Heterotic Supergravity, Moduli, and Connections

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based on work together with Xenia de la Ossa

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Introduction
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- I will show how the Strominger System can be put in terms of a holomorphic structure $\overline{D}$ on a bundle $Q \to X$.
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- Easy to find the first order deformation space $T\mathcal{M}$, computed as a cohomology $H^{(0,1)}_{\overline{D}}(Q)$. Obstructions lie in $H^{(0,2)}_{\overline{D}}(Q)$, etc.
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- Gives a natural candidate for the Kähler potential and Kähler metric on $T\mathcal{M}$.

See also [Anderson, Gray, Sharpe 2014], and [Garcia-Fernandez 2013, Baraglia, Hekmati 2013] where similar structures were studied from the perspective of heterotic generalised geometry and Courant algebroids.
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- There is a holomorphic connection \( \nabla \) on \( TX \) whose curvature \( R \) also satisfies the Yang-Mills condition [Ivanov 2009].
Holomorphic Structures
Deformations of Complex Structure

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Holomorphic Structures

Holomorphic Bundle
Holomorphic Tangent Bundle
Conditions from Bianchi Identity

Conclusions
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We impose the condition $H^{(0,1)}(X) = 0$. This ensures equality of the cohomologies, and also ensures a well-defined dilaton.

This condition also implies that $H^{(0,2)}(X) = 0$, which we will need later when considering the full Strominger system.
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$$0 \rightarrow \text{End}(V) \rightarrow Q_1 \rightarrow TX \rightarrow 0 \quad (*)$$

$$\overline{\partial}_1 = \overline{\partial} + F,$$

where $\overline{\partial}$ is the holomorphic structure on the individual bundles, and $F = F \in \Omega^{(0,1)}(\text{End}(V) \otimes T^*X)$. 
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Note: $\overline{\partial}_1^2 = 0 \iff \overline{\partial}\mathcal{F} = 0 \Rightarrow F \in H^{(0,1)}(\text{End}(V) \otimes T^*X)$, i.e. $\mathcal{F}$ is the field strength of some bundle (the Atiyah class).
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It follows that $F$ defines a map between cohomologies:

\[ F : H^{(q,p)}(TX) \rightarrow H^{(q,p+1)}(\text{End}(V)). \]
$H^{(0,1)}_{\d_{1}}(Q_{1})$ then computes simultaneous deformations of $\bar{\partial}_{A}$ and $\bar{\partial}$ [Atiyah 1957].
\[ H^{(0,1)}_{\overline{\partial}_1}(Q_1) \] then computes simultaneous deformations of \( \overline{\partial}_A \) and \( \overline{\partial} \) [Atiyah 1957]. Computed by long exact sequence of \( (\ast) \) [Anderson, Gray, Lukas, Ovrut 2010]:
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0 \to H^{(0,1)}(\text{End } V) \to H^{(0,1)}(Q_1) \to H^{(0,1)}(TX) \xrightarrow{\mathcal{F}} H^{(0,2)}(\text{End } V) \to \ldots ,
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where we have set $H^0(TX) = 0$, as $TX$ is stable of degree zero ($\omega \cdot R = 0$).
Holomorphic Bundle

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By exactness, we find

$$H^{(0,1)}(Q_1) \cong H^{(0,1)}(\text{End } V) \oplus \ker(F),$$

where $\ker(F) \subseteq H^{(0,1)}(TX)$ are cplx. structure moduli, deformations $\bar{\partial}$, and $H^{(0,1)}(\text{End } V)$ are bundle moduli.
Holomorphic Bundle

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where $\ker(\mathcal{F}) \subseteq H^{(0,1)}(TX)$ are cplx. structure moduli, deformations $\bar{\partial}$, and $H^{(0,1)}(\text{End } V)$ are bundle moduli.

Allowed complex structure moduli $= \ker(\mathcal{F})$ can be derived from F-terms of a Gukov-Vafa-Witten type superpotential [Anderson et al 2010]:

$$W = \int_X (H + i \omega) \wedge \Omega = \int_X \left( dB + \frac{\alpha'}{4} (\omega^A_{CS} - \omega^\nabla_{CS}) + i \omega \right) \wedge \Omega .$$

Heterotic Supergravity and Moduli – 8
Compatibility with the equations of motion requires the existence of a holomorphic Yang-Mills connection $\overline{\partial} \nabla$ on $TX$ [Ivanov 2009].
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$$0 \rightarrow \text{End}(TX) \rightarrow Q_2 \rightarrow Q_1 \rightarrow 0 \quad (*)$$

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- $H_{\overline{\partial}_2}^{(0,1)}(Q_2)$ computes simultaneous deformations of $\overline{\partial}_\nabla$ and $\overline{\partial}_1.$ Computed by long exact sequence of $(*)$. 

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Holomorphic Tangent Bundle
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- Atiyah class: $\mathcal{R} = R \in \Omega^{(0,1)}(\text{End}(TX) \otimes T^*X)$. $\bar{\partial}_2^2 = 0$ iff $\bar{\partial}R = 0$.
- $H^{(0,1)}_{\bar{\partial}_2}(Q_2)$ computes simultaneous deformations of $\bar{\partial}_\nabla$ and $\bar{\partial}_1$. Computed by long exact sequence of $\ast$.

- Note: "Moduli" related to deformations of $\bar{\partial}_\nabla$ are not physical. Correspond to field [Sen 1986; de la Ossa, Svanes 2014].
Conditions from Bianchi Identity

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We now want to include the heterotic Bianchi Identity (from heterotic anomaly)

\[ dH = 2\partial \partial^\dagger \rho = \frac{\alpha'}{4} \left( \text{tr} F^2 - \text{tr} R^2 \right) \ast, \]

where we have set \( \rho = \ast \omega \), and chosen a gauge where \( d\phi = 0 \).
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Note that as connections, \( \mathcal{F} \) and \( \mathcal{R} \) also act naturally as
\[ \mathcal{F}, \mathcal{R} : H^{(q,p)}(\text{End}(\ast)) \rightarrow H^{(q,p+1)}(T^* X) . \]
We now want to include the heterotic Bianchi Identity (from heterotic anomaly)

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Natural to add \( T^*X \), and consider a holomorphic structure

\[ D = \overline{\partial} + \mathcal{F} + \mathcal{R} + h \]

on \( Q = T^*X \oplus \text{End}(TX) \oplus \text{End}(V) \oplus TX \). Here \( h \in \Omega^{(0,1)}(T^*X \otimes T^*X) \).
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on \( Q = T^* X \oplus \text{End}(TX) \oplus \text{End}(V) \oplus TX \). Here \( h \in \Omega^{(0,1)}(T^* X \otimes T^* X) \).

Note: \( \overline{D}^2 = 0 \iff \mathcal{F} \wedge \mathcal{F} + \mathcal{R} \wedge \mathcal{R} + \overline{\partial} h = 0 \).
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Note: \( \overline{D}^2 = 0 \iff F \wedge F + R \wedge R + \overline{\partial} h = 0 \). Rescaling \( F \) and \( R \) appropriately (previos deformation problems are invariant under such rescalings), and sett \( h = \overline{\partial}^\dagger \rho + \overline{\partial} \) — closed, as a three-form to get \( (\star) \).
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Note: \( \overline{D}^2 = 0 \iff F \wedge F + R \wedge R + \overline{\partial}h = 0 \). Rescaling \( F \) and \( R \) appropriately (previous deformation problems are invariant under such rescalings), and set \( h = \overline{\partial}^\dagger \rho + \overline{\partial} - \) closed, as a three-form to get \( (\ast) \).

Hence, heterotic supergravity corresponds to a holomorphic structure \( \overline{D} \) on \( Q \).
Conditions from Bianchi Identity

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with extension class $\mathcal{H}$. 

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Note again that $\mathcal{H}$ defines a map between cohomologies:

$$\mathcal{H} : H^{(p,q)}(Q_2) \to H^{(p,q+1)}(T^*X).$$
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\( H^{(0,1)}_{\overline{D}}(Q) \) computes the first order heterotic moduli space \( TM \), at least when \( H^{(0,2)}(X) = 0 \). Technicality to ensure that deformations of \( \overline{D}, H^{(0,1)}_{\overline{D}}(Q) \), and deformations of anomaly cancellation agree [Anderson et al 2014, de la Ossa et al 2014].
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Long exact sequence in cohomology:

\[ \cdots \rightarrow H^0(Q_2) \xrightarrow{\mathcal{H}} H^{(0,1)}(T^*X) \rightarrow H^{(0,1)}(Q) \rightarrow H^{(0,1)}(Q_2) \xrightarrow{\mathcal{H}} H^{(0,2)}(T^*X) \rightarrow \cdots, \]
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\[ T\mathcal{M} = H^{0,1}(Q) \cong \left( H^{0,1}(T^*X)/\text{Im}(\mathcal{H}) \right) \oplus \ker(\mathcal{H}) \]

\[ \ker(\mathcal{H}) \subseteq H^{0,1}(Q_2) \]

\[ = \left( \ker(\mathcal{F}) \cap \ker(\mathcal{R}) \right) \oplus H^{0,1}(\text{End}(V)) \oplus H^{0,1}(\text{End}(TX)) \]

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Long exact sequence in cohomology:

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\[ TM = H^{(0,1)}(Q) \cong \left( H^{(0,1)}(T^*X) / \text{Im}(\mathcal{H}) \right) \oplus \ker(\mathcal{H}) \]

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\[ = \left( \ker(\mathcal{F}) \cap \ker(\mathcal{R}) \right) \oplus H^{(0,1)}(\text{End}(V)) \oplus H^{(0,1)}(\text{End}(TX)) \]

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For the hermitian moduli, we also mod out by \( \text{Im}(\mathcal{H}) \cong \text{Im}(\mathcal{F}) = \{ \mathcal{F}(\alpha) \mid \alpha \in H^0(\text{End}(V)) \} \), which automatically incorporates the Yang-Mills condition. In the case polystable bundles, this set is non-trivial.
Long exact sequence in cohomology:

\[ \cdots \to H^0(Q_2) \xrightarrow{\mathcal{H}} H^{(0,1)}(T^* X) \to H^{(0,1)}(Q) \to H^{(0,1)}(Q_2) \xrightarrow{\mathcal{H}} H^{(0,2)}(T^* X) \to \cdots, \]

\[ T\mathcal{M} = H^{(0,1)}(Q) \cong \left( \frac{H^{(0,1)}(T^* X)}{\operatorname{Im}(\mathcal{H})} \right) \oplus \ker(\mathcal{H}) \]

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\[ = \left( \ker(\mathcal{F}) \cap \ker(\mathcal{R}) \right) \oplus H^{(0,1)}(\operatorname{End}(V)) \oplus H^{(0,1)}(\operatorname{End}(TX)) \]

\[ \ker(\mathcal{F}) \cap \ker(\mathcal{R}) \subseteq H^{(0,1)}(TX), \]

\[ H^{(0,1)}(\operatorname{End}(TX)) \] are unphysical, corresponding to field redefinitions, and

\[ H^{(0,1)}(T^* X) \] are hermitian moduli.

For the hermitian moduli, we also mod out by \( \operatorname{Im}(\mathcal{H}) \cong \operatorname{Im}(\mathcal{F}) = \{ \mathcal{F}(\alpha) \mid \alpha \in H^0(\operatorname{End}(V)) \} \), which automatically incorporates the Yang-Mills condition. In the case polystable bundles, this set is non-trivial.

Note that the Bianchi Identity imposes the constraint that the moduli are in \( \ker(\mathcal{H}) \).
Conclusions
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- Easy to find Moduli, given in terms of deformations of $D$,

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Conclusions:

- The Strominger System can be put in terms of a holomorphic structure $\overline{D}$ on a bundle $Q$.
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Outlook, and work in progress:
Conclusions and Outlook

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Outlook, and work in progress:

- $\overline{D}$ gives a nice venue to study the moduli space $\mathcal{M}$. Obstructions live in $H^{(0,2)}(Q)$. Comes with a natural Kähler potential.
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Conclusions:

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- Higher orders in $\alpha'$? Does this structure survive? Work in progress with Xenia de la Ossa.
Thank you for your attention!