Circling the Square - Deformations of sLag Cycles in type II Orbifold Models

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Based on: MB, G. Honecker, I.Koltermann, 1403.2394

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Motivation

- Type IIB on $T^4/\mathbb{Z}_N \times \Omega\mathcal{R}$ with D7-branes

  Gimon,Polchinski; Blumenhagen, Braun, Kors, Lust;...

- Type IIA on $T^6/\mathbb{Z}_N \times \mathbb{Z}_M \times \Omega\mathcal{R}$ (+discrete torsion) with D6-branes

  Bailin, Blumenhagen, Cvetic, Förste, Honecker, Love, Lust, Ott, Staessens,...

- special Lagrangian (*sLag*) cycles support O-planes and D-branes for $\mathcal{N}=1$ SUSY

$$J_{1,1}^{\text{Kähler}}\big|_{sLag} = 0, \qquad \Im(\Omega_n)\big|_{sLag} = 0, \qquad \Re(\Omega_n)\big|_{sLag} > 0.$$

- How do they react to deformations of Orbifold singularities?
  - What deformations are allowed?
  - What cycles remain *sLag*?
  - Study more generic regions in Moduli Space
  - Deformation moduli may be stabilized
  - More possibilities for orientifolds

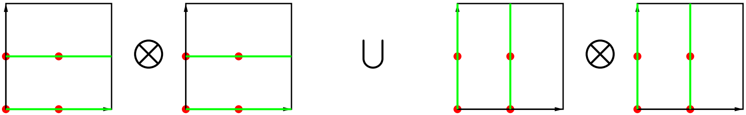
Special Lagrangeans in $T^4/\mathbb{Z}_2$

On $T^4/\mathbb{Z}_2$

- Bulk cycles $\Pi_{ij} = \pi_i \otimes \pi_j$ inherited from Torus
- Exceptional cycles $e_{\alpha\beta}$ hidden in $\mathbb{Z}_2$ singlarities
- Fractional cycles

$$\Pi^{\text{frac}} = \frac{1}{2}\left(\Pi_{ij} + \sum (-1)^{\tau_{\alpha\beta}} e_{\alpha\beta}\right)$$

- e.g. O-plane from $\Omega\mathcal{R} : z_i \longmapsto \bar{z}_i$:



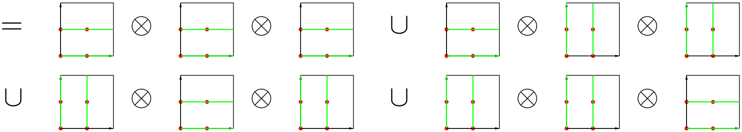
Special Lagrangeans in $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

On $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- Bulk cycles $\Pi_{ijk} = \pi_i \otimes \pi_j \otimes \pi_k$
- Exceptional cycles $E_{\alpha\beta} = e_{\alpha\beta} \otimes \pi_i$
- Fractional cycles

$$\Pi^{\text{frac}} = \frac{1}{4}\left(\Pi_{ijk} + \sum_{i,\alpha\beta} (-1)^{\tau^{(i)}_{\alpha\beta}} E^{(i)}_{\alpha\beta}\right)$$

$\Pi_{\Omega\mathcal{R}} =$



Gimon–Polchinski Model

- type IIB on square $T^4/\mathbb{Z}_2 \times \Omega$

  Giomon,Polchinski

  - O9 plane from Ω
  - 16 O5 planes at $\mathbb{Z}_2$ singularities
- add 16$\times$ D9 and 16$\times$D5 for tadpole cancellation
- T-Dualize along 7- and 9- direction:
  - IIB on $T^4/\mathbb{Z}_2 \times \Omega\mathcal{R}$ with $\mathcal{R} : z_i \mapsto \bar{z}_i$
  - O7 planes and D7 branes on *sLag* cycles

Gimon–Polchinski Spectrum

- From CFT calculation:
  - Gauge group U(16) times U(16)
  - HyperMultiplets: $2 \times (\mathbf{120} \times \mathbf{1}) + 2 \times (\mathbf{1} \times \mathbf{120}) + (\mathbf{16} \times \mathbf{16})$

- from Geometric considerations:

$(\mathbf{N_a} \times \mathbf{N_b})$	$\Pi_a \circ \Pi_b$
$\mathrm{Sym}(\mathbf{N_a})$	$\Pi_a \circ \Pi_{a'} - \Pi_a \circ \Pi_{O7}$
$\mathrm{Asym}(\mathbf{N_a})$	$\Pi_a \circ \Pi_{a'} + \Pi_a \circ \Pi_{O7}$

$\Rightarrow$ O7 planes wrap bulk cycles

$\Rightarrow$ D7 branes wrap fractional cycles $\Pi_{a/a'} = \frac{1}{2}\left(\Pi_{\mathrm{bulk}} \pm \Pi_{\mathbb{Z}_2}\right)$

$T^4/\mathbb{Z}_2$ as Hypersurface

Deformable $T^4/\mathbb{Z}_2$ Orbifold is hypersurface in toric variety:
Start with 2 elliptic curves (y_i, x_i, v_i), $i = 1, 2$, identify $y := y_1 y_2$

$$\mathrm{Def}(T^4/\mathbb{Z}_2) = \left\{ y^2 = F_1 \cdot F_2 + \sum_{\alpha\beta=1}^{4} \varepsilon_{\alpha\beta} \delta F_1^{(\alpha)} \cdot \delta F_2^{(\beta)} \right\} .$$

- F_i encode torus complex structure
- $\varepsilon_{\alpha\beta} \in \mathbb{C}$ encode deformation of 16 $\mathbb{Z}_2$ singularities
- Holomorphic two-form:

$$\Omega_2 = \frac{\mathrm{d}z_1 \wedge \mathrm{d}z_2}{y}$$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ as Hypersurface

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold w/ discrete torsion is hypersurface in toric variety: Vafa, Witten

3 elliptic curves (y_i, x_i, v_i), $i = 1, 2, 3$, identify $y := y_1 y_2 y_3$

$$\mathrm{Def}(T^6/\mathbb{Z}_2 \times \mathbb{Z}_2) =$$

$$\left\{ y^2 = F_1 \cdot F_2 \cdot F_3 + \sum \varepsilon^i_{\alpha\beta} F_i \delta F_j^{(\alpha)} \delta F_k^{(\beta)} + \sum \varepsilon_{\alpha\beta\gamma} \delta F_1^{(\alpha)} \delta F_2^{(\beta)} \delta F_3^{(\gamma)} \right\} .$$

- F_i encode torus complex structure
- $\varepsilon_{\alpha\beta} \in \mathbb{C}$ encode deformation of 3×16 $\mathbb{Z}_2$ fixed planes
- $\varepsilon_{\alpha\beta\gamma}$ ensure $4 \times 4 \times 4$ codimension-three (conifold) singularities

$$\Omega_3 = \frac{dz_1 \wedge dz_2 \wedge dz_3}{y}$$

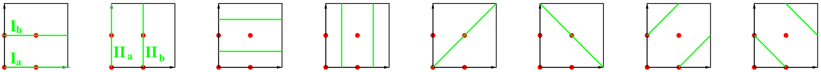
sLags on Elliptic curves

$$T^2 \cong \{y^2 = 4vx^3 - g_2 v^3 x - g_3 v^4\} \subset \mathbb{P}_{112}$$

- Orientifold symmetry requires $g_2, g_3 \in \mathbb{R}$:
  - untilted (rectangular) tori $j \geq 1$
  - tilted tori $j \leq 1$
- sLags from fixed sets of antilinear involutions σ :

$$\begin{pmatrix} x \\ v \end{pmatrix} \longmapsto A \begin{pmatrix} \bar{x} \\ \bar{v} \end{pmatrix} , \qquad y \longmapsto e^{i\beta} \bar{y} ,$$

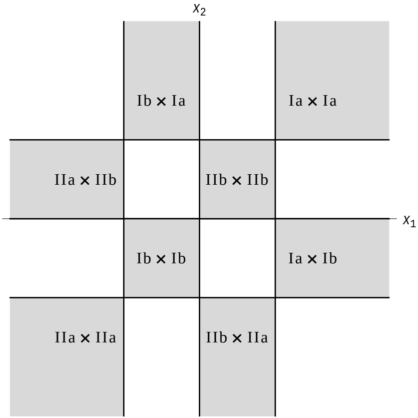
- On square torus:



sLags on $T^4/\mathbb{Z}_2$

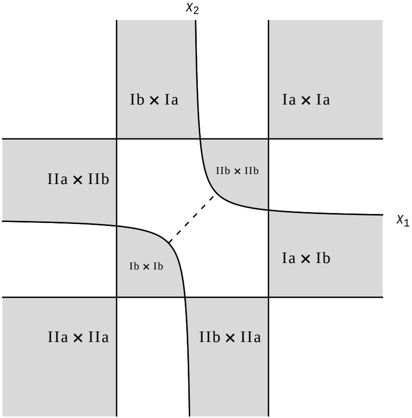
frac. horizontal & vertical cycles from $\sigma_{\mathcal{R}} : (x_i, v_i) \mapsto (\overline{x}_i, \overline{v}_i)$

- $\sigma_{\mathcal{R}} : y \mapsto \overline{y}$: *sLags* calibrated with $\mathrm{Re}(\Omega_2)$ (gray)
- $\sigma_{\mathcal{R}} : y \mapsto -\overline{y}$: *sLags* calibrated with $\mathrm{Im}(\Omega_2)$ (white)

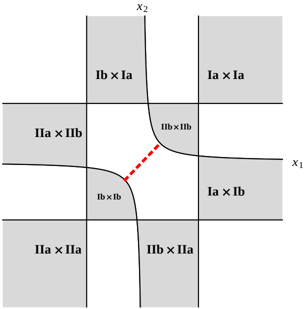


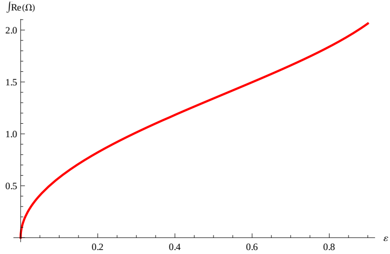
sLags on Deformed $T^4/\mathbb{Z}_2$

- $\Omega\mathcal{R}$ restricts $\varepsilon \in \mathbb{R}$
- Exceptional cycle
  $e_{33} = \{x_1 = \overline{x}_2\}$:
  - $\varepsilon > 0$: *sLag* w.r.t. $\mathrm{Re}(\Omega_2)$
  - $\varepsilon < 0$: *sLag* w.r.t. $\mathrm{Im}(\Omega_2)$
- fractional cycles shrink
  $(\mathbf{I_b} \otimes \mathbf{I_b}, \mathbf{II_b} \otimes \mathbf{II_b})$
- Gimon-Polchinski cycles
  loose *sLag* property
  $\rightarrow$model not deformable?



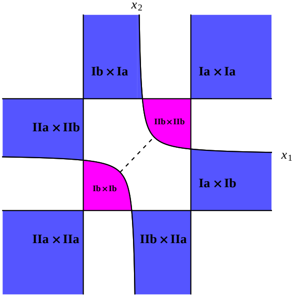
Integral of Ω_2 on Exceptional Cycle

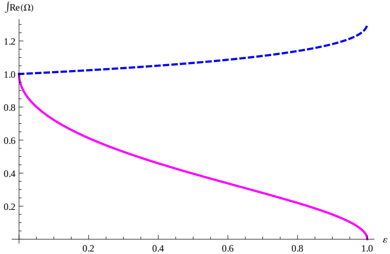




$$\mathrm{Vol}(e_{33}) = 4\pi\sqrt{\varepsilon_{33}} + \mathcal{O}(\varepsilon_{33})$$

Integral of Ω_2 on Fractional Cycles



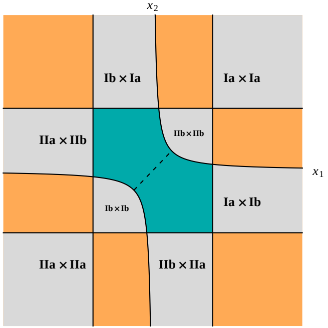


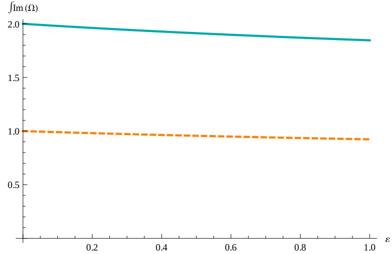
$$\mathrm{Vol}(\mathbf{I_b} \otimes \mathbf{I_b}) = 4K(1/\sqrt{2})^2 - 2\pi \cdot \sqrt{\varepsilon_{33}} + \mathcal{O}(\varepsilon_{33})$$

$$\Rightarrow \mathbf{I_b} \otimes \mathbf{I_b} = \tfrac{1}{2}\left(\Pi_{13} - e_{33} \pm \ldots\right)$$

$$\mathrm{Vol}(\mathbf{I_a} \otimes \mathbf{I_a}) = 4K(1/\sqrt{2})^2 + 2\pi \frac{\Gamma(3/4)^2}{\Gamma(1/4)^2} \cdot \varepsilon_{33} + \mathcal{O}(\varepsilon_{33}^2)$$

Integral of Ω_2 on Fractional Cycles



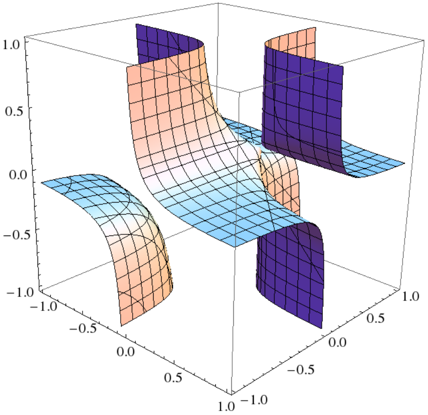


$$\text{Vol}(\mathbf{I_a} \otimes \mathbf{II_a}) = 4K(1/\sqrt{2})^2 - 2\pi \frac{\Gamma(3/4)^2}{\Gamma(1/4)^2} \cdot \varepsilon_{33} + \mathcal{O}(\varepsilon_{33}^2)$$

$$\text{Vol}(\mathbf{I_b} \otimes \mathbf{II_b} + \mathbf{II_b} \otimes \mathbf{I_b}) = 2\text{Vol}(\mathbf{I_a} \otimes \mathbf{II_a})$$

sLags on Deformed $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- In general: difficult due to requirement of 64 conifolds
- Exceptional cycle: S^2 ($x_2 = \overline{x}_3$) fibration over Interval ($0 \leq x_1 \leq 1$)
- Cycle structure more involved
- Volumes are harder to compute

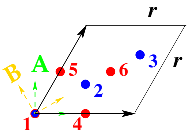


Conclusion

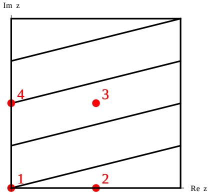
- $\Omega\mathcal{R}$ restricts $\varepsilon \in \mathbb{R}$, further restrictions from D-branes
- O-planes without / with / with only exceptional cycles
- Non-trivial structure of *sLag* cycles
- Deformation dependence for volumes of all cycles

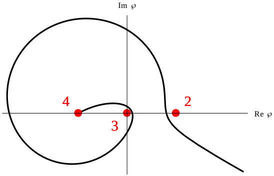
Outlook

- Tilted tori and higher orbifold groups



- General (n, m) cycles





sLags on Fully Deformed $T^4/\mathbb{Z}_2$

Two cases: $\sigma_{\mathcal{R}} : y \mapsto \pm\bar{y}$

- *sLag* w.r.t $\mathrm{Re}(\Omega)$: gray

$$\Pi_{\Omega\mathcal{R}} = \sum \Pi^{\text{frac}}$$
$$= 2\Pi_{13} - 2\Pi_{24} - \sum_{\alpha\beta} e_{\alpha\beta}$$

- *sLag* w.r.t $\mathrm{Im}(\Omega)$: white

$$\Pi_{\Omega\mathcal{R}} = \sum \Pi^{\text{frac}}$$
$$= 2\Pi_{13} - 2\Pi_{24}$$