Comments on massless singlets from decomposed spectral covers

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Work in Progress

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Motivation

F-theory is a nice framework to construct grand unified theories. A lot of progress has been made so far. In particular, the understanding of the Abelian sectors has been developed recently.

Mayrhofer, Palti, Weigand 12; Cvetic , Grimm, Klevers 12; Braun, Grimm, Keitel 13; Borchmann, Mayrhofer, Palti, Weigand 13; Cvetic, Klevers, Piragua 13; Cvetic, Grassi, Klevers, Piragua 13; Cvetic, Grassi, Piragua, Song 13; Braun, Collinucci, Valandro 14; Bies, Mayrhofer, Pehle, Weigand 14; Morrison, Taylor 14; Mayrhofer, Morrison, Till, Weigand 14; Anderson, García-Etxebarria, Grimm, Keitel 14; Kuntzler, Schäfer-Nameki 14; Lin, Weigand 14, …

In fact, the global structure is important for realizing a U(1) symmetry since it is associated with a global section. Morrison, Vafa 96

In models with U(1) symmetries, we typically have fields which are not charged under the GUT group.

Since such singlets are not localized on the GUT surface, the understanding of their property requires the global analysis.

1. Some singlets are localized along the SU(2) singularity locus which is not localized on the GUT surface.

Grimm, Weigand 10

2. The massless singlets are characterized by certain cohomology

Bies, Mayrhofer, Pehle, Weigand 14

$$H^i(\Sigma_{\mathbf{R}}, K_{\Sigma_{\mathbf{R}}}^{\frac{1}{2}} \otimes L_{\mathbf{R}}), \quad i = 0, 1$$

In this talk, we revisit the two properties of the singlets from the dual heterotic string theory.

In fact, the decomposed spectral cover which yields a U(1) symmetry has been constructed. Choi, HH 12

By building a vector bundle from the decomposed spectral cover, one may study the localization of the singlets and their massless spectrum in heterotic string theory, and check the agreement with the F-theory results.

Singlets from decomposed spectral cover

Heterotic string theory on CY_3 with a vector bundle V whose structure group H may yield $\boldsymbol{N} = 1$ supersymmetric gauge theory with a gauge group G ($G \times H \subset E_8$)

The vector bundle V needs to be holomorphic and "stable".

The stability condition is in general difficult to satisfy. However, if the CY_3 has an elliptic fibration with a section, there is a nice way to construct holomorphic stable bundles.

→ "spectral cover construction"

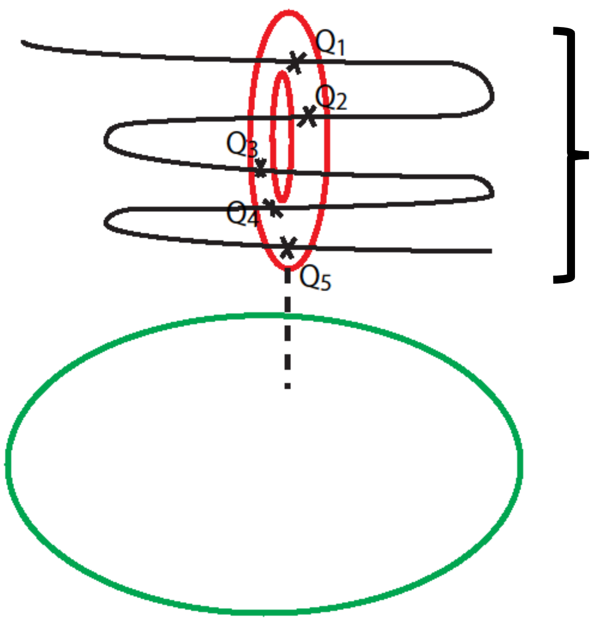
Friedman, Morgan, Witten 97
Donagi 97

Stable holomorphic SU(n) bundle V

$$\boxed{(C_V, \mathcal{N}_V) \quad \leftrightarrow \quad V}$$

- C_V is a n-fold cover over the base S
- $\mathcal{N}_V$ is a line bundle over C_V

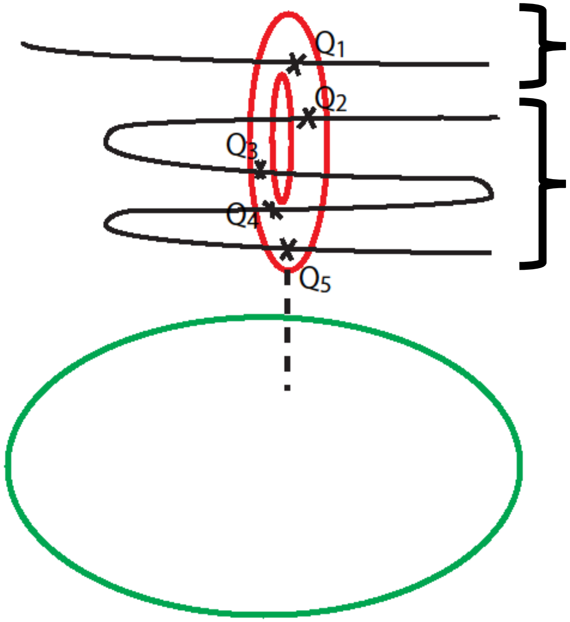
Ex. SU(5) vector bundle



5-fold cover

The condition of det V = O can be realized by $Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = \sigma$.

S(U(4) x U(1)) vector bundle



$Q_1 \neq \sigma$

$Q_2 + Q_3 + Q_4 + Q_5 \neq \sigma$

$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = \sigma$

This configuration can be achieved by requiring another global section of the elliptic fibration of the CY_3 and putting Q_1 on the global section.

Choi, HH 12

Cf: Other constructions

Blumenhagen, Honecker, Weigand 05
Blumenhagen, Moster, Reinbacher, Weigand 06
Buchbinder, Constantin, Lukas 14

Practically, we can do the following.

· Elliptically - fibered CY_3: $y^2 = x^3 + fx + g$

· Spectral cover : $a_0 + a_2x + a_3y + a_4x^2 + a_5xy = 0$

We find a solution which satisfies

1. $y_1{}^2 = x_1{}^3 + fx_1 + g$,
2. $a_0 + a_2x_1 + a_3y_1 + a_4x_1{}^2 + a_5x_1y_1 = 0$,

for a given $(x, y) = (x_1, y_1)$ which describes a global section of the elliptic fibration.

We may then construct the S(U(4) x U(1)) bundle by the 4 + 1 decomposed spectral cover : $C_V = C_U + C_1$

· $V = U \oplus L$

Rank 4 bundle U $\leftrightarrow$ $(C_U, \mathcal{N}_U)$

Rank 1 bundle L $\leftrightarrow$ $(C_1, \mathcal{N}_1)$

The massless spectra are :

$(4, 10)_1$	$(1, 10)_{-4}$	$(6, \bar{5})_2$	$(4, \bar{5})_{-3}$
$H^1(Z, U)$	$H^1(Z, L)$	$H^1(Z, \Lambda^2 U)$	$H^1(Z, U \otimes L)$

$(4, 1)_5$
$H^1(Z, U \otimes L^{-1})$

Blumenhagen, Honecker, Weigand 05,
Blumenhagen, Moster, Reinbacher, Weigand 06
Tatar, Watari 06

A key observation to compute $H^1(Z, U \otimes L^{-1})$ is that the base S is isomorphic to C_1 by the very definition of the global section.

Then, one can show :

$$U \otimes L^{-1} \cong \tilde{U} \otimes \pi^* O_{C_1}(D) \otimes \pi^* N_1^{-1}$$

$\tilde{U}$ is a vector bundle which is constructed by the Fourier-Mukai transformation of $(C_U, \tilde{N}_U)$ with respect to the elliptic fibration $\pi : Z \to C_1$.

Then, one can apply the standard argument to this case.

Diaconescu, Ionesei 98
Donagi, He, Ovrut, Reinbacher 04
Blumenhagen, Moster, Reinbacher, Weigand 06
HH, Tatar, Toda, Watari, Yamazaki 08

$$H^1(Z, U \otimes L^{-1}) \cong H^0(C_1, R^1\pi_*(\tilde{U} \otimes \pi^* O_{C_1}(D) \otimes \pi^* N_1^{-1}))$$
$$\cong H^0(C_1, \underline{R^1\pi_*\tilde{U}} \otimes O_{C_1}(D) \otimes N_1^{-1})$$

This sheaf is localized along $C_U \cap C_1 = \Sigma$

After further computation, we arrive at the expression

$$\begin{aligned} H^1\left(Z, U \otimes L^{-1}\right) &\simeq H^0\left(\Sigma, (\mathcal{N}_U \otimes \mathcal{N}_1^{-1} \otimes K_{C_1})|_\Sigma\right) \\ &\simeq H^0\left(\Sigma, K_\Sigma^{\frac{1}{2}} \otimes \mathcal{O}_\Sigma\left(\underbrace{k^*\gamma_U + \frac{1+n}{n} h^*\zeta}_{\text{"flux" part}}\right)\right) \end{aligned}$$

Υ_U is a divisor in C_U which satisfies $\pi_{C_U *}\gamma_U = 0$.

ζ is defined as $c_1(\sigma^* U) = \zeta$.

F-theory perspective

Let us move on to the F-theoretic version of the decomposed spectral covers. We consider a CY_4 which is elliptic K3-fibration over the same base S.

In the stable degeneration limit, the geometry becomes

Katz, Mayr, Vafa 97
Berglund, Mayr 98

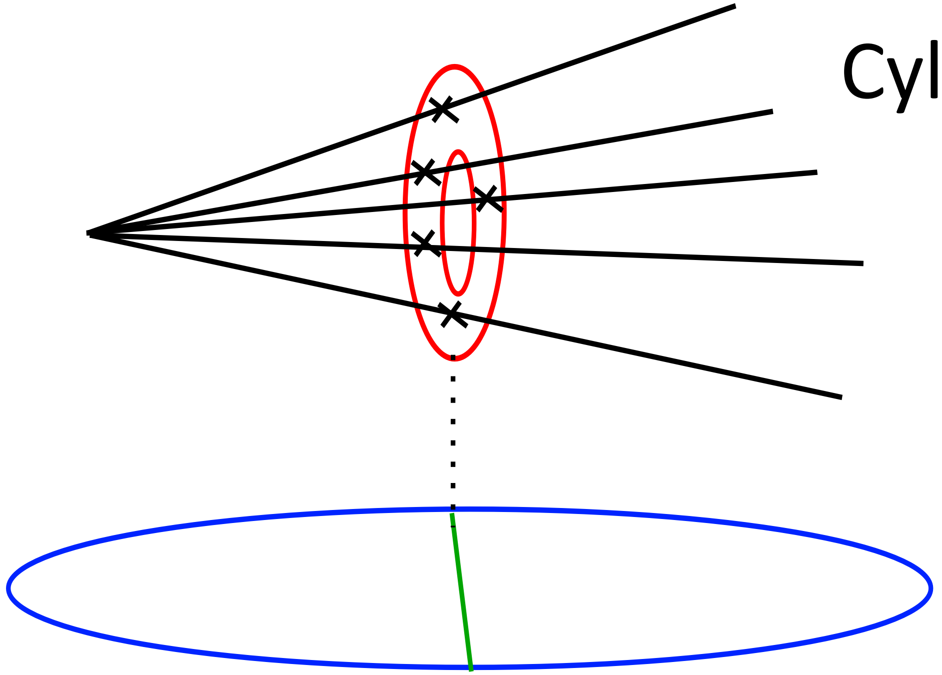
$$
\begin{aligned}
y^2 = x^3 &+ a_0 z^5 + g z^6 \\
&+ a_2 z^3 x + \underbrace{f z^4 x}_{\text{complex structure moduli}} \\
&+ a_3 z^2 y \\
&+ a_4 z x^2 \\
&+ \underbrace{a_5 x y}_{\text{bundle moduli}}
\end{aligned}
$$

In heterotic string theory, the (4+1)-decomposition is realized by tuning the coefficients such that

1. $y_1^2 = x_1^3 + fx_1 + g$,
2. $a_0 + a_2x_1 + a_3y_1 + a_4x_1^2 + a_5x_1y_1 = 0$

are automatically satisfied.

Therefore, the same tuning guarantees that the elliptic fibration of the CY_4 also has another section at $(x, y) = (x_1z^2\ y_1z^3)$, which is obtained by the cylinder map of the global section of the heterotic elliptic fibration.



Cylinder map

Curio, Donagi 98
Donagi, Wijnholt 08

When C_1 intersects with C_U, the two lines coincide with each other, which implies that two 7-branes away from the GUT 7-branes intersect at a curve. Then, the locus acquires a double-point singularity.

This agrees with the result in F-theory.

Grimm, Weigand 10,
Mayrhofer, Palti, Weigand 12

One might identify the location of the singlets by looking at the discriminant of the discriminant of the Weierstrass equation of the CY_4 .

$$\tilde{\Delta} = 16777216 a_5^5 A^3 B$$

When one inserts the parameterization of the (4+1) decomposition, the discriminant factorizes as follows.

$$B \to B_1^2 B_2$$

Then, $B_1 = 0$ is exactly the location of the singlets, which can be also checked in explicit examples.

Conclusion:

We study the properties of the singlets from the decomposed spectral covers. They are localized along the intersection of the spectral covers, and their massless degrees of freedom are characterized by certain cohomology. Those results agree with the F-theory results.

Further directions:

· Matching with the flux part

·(3+2)-decomposition, …..