de Sitter Vacua in Type IIB String Theory

http://arxiv.org/pdf/1402.5112.pdf

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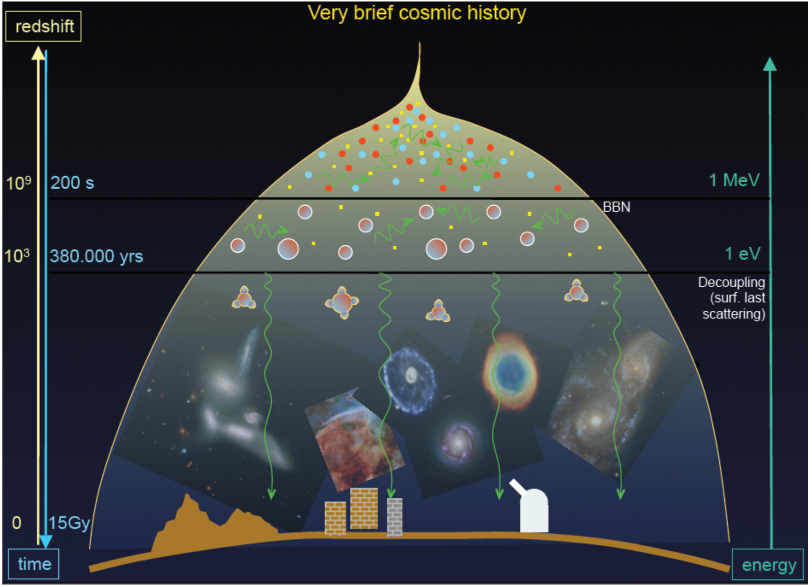
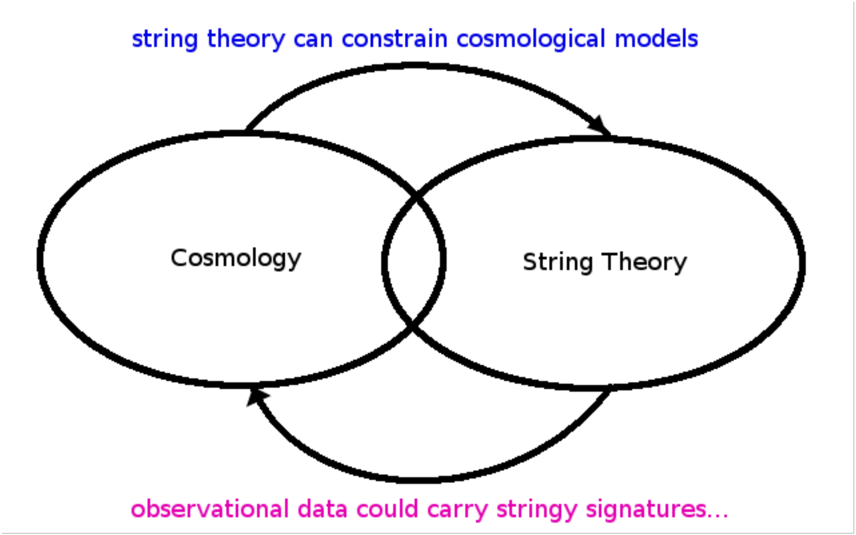
String theory, if correct, describes the very early universe:



Cosmology is a unique potential observational window on string theory:



Obsservational constraints on string theory

The data suggests

1. we are in a homogeneous and isotropic universe dominated by a **positive cosmological constant**.
2. there was a period of exponential expansion in the early universe (inflation)

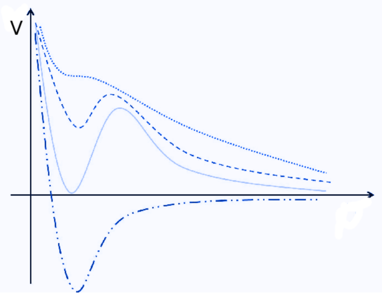
Both imply a quasi (time-dependent) **de Sitter spacetime**.

If string theory is the correct UV description of our universe, it must be possible to realize de Sitter in string theory!

The Usual Approach

The usual approach is to construct a string compactification which is AdS in 4D and then **uplift**:

- start with known SUSY AdS solution
- uplift to de Sitter



- generally done in 4D EFT or at $\mathcal{N} = 1$ supergravity level

Our focus

For a consistent compactification, the EOM derived from the EFT must be equivalent to the 10D EOM [e.g. Buchel, 0312076]

so we asked:

- **What do things look like from the 10 D point of view?**
- **What can source this uplift?**

Direct Product Space

Consider EH action coupled to matter:

$$S_{\text{total}} = \frac{1}{\mathcal{K}_D} \int d^D x \sqrt{-g_D} R_D + \int d^D x \mathcal{L}_{\text{int}}, \quad (1)$$

which has Einstein equation

$$R_{MN} \;=\; \frac{\mathcal{K}_D}{2} \left(T_{MN} - \frac{1}{D-2} g_{MN} T \right). \quad (2)$$

For $\mathcal{M} = \mathcal{M}_4 \times \mathcal{M}_{D-4}$ and $ds_D^2 \equiv g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n$, we find

$$R_4 = -\frac{\mathcal{K}_D}{2(D-2)} \left[T^\mu_\mu (6-D) + 4 T^m_m \right], \quad (3)$$

$R_4 = g^{\mu\nu} R_{\mu\nu}$ is the 4 D curvature.

Condition for dS

$$R_4 = -\frac{\mathcal{K}_D}{2(D-2)}\left[T^{\mu}_{\mu}(6-D) + 4T^{m}_{m}\right], \tag{4}$$

implies the following condition for de Sitter:

$$R_4 > 0 \Rightarrow (D-6)\,T^{\mu}_{\mu} > 4T^{m}_{m}.$$

$$D = 10 \Rightarrow T^{\mu}_{\mu} > T^{m}_{m}$$

Fluxes

For fluxes and scalar fields the no-go theorem was formulated by Gibbons [0301117] and Maldacena-Nunez [0007018] :

$$\mathcal{L}^F_{\rm int} = -\sqrt{-G_D} F_{a_1 \dots a_q} F^{a_1 \dots a_q} \tag{5}$$

$$R_4 > 0 \Rightarrow 4(1-q)F^2 > -F_{\mu a_2 .. a_q} F^{\mu a_2 .. a_q} q\,(D-2)\,. \tag{6}$$

- **Case 1**: All legs in $\mathcal{M}_6$. Then $F^2 > 0$ and $F_{\mu a_1 \dots} F^{\mu a_1 \dots} = 0$ so the condition cannot be satisfied for $q > 1$.
- **Case 2**: $q \geq 4$ and legs in all 4 $\mathcal{M}_4$ directions. Eq (6) is only satisfied for $D < q + 1$ i.e. $q > 9$ - but no 10 form fluxes in ST!

(note that we demand Poincaré invariance in the noncompact directions)

Scalar fields

$$\mathcal{L}^{\phi}_{\text{int}} = -\sqrt{-g_D}\left(\partial_M\phi\partial^M\phi + V(\phi)\right) \quad (7)$$

$$T^{\phi}_{MN} = -g_{MN}\left(\partial_K\phi\partial^K\phi + V(\phi)\right) + 2\partial_M\phi\partial_N\phi. \quad (8)$$

The condition for $R_4 > 0$ becomes

$$\partial_\mu\phi\partial^\mu\phi + V(\phi) > 0 \quad (9)$$

$$\Rightarrow V(\phi) > 0 \quad (10)$$

But no available scalars in 10 D IIB string theory!

The same conclusion holds for D branes and O planes in direct product space. But we should really be looking at a warped metric ansatz in these cases....

Warped Product Space

We look for warped solutions

$$ds^2 = e^{2A(y)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n, \qquad (11)$$

finding

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - \tilde{g}_{\mu\nu}e^{4A}\tilde{\nabla}^2 A \qquad (12)$$

$$\tilde{\nabla}^2 e^{4A} = \tilde{R}_4 + \frac{e^{2A}G_3\cdot\bar{G}_3}{12\,\mathrm{Im}\tau} - \frac{e^{2A}\hat{F}_{\mu abcd}\hat{F}^{\mu abcd}}{4\cdot 4!} + e^{-6A}\partial_m e^{4A}\partial^m e^{4A}$$

$$+ \frac{\kappa_{10}^2}{2}e^{2A}\sum_i\left[T^m_{m\ (Op/\bar{O}p)i} - T^\mu_{\mu\ (Op/\bar{O}p)i}\right] \qquad (13)$$

$$+ \frac{\kappa_{10}^2}{2}e^{2A}\sum_j\left[T^m_{m\ (Dp/\bar{D}p)j} - T^\mu_{\mu\ (Dp/\bar{D}p)j}\right]. \qquad (14)$$

D-branes/anti D-branes

The action for a *Dp*- brane in the Einstein frame is

$$S_{Dp} = -\int d^{p+1}\sigma\ T_p\ e^{\frac{\phi(p+1)}{4}} \sqrt{-\det(g_{ab}+\tilde{F}_{ab})} + \mu_p \int \left(C \wedge e^{\hat{F}}\right)_{p+1} \quad (15)$$

where $\tilde{F}_{ab} = F_{ab} + B_{ab}$, F_{ab} is the gauge field on the brane, and g_{ab}, B_{ab} are the pullbacks of the metric and the antisymmetric 2 form.

- $T_p > 0, \mu_p > 0$: branes
- $T_p > 0, \mu_p < 0$: anti-branes

Note that the CS term is topological and therefore doesn't enter the Einstein equations:

$$T^{CS}_{mn} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{CS}}{\delta g^{mn}} = 0 \quad (16)$$

D-branes/ anti D-branes

Calculate the stress-energy tensor to find

$$T^{\mu}_{\ \mu\,(Dp/\overline{Dp})} = -4\,T_p N \tag{17}$$

$$T^{m}_{\ m\,(Dp/\overline{Dp})} = -(p-3)\,T_p N \tag{18}$$

where

$$N = e^{\phi(p+1)/4} \frac{\sqrt{-\mathrm{det}(g_{ab} + \tilde{F}_{ab})}\sqrt{\det g_{9-p}}}{\sqrt{-\mathrm{det} g_{10}}} \delta^{9-p}(x - \overline{x}) \tag{19}$$

Thus there is no way to get $T^{\mu}_{\mu} > T^{m}_{m} \iff \tilde{R}_4 > 0$ for $p = 3, 5, 7$ (brane or antibrane).

Orientifold planes

$$S_{Op} = -\int d^{p+1}\sigma \; T_{Op} e^{\frac{\phi(p+1)}{4}} \sqrt{-f} + \mu_{Op} \int C_{p+1}, \qquad (20)$$

where $T_{Op} < 0$. Calculate the stress-energy tensor to find

$$T^{\mu}_{\mu\,(Op)} = 4|T_{Op}|N_O \qquad (21)$$

$$T^{m}_{m\,(Op)} = -(p-3)|T_{Op}|N_O \qquad (22)$$

where

$$N_O = e^{\phi(p+1)/4} \frac{\sqrt{-\mathrm{det} g_{ab}}\sqrt{\det g_{9-p}}}{\sqrt{-\mathrm{det} g_{10}}} \delta^{9-p}(x-\bar{x}) \qquad (23)$$

Looks like these might do the job...
However, O-planes are inherently local

Orientifold planes and warping

Integrate over the internal manifold $\tilde{\mathcal{M}}_6$:

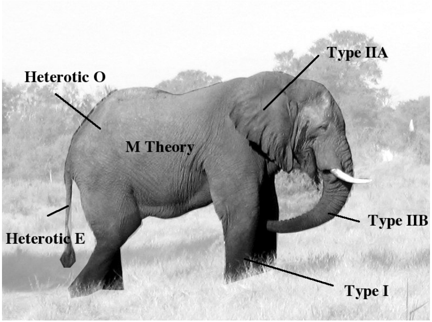
$$C = \tilde{V}_6 \tilde{R}_4 + \int d^6x \sqrt{\tilde{g}_6} \mathcal{I}_{\text{global}} + \int d^6x \sqrt{\tilde{g}_6} \frac{\kappa_{10}^2}{2} e^{2A} \sum_i \left[T^m_m - T^\mu_\mu \right] ,$$

If $\mathcal{M}_6$ smooth, $C = 0$. But we have local sources!

- can smear branes $\rightarrow \tilde{R}_4 < 0$.
- can't smear Orientifolds: find divergent warp factor and Riemann tensor
- known localized solutions have $\tilde{R}_4 = 0$ [Blaback et al., 1009.1877]]
- pretend I can smear O-planes-> need solution. Sign of $\tilde{R}_4$ ambiguous.
- points to a need to go beyond classical theory

M theory

- M theory is a cleaner set-up for investigating the role of curvature corrections R^n:
- O planes taken into account by geometry in M theory
- For nonsingular manifolds need HD corrections for anomaly cancellation
- $G_3, \tau, F_5 \rightarrow G_4$



M theory

$$
\begin{aligned}
S &= S_{bulk} + S_{brane} + S_{corr} \\
S_{bulk} &= \frac{1}{2\kappa^2}\int \mathrm{d}^{11}x\,\sqrt{-g}\left[R - \frac{1}{48}G^2\right] - \frac{1}{12\kappa^2}\int C\wedge G\wedge G, \\
S_{brane} &= -\frac{T_2}{2}\int \mathrm{d}^3\sigma\sqrt{-\gamma}\left[\gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N g_{MN} - 1\right. \\
&\quad \left. + \frac{1}{3!}\tilde{\epsilon}^{\mu\nu\rho}\partial_\mu X^M\partial_\nu X^N\partial_\rho X^P C_{MNP}\right],
\end{aligned}
$$

There are multiple possible sources for the corrections:

$$
S_{corr} \supset -T_2\int C\wedge X_8, \left(J_0 - \frac{1}{2}E_8\right), \ldots
$$

A parametrization

The M theory uplift of our metric ansatz is

$$\begin{aligned} ds^2 &= e^{2A(y,t)}(-dt^2 + \eta^{ij} dz_i dz_j) + e^{2B(y,t)} \tilde{g}_{mn} dy^m dy^n + e^{2C(y,t)} |dz|^2 \\ &= \frac{1}{(\Lambda(t)\sqrt{h})^{4/3}}(-dt^2 + \eta^{ij} dz_i dz_j) \\ &\quad + h^{1/3}\left[\frac{\tilde{g}_{mn} dy^m dy^n}{(\Lambda(t))^{1/3}} + (\Lambda(t))^{2/3}|dz|^2\right] \end{aligned}$$

which is the uplift of

$$ds^2 = \frac{1}{\Lambda t^2 \sqrt{h}}(-dt^2 + \eta^{ij} dz_i dz_j + dx_3^2) + \sqrt{h}\tilde{g}_{mn} dy^m dy^n,$$

where $\Lambda(t) = \Lambda |t|^2$.

Correction ansatz

Rather than calculate T^{corr}_{MN} explicitly (for unknown R^n, G^n terms) , we parametrize the contributions of S_{corr} with an ansatz:

$$T^{MN}_{corr} = \frac{-2}{\sqrt{-g}} \frac{\delta \hat{S}_{ntop}}{\delta g_{MN}}\Big|_{g,C} \equiv \sum_i [\Lambda(t)]^{\alpha_i + 1/3} \mathcal{C}^{MN,\, i} \qquad (24)$$

where

$$\mathcal{C}^i_{MN} = g_{MN} \tilde{\mathcal{C}}_i - 2 \frac{\delta \tilde{\mathcal{C}}_i}{\delta g^{MN}} . \qquad (25)$$

M theory condition

In the absence of correction terms, we find

$$0 = \frac{1}{12}\int d^8x\sqrt{\tilde{g}}\,\widetilde{G}_{mnpa}\widetilde{G}^{mnpa} + 12\Lambda\int d^8x\sqrt{\tilde{g}}\,h^2 + 2\kappa^2 T_2 n_3 \quad (26)$$

All terms are positive definite - **no way to get dS!**

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Including corrections, we find conditions

$$\frac{1}{2}\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}_a^{a,\,i}\rangle + \frac{1}{4}\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}_m^{m,\,i}\rangle < \frac{2}{3}\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}_\mu^{\mu,\,i}\rangle, \qquad (27)$$

$$\sum_{\{\alpha_i\}\neq 0} a^{\alpha_i}\langle\tilde{\mathcal{C}}_\mu^{\mu,\,i}\rangle \;>\; 0. \qquad (28)$$

Conclusions

- It is difficult to see how to construct dS solutions in String Theory from the 10 POV.
- Fluxes, D branes, anti D branes and orientifolds do not work at the classical level.
- Including R^n corrections may help (calculation in progress)

Questions

- Which curvature corrections, if any, satisfy these conditions?
- Which curvature corrections should be included?
- Result should be consistent with e.g. Kähler uplifting (4 D theory), heterotic no-go, etc...
- Which assumptions to drop?

Orientifold planes in direct product space

With the metric ansatz

$$ds^2 = g_{\mu\nu}(x^\mu)dx^\mu dx^\nu + g_{mn}(x^m)dx^m dx^n, \tag{29}$$

we find

$$R_4(x^\mu) = -\frac{G_3 \cdot \bar{G}_3}{12\,\mathrm{Im}\tau} + \frac{\hat{F}_{\mu abcd}\hat{F}^{\mu abcd}}{4 \cdot 4!} + \frac{\kappa_{10}^2 N_f}{2}\left(T^{\mu\,\mathrm{loc}}_{\mu} - T^{m\,\mathrm{loc}}_{m}\right). \tag{30}$$

The RHS is independent of the internal coordinates, so can be evaluated at any point, including away from the Oplanes. Then we have

$$R_4 \leq 0. \tag{31}$$