Ensembles of F-theory Flux Vacua and the Middle Cohomology of Calabi-Yau Fourfolds

based on [1401.5908] with Y.Kimura and T.Watari
and [1407.x] with T.Watari

aim: bring together

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- number of generations ?
- extra U(1)s ?
- discrete symmetries ?

For different tweaks of my background, which one is more natural ?

Let's set up an ensemble of vacua.

- Elliptic Calabi-Yau fourfold Z with base B_3
- fixed gauge group (= degenerate fibre type) along S in B_3
- $G_4 = G_{\rm fix} + G_{\rm scan}$
- use $G_{\rm fix}$ for chirality (... GUT breaking ...),
- vary $G_{\rm scan}$ in $H_{\rm scan}$; talks to complex structure moduli

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For a SUSY minimum, G_4 is in

$$H^{2,2}(Z) = H_V^{2,2}(Z) \oplus H_{RM}^{2,2}(Z) \oplus H_H^{2,2}(Z)$$

G_{scan} should not be in $H_V^{2,2}(Z) \oplus H_{RM}^{2,2}(Z)$; want ensemble without (further) gauge symmetry breaking

Count vacua using [Ashok,Denef,Douglas] method (case by case counting already very difficult for $K3 \times K3$). Index density on moduli space:

$$d\mu_I = \frac{(2\pi L_*)^{K/2}}{(K/2)!} \rho_I,$$

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Here:

$$L_* = \chi(Z)/24 - \frac{1}{2}\int_Z G_4 \wedge G_4$$
$$K = \dim_{\mathbf{R}}(H_{\text{scan}} \otimes \mathbf{R}) \,.$$

Note: this approx. is bad if $K \gg L_*$, [Ashok,Douglas] suggests $e^{\sqrt{2\pi K L_*}}$ as a better estimate.

How to compute

$$\dim_{\mathbf{R}}(H_{\text{scan}} \otimes \mathbf{R}) = h^4(Z) - h^{2,2}_V - h^{2,2}_{RM}$$

?????????

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Technique to find $h_V^{2,2}$ and $h_H^{2,2}$ in $h^{2,2}$: stratification

stratification = decomposition of Z into open even-dimensional submanifolds, **strata**,

$$Z = \amalg Z_{\Theta_n} \amalg Z_{\Theta_{n-1}} \cdots \amalg \text{pt}$$

Then compute Hodge-Deligne numbers of strata and use additivity of

$$e^{p,q}(Z_\Theta) = \sum_k (-1)^k h^{p,q}\left(H_C^k(Z_\Theta)\right)$$

to compute hodge numbers of Z !

This is particularly easy if Z is embedded into an ambient toric space:

p **– dimensional cone** σ **in fan** $\Sigma \sim (n-p)$ **– dim. stratum** $A_\sigma = (\mathbb{C}^*)^{n-p}$

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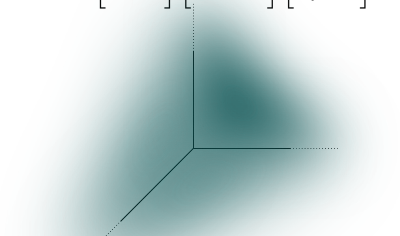
gives rise to stratification $Z = \amalg_{\sigma \in \Sigma} Z_\sigma$ via intersection, e.g.

$$\mathbb{P}^2 = \left[(\mathbb{C}^*)^2\right] \left[\amalg_{i=1}^3 \mathbb{C}^*\right] \left[\amalg_{j=1}^3 \mathrm{pt}\right]$$



$e^{p,q}(Z_\sigma)$ easy to compute (in principle) [Danilov-Khovanskii 1987], e.g.

$$e^{p,q}(T^2) = \left| \begin{array}{cc} -1 & 1 \\ -8 & -1 \\ \hline \end{array} \right. + 3 \left| \begin{array}{cc} & \\ 3 & \\ \hline \end{array} \right. + 0 = \left| \begin{array}{cc} -1 & 1 \\ 1 & -1 \\ \hline \end{array} \right.$$

using Newton polytopes.

Using these rechniques we can compute $h^{2,2}$ from combinatorics and identify $h_V^{2,2}$ for toric hypersufaces (similar to [Batyrev '93]) !

Trick: $Z = (Z \setminus Y) \amalg Y$, $Y = \cup D_i$, for toric divisors D_i, vertical contributions all come from Y.

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\begin{aligned}
h_V^{2,2}(Z) = & \ell_1(\Delta_{\leq n-2}) + \frac{n(n-1)}{2} \\
& - (n-1)\left[\ell(\Delta) - \sum_{\Theta^{[n-1]} \leq \Delta} \ell^*(\Theta^{[n-1]}) - \sum_{\Theta^{[n-2]} \leq \Delta} \ell^*(\Theta^{[n-2]}) - 1\right] \\
& + \cdots
\end{aligned}
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It turns out that $h_V^{2,2} + h_H^{2,2} = h^{2,2}$ holds in this case.

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Lets apply this to some examples !

F-theory with SU(5) along $S = \mathbb{P}^2$ in $B_3 = \mathbb{P}^1$ bundle over $\mathbb{P}^2$.

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Switch on G_{fix} to generate chirality as done in [Marsano, Saulina, Schäfer-Nameki; Grimm, Hayashi]

$$L_* = \frac{2163}{4} + \frac{125}{8} n(n+7) - \frac{5 N_{\text{gen}}^2}{2(18-n)(3-n)}$$

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n	-3	-2	-1	0	1	2	3
$h^{3,1}_*$	1249	1423	1723	2148	2698	3373	4173
$h^{2,2}_{H*}$	5057	5755	6955	8655	10855	13555	16756
$h^{2,2}_{V*}$	9	9	9	9	9	9	8
$L^{\max}_*$	237	297	387	507	657	837	[1047]
K	7557	8603	10403	12953	16253	20303	25104

for $n = 0$:

	$h^{1,1}$	$h^{3,1}$	$h^{2,2}_{V*}$	$h^{2,2}_{H*}$	$\chi(Y)$	K
no gauge group	3	3277	4	13160	19728	19716
SU(5) model	7	2148	9	8655	12978	12953
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- Comparing 'cost' of enhancing gauge group, D_n (and E_k) are favoured starting from *SU* groups.

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