Geometry of F-theory 'Standard Models'

based on arXiv:1406.6071

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13$^{\text{th}}$ String Phenomenology
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Motivation

- String theory as a unifying theory should reproduce the Standard Model at low energies.
- Alternative to GUT model building: directly realise Standard Model gauge group.
- Well analysed examples in type II string theories, but only few attempts in F-theory. [Choi '10, '11, '13]
- First step towards phenomenologically interesting models: understand the geometry.

Outline

Elliptic Fibrations with two extra Sections

- Elliptic fibration with extra (non-torsional) section $\Rightarrow U(1)$ gauge symmetry. [Grimm, Weigand '10], [Morrison, Park '12], [Braun, Keitel, Grimm '13], ...
- Need hypercharge and one further $U(1)$ as selection rule, realised by hypersurface inside $\mathrm{Bl}_2\mathbb{P}^2$-fibration:
  [Borchmann, Mayrhofer, Palti, Weigand '13], [Cvetic, (Grassi), Klevers, Piragua '13]

$$P_T = \mathrm{v}\,\mathrm{w}(c_1\,\mathrm{w}\,s_1 + c_2\,\mathrm{v}\,s_0) + \mathrm{u}\,(b_0\,\mathrm{v}^2\,s_0^2 + b_1\,\mathrm{v}\,\mathrm{w}\,s_0\,s_1 + b_2\,\mathrm{w}^2\,s_1^2) + \mathrm{u}^2(d_0\,\mathrm{v}\,s_0^2\,s_1 + d_1\,\mathrm{w}\,s_0\,s_1^2 + d_2\,\mathrm{u}\,s_0^2\,s_1^2)$$

- Independent rational sections:
  $S_0 = \{s_0\},\ S_1 = \{s_1\},\ U = \{\mathrm{u}\} \xrightarrow{\text{Shioda-map}} U(1)$-generators.
  [for details see Timo Weigand's talk]

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2$ Gauge Symmetry

- Non-abelian singularity enhancement over divisor $X = \{x\} \subset \mathcal{B}$ by restricting the coefficients $\{b_i, c_j, d_k\} \ni g_m = g_{m;l}\, x^l$.
- After resolution fibre over X factorises into $\mathbb{P}^1$-components $\leftrightarrow$ simple roots; further factorisation in higher codimension $\leftrightarrow$ matter.
- Different singularity types along different divisors $X, Y \subset \mathcal{B} \Rightarrow$ product structure of gauge symmetry
- Toric geometry determines (subclass of) valid restrictions $\{m; l\}$ *and* resolution of singularity
  $\Rightarrow 2\times2$ inequivalent (toric) $SU(3) \times SU(2)$ models

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2$ Gauge Symmetry

Example:

$U(1)_1 \times U(1)_2$

$$\begin{aligned} P_T = & \, v\, w (c_1 \quad w\, s_1 + c_2 \quad v\, s_0) \\ & + u\, (b_0 \quad v^2\, s_0^2 + b_1\, v\, w\, s_0\, s_1 + b_2 \quad w^2\, s_1^2) \\ & + u^2 (d_0 \quad v\, s_0^2\, s_1 + d_1 \quad w\, s_0\, s_1^2 + d_2 \quad u\, s_0^2\, s_1^2) \end{aligned}$$

Charged matter states:

Six singlets $\mathbf{1}_i$ (plus conjugates $\overline{\mathbf{1}}_i$) [Borchmann, Mayrhofer, Palti, Weigand '13], [Cvetic, (Grassi), Klevers, Piragua '13]

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2$ Gauge Symmetry

Example:

$U(1)_1 \times U(1)_2 \times SU(2)$

$$\begin{aligned} P_T = & \, \mathrm{v}\,\mathrm{w}(c_{1;0} \quad e_1 \quad \mathrm{w}\, s_1 + c_{2;0} \qquad \mathrm{v}\, s_0) \\ & + \mathrm{u}\,(b_{0;1} \quad e_0 \quad \mathrm{v}^2\, s_0^2 + b_1\, \mathrm{v}\,\mathrm{w}\, s_0\, s_1 + b_{2;0} \quad e_1 \qquad \mathrm{w}^2\, s_1^2) \\ & + \mathrm{u}^2(d_{0;1} \quad e_0 \qquad \mathrm{v}\, s_0^2\, s_1 + d_{1;0} \qquad \mathrm{w}\, s_0\, s_1^2 + d_{2;1} \quad e_0 \qquad \mathrm{u}\, s_0^2\, s_1^2) \\ W_2 = & \, \pi(\{e_0\, e_1\}), \qquad\qquad (\pi : \hat{Y}_4 \to \mathcal{B}) \end{aligned}$$

blow-down: $e_0 \to w_2$, $e_1 \to 1$

Charged matter states:
Six singlets $\mathbf{1}_i$ (plus conjugates $\overline{\mathbf{1}}_i$)
three $\mathbf{2}_j$ (plus conjugates $\overline{\mathbf{2}}_j$)

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2$ Gauge Symmetry

Example:

$U(1)_1 \times U(1)_2 \times SU(2) \times SU(3)$

$$\begin{aligned} P_T = & \, v\, w(c_{1;0,0}\, e_1\, f_2\, w\, s_1 + c_{2;0,1}\, f_0\, f_2\, v\, s_0) \\ & + u\,(b_{0;1,1}\, e_0\, f_0\, v^2\, s_0^2 + b_1\, v\, w\, s_0\, s_1 + b_{2;0,0}\, e_1\, f_1\, f_2\, w^2\, s_1^2) \\ & + u^2(d_{0;1,1}\, e_0\, f_0\, f_1\, v\, s_0^2\, s_1 + d_{1;0,0}\, f_1\, w\, s_0\, s_1^2 + d_{2;1,1}\, e_0\, f_0\, f_1^2\, u\, s_0^2\, s_1^2) \end{aligned}$$

$$W_2 = \pi(\{e_0\, e_1\}),\ W_3 = \pi(\{f_0\, f_1\, f_2\}) \quad (\pi : \hat{Y}_4 \to \mathcal{B})$$

blow-down: $e_0 \to w_2,\ e_1 \to 1,\ f_0 \to w_3,\ f_1, f_2 \to 1$

Charged matter states:
Six singlets $\mathbf{1}_i$ (plus conjugates $\overline{\mathbf{1}}_i$)
three $\mathbf{2}_j$ (plus conjugates $\overline{\mathbf{2}}_j$)
five $\mathbf{3}_k$ (plus conjugates $\overline{\mathbf{3}}_k$)

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2$ Gauge Symmetry

Example:

$U(1)_1 \times U(1)_2 \times SU(2) \times SU(3)$

$$\begin{aligned} P_T = {} & \mathrm{v}\,\mathrm{w}(c_{1;0,0}\, e_1\, f_2\, \mathrm{w}\, s_1 + c_{2;0,1}\, f_0\, f_2\, \mathrm{v}\, s_0) \\ & + \mathrm{u}\,(b_{0;1,1}\, e_0\, f_0\, \mathrm{v}^2\, s_0^2 + b_1\, \mathrm{v}\,\mathrm{w}\, s_0\, s_1 + b_{2;0,0}\, e_1\, f_1\, f_2\, \mathrm{w}^2\, s_1^2) \\ & + \mathrm{u}^2(d_{0;1,1}\, e_0\, f_0\, f_1\, \mathrm{v}\, s_0^2\, s_1 + d_{1;0,0}\, f_1\, \mathrm{w}\, s_0\, s_1^2 + d_{2;1,1}\, e_0\, f_0\, f_1^2\, \mathrm{u}\, s_0^2\, s_1^2) \end{aligned}$$

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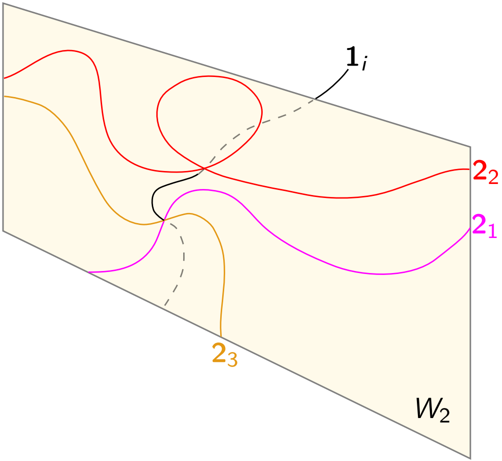
three $\mathbf{2}_j$ (plus conjugates $\overline{\mathbf{2}}_j$)

five $\mathbf{3}_k$ (plus conjugates $\overline{\mathbf{3}}_k$)

one $(\mathbf{3}, \mathbf{2})$ (plus conjugate $(\overline{\mathbf{3}}, \mathbf{2})$)

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2$ Gauge Symmetry



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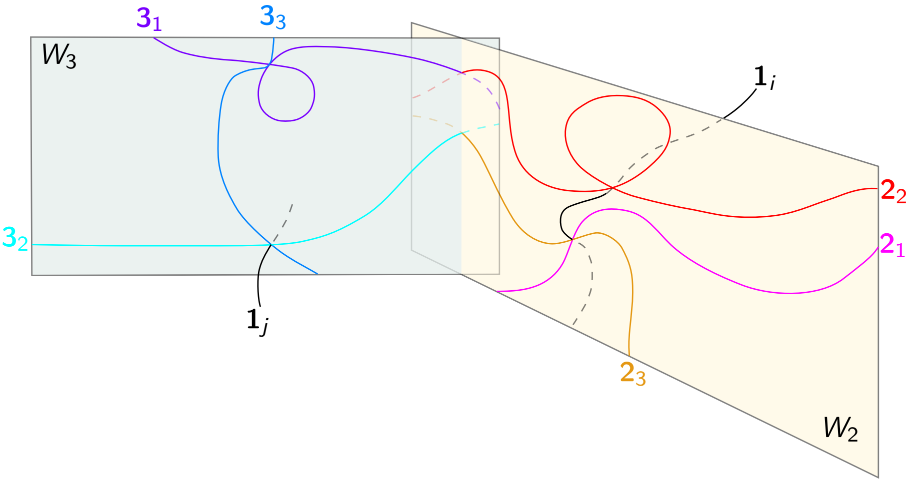

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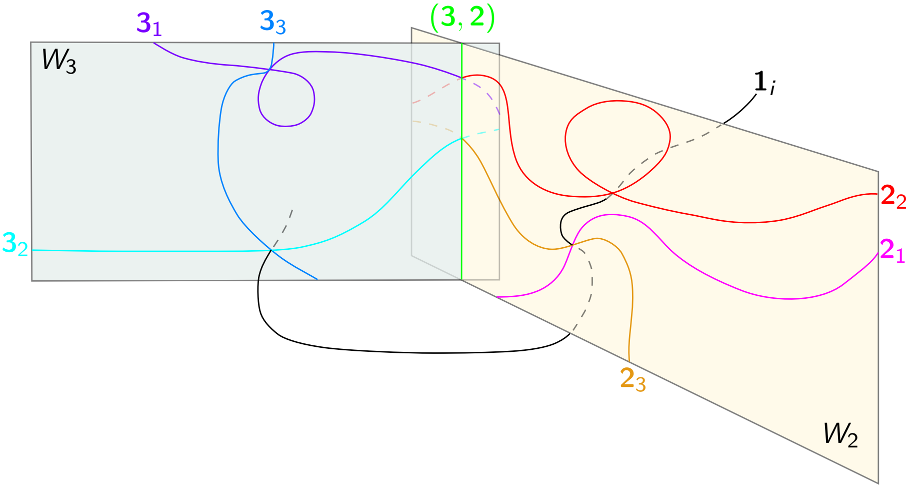

Yukawa Couplings

Gauge invariant coupling types:

$(\mathbf{3},\mathbf{2})$-$\overline{\mathbf{3}}$-$\overset{(-)}{\mathbf{2}}$, $(\mathbf{3},\mathbf{2})$-$(\mathbf{3},\mathbf{2})$-$\mathbf{3}$, $\mathbf{3}$-$\overline{\mathbf{3}}$-$\overset{(-)}{\mathbf{1}}$, $\mathbf{2}$-$\overline{\mathbf{2}}$-$\overset{(-)}{\mathbf{1}}$, $\mathbf{2}$-$\mathbf{2}$-$\overset{(-)}{\mathbf{1}}$, $\mathbf{3}$-$\mathbf{3}$-$\mathbf{3}$, $\overset{(-)}{\mathbf{1}}$-$\overset{(-)}{\mathbf{1}}$-$\overset{(-)}{\mathbf{1}}$

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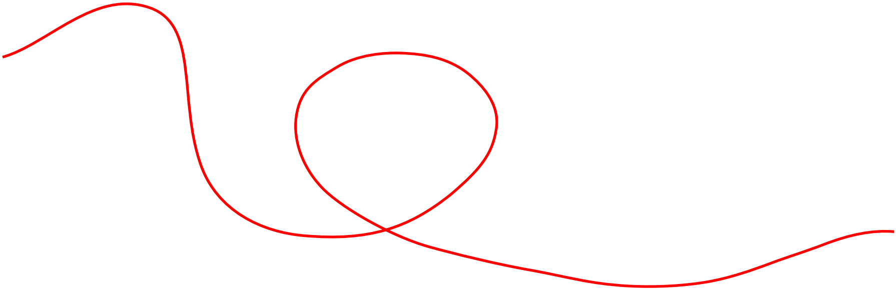
Possible self-coupling $\leftrightarrow$ self-intersection of the matter curve; encoded in the geometry of the curve.

Yukawa Couplings

Example: $\mathbf{2}_2$-curve



$$c_{1;0,0}^2\, d_{1;0,0} - b_1\, b_{2;0,0}\, c_{1;0,0} + b_{2;0,0}^2\, c_{2;0,1}\, w_3$$

Yukawa Couplings

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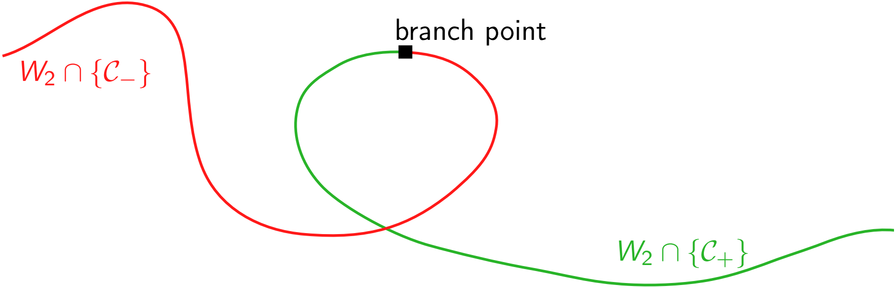

$$c_{1;0,0}^2\, d_{1;0,0} - b_1\, b_{2;0,0}\, c_{1;0,0} + b_{2;0,0}^2\, c_{2;0,1}\, w_3 = \mathcal{C}_+\, \mathcal{C}_- / d_{1;0,0},$$

$$\mathcal{C}_\pm = c_{1;0,0}\, d_{1;0,0} - b_{2;0,0} \left(\frac{b_1}{2} \pm \sqrt{\frac{b_1^2}{4} - d_{1;0,0}\, c_{2;0,1}\, w_3} \right) \text{ for } d_{1;0,0} \neq 0$$

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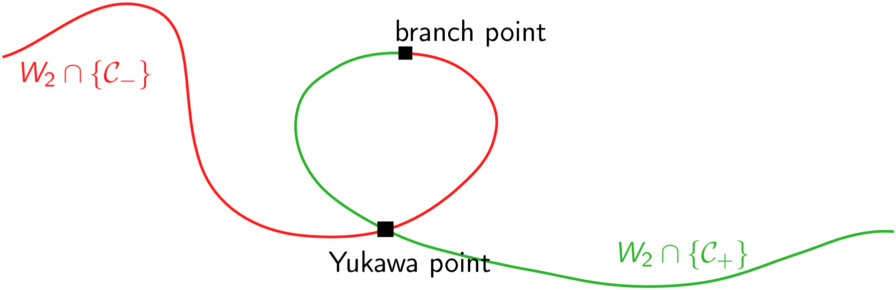

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Yukawa pt $= W_2 \cap \{\mathcal{C}_+\} \cap \{\mathcal{C}_-\} = W_2 \cap \{b_{2;0,0}\} \cap \{c_{1;0,0}\}$

Yukawa Couplings

Solve $\mathcal{C}_\pm = 0$ for $c_{1;0,0} = b_{2;0,0}\,(\ldots)$ and insert into hypersurface equation
$\Rightarrow$ over $W_2 \cap \{\mathcal{C}_\pm\}$ fibre structure enhances:

$$P_T|_{(e_0=0,\,\mathcal{C}_\pm=0)} =$$

$$\underbrace{\frac{1}{d_1}\left[d_1\, s_1\, \mathrm{u} + \mathrm{v}\left(\frac{b_1}{2} \pm \sqrt{\frac{b_1^2}{4} - d_1\, c_2\, w_3}\right)\right]}_{\mathbb{P}^1_{0B}} \underbrace{\left[b_2\, e_1\, s_1 + d_1\, s_0\, s_1\, \mathrm{u} + s_0\, \mathrm{v}\left(\frac{b_1}{2} \mp \sqrt{\frac{b_1^2}{4} - d_1\, c_2\, w_3}\right)\right]}_{\mathbb{P}^1_{0C}}$$

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At $b_{2;0,0} = 0 = c_{1;0,0}$, $\mathbb{P}^1_{0C}$ factorises, realising $\mathbf{2}_3\, \mathbf{2}_3\, \overline{\mathbf{1}}_3$-coupling

Yukawa Couplings

Works for more complicated curves, e.g. $\mathbf{2}_3$ over $W_2 \cap$
$$\underbrace{\{b_0^2\, d_1^2 + b_0\,(b_1^2\, d_2 - b_1\, d_0\, d_1 - 2\, c_2\, d_1\, d_2\, w_3) + c_2\, w_3\,(d_0^2\, d_1 - b_1\, d_0\, d_2 + c_2\, d_2^2\, w_3)\}}_{\mathcal{D}_+\,\mathcal{D}_-/\boldsymbol{d_1^2}}.$$

- $\mathbf{2}_3\,\mathbf{2}_3\,\overline{\mathbf{1}}_2$-coupling over
  $W_2 \cap \{\mathcal{D}_+\} \cap \{\mathcal{D}_-\} = W_2 \cap \{b_0\, d_1 - c_2\, d_2\, w_3\} \cap \{b_1\, d_2 - d_0\, d_1\}$

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- $\mathbf{2}_3\,\mathbf{2}_3\,\overline{\mathbf{1}}_2$-coupling over $W_2 \cap \{\mathcal{D}_+\} \cap \{\mathcal{D}_-\} = W_2 \cap \{b_0\, d_1 - c_2\, d_2\, w_3\} \cap \{b_1\, d_2 - d_0\, d_1\}$
- Distinguish $\mathbf{2}_2\,\mathbf{2}_3\,\overline{\mathbf{1}}_4$- and $\mathbf{2}_2\,\overline{\mathbf{2}}_3\,\overline{\mathbf{1}}_6$-coupling geometrically: First arises over $W_2 \cap \{\mathcal{C}_\pm\} \cap \{\mathcal{D}_\pm\}$, second over $W_2 \cap \{\mathcal{C}_\pm\} \cap \{\mathcal{D}_\mp\}$

Yukawa Couplings

Apply directly to codimension 3 loci, e.g. to verify existence of $\overline{\mathbf{1}}_5\,\mathbf{1}_6\,\mathbf{1}_6$-coupling by making fibre enhancement explicit:
'Prime ideal technique' [Cvetic, Grassi, Klevers, Piragua '13; cf. talk by Mirjam Cvetic]
$\Rightarrow$ curves intersect over
$\{c_1\} \cap \{c_2\} \cap \{b_1\, d_0\, d_1 - b_2\, d_0^2\, w_2\, w_3 - b_0\, d_1^2 - b_1^2\, d_2 + 4\, b_0\, b_2\, d_2\, w_2\, w_3\}$,
explicitly see factorisation of fibre by inserting $c_1 = c_2 = 0, b_1 = \ldots$ into hypersurface equation.

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explicitly see factorisation of fibre by inserting $c_1 = c_2 = 0, b_1 = \ldots$ into hypersurface equation.
$\Rightarrow$ All couplings allowed by $U(1)$ selection rules realised in geometry.

Summary & Outlook

- Using toric geometry we constructed F-theory compactifications with Standard Model gauge group and a further $U(1)$ $\Rightarrow$ Which allow weak coupling limit, which are genuinely non-perturbative? Which can be embedded into an ($SU(5)$-)GUT model?
- We find an abundance of matter states with all gauge invariant Yukawa couplings present $\Rightarrow$ Which (if any) fluxes give correct chiral spectrum and/or forbid problematic couplings? Are the mixed $U(1)$ anomalies cancelled?
- Is it possible reduce the gauge group's centre (e.g. by $\mathbb{Z}_6$) to restrict matter spectrum?

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Thank you for your attention!