Hierarchy from the product landscape

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Lv, Sun, Wu, [1405.0440] and more.
Sumitomo, Tye, [1204.5177], [1209.5086], [1211.6858].

Outline

Hierarchy problems from vacuum distribution point of view

The product landscape and preferred scales

Efficiency and practicality

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Hierarchy and naturalness

What is Hierarchy?

- A fundamental theory gives everything naturally.
- Dimensionless parameters have natural values of order 1.
- Parameters with mass dimensions have natural values around the fundamental mass scale.
- But their observed values may be too small.
- e.g., $m_{\text{Higgs}} \ll m_{\text{bare}}$, $\Lambda \ll m_{\text{SUSY}} \ll m_{\text{Planck}}$.

Relation to fine tuning

- Hierarchy: Large $\to$ small.
- Symmetries may help to get 0, but not a small value.
- Canceling large values $\Rightarrow$ a small value needs fine tuning.
- e.g., $m_{\text{Higgs}} \Leftarrow$ tuning of m_{bare} and quantum corrections.

Stabilization and Selection

Tuning or stabilization?

- Free parameters are bad for fundamentalists.
- Parameters fixed $\rightarrow$ dynamical fields stabilized at low energy.
- Eventually, parameters are fixed by symmetries, topological invariants, quantizations, etc. (eg. flux compactification)
- Parameters fixed OK, but why at small values (if possible)?

The string landscape (anthropic or not)

- Metastable vacua, different physics $\leftarrow$ string theory or others.
- e.g., flux compactification, Calabi-Yau's, quantized fluxes $\rightarrow$ low energy superpotentials, moduli stabilization, vacua.
- We hope there is at least one vacuum with the correct world.
- Hierarchy is rare in a trivial distribution (flat or smooth) $\Rightarrow$ possibly no "correct" vacuum $\Rightarrow$ hierarchy problem persists.

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Ways to hierarchy

Non-trivial distributions

- Exponential factors: Order 1 on powers $\rightarrow$ exponentially small.
- e.g., dynamical SUSY breaking $F = e^{-8\pi^2/g^2}$.
- e.g., Randall-Sundrum models $m = e^{-ky} m_0$.

More hierarchy from original hierarchy

- Transfer and accumulation of hierarchy.
- e.g., the seesaw mechanism $m = m_0^2/M$.
- e.g., large extra dimensions, $M_{\text{Planck}}^2 = M_{\text{Planck}(4+n)}^{2+n} R^n$.
- e.g., loop factors $1/(16\pi^2) < 1$, many loops.

New attempt: The product landscape

- Multiplying random variables $\rightarrow$ non-trivial distributions.
- Hierarchy may be preferred by distribution peaks.

The distribution of multiplying ramdom variables

The distribution of $z = x_1 \cdots x_n$

- $x_1, \ldots, x_n$ with distribution $P(x_1), \ldots, P(x_n)$,

$$P(z) = \int P(x_1) \cdots P(x_n) \delta(z - x_1 \cdots x_n) dx_1 \cdots dx_n$$
$$= \int P(x_1) \cdots P(x_{n-1}) \frac{P(z/(x_1 \cdots x_{n-1}))}{x_1 \cdots x_{n-1}} dx_1 \cdots dx_{n-1}.$$

- $P(x_i) = 1$ for $x_i \in (0, 1) \Rightarrow P(z) = (-\log z)^{n-1}/(n-1)!$.

Other distributions (assuming $P(x_i) = 1$ for $x_i \in (0, 1)$)

- $z = x_1^n \Rightarrow P(z) = z^{-1+1/n}/n$.
- $z = x_1^n \cdots x_m^n \Rightarrow P(z) = z^{-1+1/n}(-\log z)^{m-1}/(n^m(m-1)!)$.
- $z = x_1^m x_2^n \Rightarrow P(z) = (z^{-1+1/n} - z^{-1+1/m})/(n - m)$.

Can we get hierarchy?

The distribution of scales

- All previous $P(z)$'s are singular at the origin $\Rightarrow$ lowest values seem to be preferred in the distribution.
- We must look at logarithmic distributions, e.g. $(10^{-11}, 10^{-9})$ versus $(10^{-2}, 1)$ to compare 10^{-10} versus 10^{-1}.
- $P(z) = z^{-1}$ is the actual distribution uniform on all scales.
- All previous $P(z)$'s are less singular than z^{-1}, so they have preferred scales (which are not the lowest).

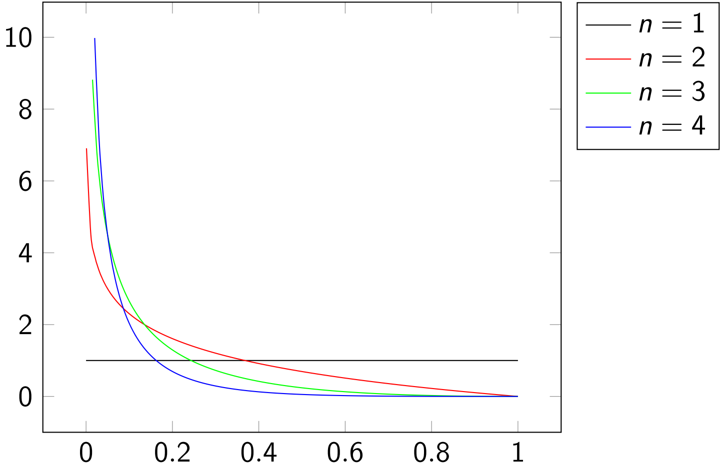
The preferred scale

- $P(z)dz = P_{\log}(z)d(\log z) \Rightarrow P_{\log}(z) = zP(z)$.
- The distribution peak of $P_{\log}(z) \Leftarrow \partial_z P_{\log}(z) = 0$.
- e.g., for $z = x_1 \cdots x_n$, the peak is at $z_0 = e^{1-n}$.
- $x_i \in (0, 1)$ flat, $\bar{x}_i = 1/2$, excluding this effect, $z_0' = e^{1-n} 2^n$.

Distribution plots

Plots on linear scale

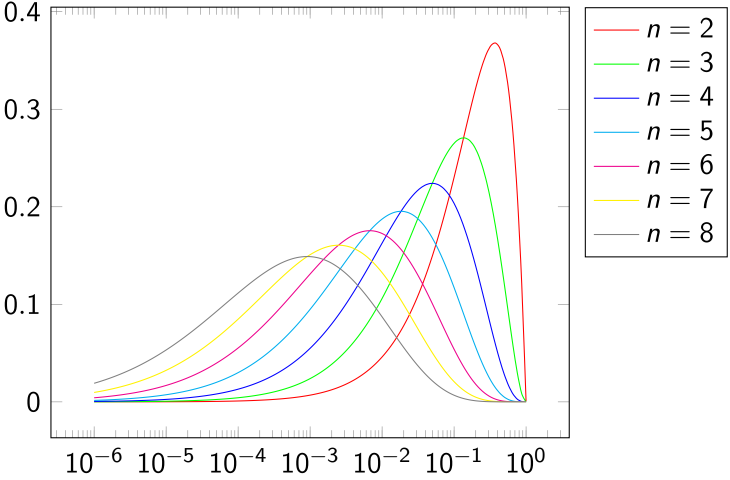
- $z = x_1 \cdots x_n$, $P(x_i) = 1$ for $x_i \in (0, 1)$, $P(z)$ is singular:



Distribution plots (2)

Plots on logarithmic scale

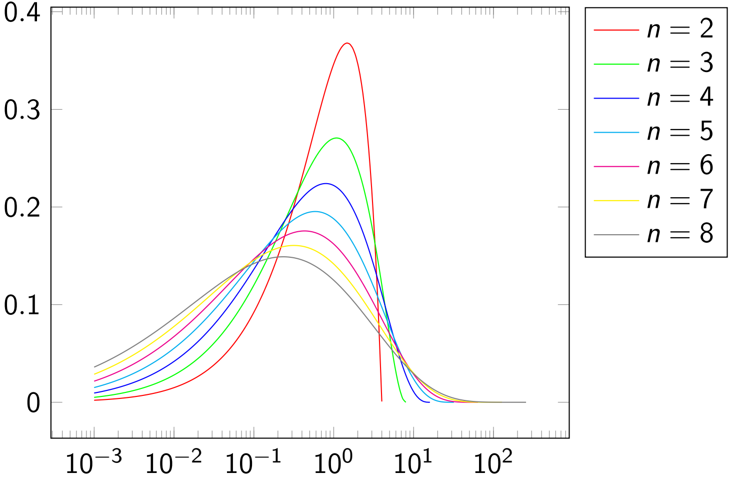
- $P_{\log}(z) = zP(z)$ (the peak is at $z = e^{1-n}$):



Distribution plots (3)

Plots on logarithmic scale

- Rescaling z to $\bar{z} = 1$, $P_{\log}(z)$ (the peak is at $z = e^{1-n}2^n$):



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Contributions to hierarchy

Effects purely from the product landscape

- $z_0' = e^{1-n}2^n$ is the "pure effect" from the product landscape.
- $n \sim 10$ for $z_0' = 0.1$, quite slow to generate hierarchy.
- If $\Lambda/M_{\text{Planck}} \sim 10^{-122}$ comes from this exclusively, $n \sim 1000$.
- Physical observables must be expressed as a single term with n factors, quite uncommon for model building.

Other effects

- $\bar{x}_i = 1/2 \Leftarrow$ flat distribution in $(0, 1)$.
- Similar small parameters are (almost) natural in any EFT, e.g., field values should be smaller than Λ_{cutoff}.
- For IIB flux vacua, $r = \Lambda_{\text{EFT}}/\Lambda_{\text{String}} \lesssim 1 \Rightarrow \bar{x}_i \sim r \lesssim 1$.
- Accumulation of small hierarchies $\Rightarrow z_0' \sim r^n \ll 1$ for large n.
- Hierarchy from both effects.

Application

Multi-moduli IIB flux vacua

- Large volume scenario, SUSY with $W_0 \neq 0 \Rightarrow \Lambda \neq 0$:

$$W_0 = -\frac{2(c_1 + sc_2)\prod_{i=1}^{N_{\rm CS}}(1 - sr)}{\prod_{i=1}^{N_{\rm CS}}(1 + sr)}.$$

- $\Lambda/M_{\rm Planck} \sim 10^{-122}$ requires $N_{\rm CS} \sim 200$, common in CY data.
- The product landscape contributes $\sim 10^{-27}$, the rest from other effects (accumulation, constraints from stability, etc.).

Summary

- Hierarchy $\leftarrow$ distributions, predictions $\leftarrow$ logarithmic plots.
- $\sim e^{1-n}2^n \leftarrow$ the product landscape.
- Combining other effects, practical model building.
- **END**