 3 \cdot \# \text{Add-Vectors} = (4 - \mathcal{N})76$$

Most (geometric) models have Orbifold phase: [Fischer,Ratz,Torrado,Vaudrevange'12]

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Matching with T^6/\mathbb{Z}_3

[Aspinwall, Plesser'10]

$$\boxed{\widehat{T^6/\mathbb{Z}_3}} \text{ Fermat LGO} \\ (h^{1,1}, h^{2,1}) = (0, 36)$$

Mirror Symmetric \longleftrightarrow

$$\boxed{T^6/\mathbb{Z}_3} \text{ Fermat LGO} \\ (h^{1,1}, h^{2,1}) = (36, 0)$$

The Additional Singlet and Vector Spectrum

Singlet	q_1	q_2	q_r
U	0	1	0
	-1	0	0
	1	-1	0
B	$\frac{1}{3}$	0	$-\frac{1}{3}$
	0	$-\frac{1}{3}$	$-\frac{1}{3}$
	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$

Symmetries:

- 8 Vectors \rightarrow Adjoint of SU(3)
- $(U(1) \times U(1) \times S_3)^3$
- Discrete R-Symmetry [Distler, Kachru]
- $q_r = (3 \underbrace{k_0}_{\text{Twist}} - 2q_L + \underbrace{q_s}_{\text{Strangeness}}) / 18 \text{ mod } 1$
with $q_r(\mathcal{W}) = 1/3$

Matching with T^6/\mathbb{Z}_3

[Aspinwall, Plesser'10]

 $\widehat{T^6/\mathbb{Z}_3}$ Fermat LGO

$$(h^{1,1}, h^{2,1}) = (0, 36)$$

↓ C.S. Deformation

 $\widehat{T^6/\mathbb{Z}_3}$ non-Fermat LGOMirror Symmetric
↔ T^6/\mathbb{Z}_3 Fermat LGO

$$(h^{1,1}, h^{2,1}) = (36, 0)$$

↓ Kähler Deformation

 T^6/\mathbb{Z}_3 Orbifold

The Additional Singlet and Vector Spectrum

Singlet	q_1	q_2	q_r
$\langle U \rangle$	0	1	0
	-1	0	0
	1	-1	0
	$\frac{1}{3}$	0	$-\frac{1}{3}$
	0	$-\frac{1}{3}$	$-\frac{1}{3}$
	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$

Symmetries:

- 8 Vectors \rightarrow Adjoint of SU(3)
- $(\mathbb{Z}_3 \times \mathbb{Z}_3 \times S_3)^3 = \Delta(54)^3$ [Beye, Kobayashi, Kuwakino'14]
- Discrete R-Symmetry [Distler, Kachru]
- $q_r = (3 \underbrace{k_0}_{\text{Twist}} - 2q_L + \underbrace{q_s}_{\text{Strangeness}}) / 18 \pmod{1}$
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Matching with T^6/\mathbb{Z}_3

[Aspinwall, Plesser'10]

 $\widehat{T^6/\mathbb{Z}_3}$ **Fermat LGO**

$$(h^{1,1}, h^{2,1}) = (0, 36)$$

↓ C.S. Deformation

 $\widehat{T^6/\mathbb{Z}_3}$ **non-Fermat LGO**

↓ C.S. Deformation

 $\widehat{T^6/\mathbb{Z}_3}$ **non-Fermat LGO**Mirror Symmetric
↔ T^6/\mathbb{Z}_3 **Fermat LGO**

$$(h^{1,1}, h^{2,1}) = (36, 0)$$

↓ Kähler Deformation

 T^6/\mathbb{Z}_3 **Orbifold**

↓ Kähler Deformation

Smooth CY

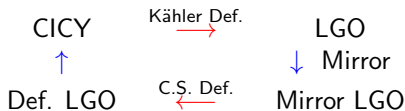
The Additional Singlet and Vector Spectrum

Singlet	q_1	q_2	q_r
$\langle U \rangle$	0	1	0
	-1	0	0
	1	-1	0
$\langle B \rangle$	$\frac{1}{3}$	0	$-\frac{1}{3}$
	0	$-\frac{1}{3}$	$-\frac{1}{3}$
	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$

Symmetries:

- ~~8 Vectors~~ → Adjoint of $SU(3)$
- $(\mathbb{Z}_3 \times \mathbb{Z}_3 \times S_3)^3 = \Delta(54)^3$ [Beye, Kobayashi, Kuwakino'14]
- ~~Discrete R-Symmetry~~ [Distler, Kachru]
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with $q_r(\mathcal{W}) = 1/3$

Summary and conclusion



Track Symmetries through various phases

i.e. track **Flavor and R-symmetries** to

- Orbifold Phase
- Large Volume Calabi-Yau phase

Works whenever we have a GLSM description of a CY (i.e. CICY)

Exhaustive Scan of all $(A_1)^9$ Gepner Models

Constructed the whole vector and singlet Spectrum

- Orbifold Match: Compactification at **self dual radius**
- Models with $\mathcal{N} = 1, 2, 4$ Susy
- A **mysterious** relation satisfied by all models

Thank You!