Heterotic Supergravity, Moduli, and Connections

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based on work togehter with Xenia de la Ossa

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Introduction

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See also [Anderson, Gray, Sharpe 2014], and [Garcia-Fernandez 2013, Baraglia, Hekmati 2013] where similar structures where studied from the perspective of heterotic generalised geometry and Courant algebroids.

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- There is a holomorphic connection ∇ on TX whose curvature R also satisfies the Yang-Mills condition [Ivanov 2009].

Holomorphic Structures

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This condition also implies that $H^{(0,2)}(X) = 0$, which we wil need later when considering the full Strominger system.

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$$0 \to \text{End}(V) \to \mathcal{Q}_1 \to TX \to 0 \qquad (*)$$
$$\overline{\partial}_1 \;=\; \overline{\partial} + \mathcal{F}\,,$$

where $\overline{\partial}$ is the holomorphic structure on the individual bundles, and $\mathcal{F} = F \in \Omega^{(0,1)}(\text{End}(V) \otimes T^*X)$.

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Note: $\overline{\partial}_1^2 = 0 \Leftrightarrow \overline{\partial}\mathcal{F} = 0 \Rightarrow F \in H^{(0,1)}(\text{End}(V) \otimes T^*X)$, i.e. $\mathcal{F}$ is the field strength of some bundle (the Atiyah class).

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It follows that $\mathcal{F}$ defines a map between cohomologies:

$$\mathcal{F}\ :\ H^{(q,p)}(TX) \to H^{(q,p+1)}(\text{End}(V))\ .$$

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By exactness, we find

$$H^{(0,1)}(\mathcal{Q}_1) \cong H^{(0,1)}(\text{End}\, V) \oplus \ker(\mathcal{F}) ,$$

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Allowed complex structure moduli $=$ $\ker(\mathcal{F})$ can be derived from F-terms of a Gukov-Vafa-Witten type superpotential [Anderson et al 2010]:

$$W = \int_X (H + i\text{d}\omega) \wedge \Omega = \int_X \left(\text{d}B + \frac{\alpha'}{4}(\omega^A_{CS} - \omega^\nabla_{CS}) + i\text{d}\omega\right) \wedge \Omega \, .$$

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- Note: ”Moduli” related to deformations of $\overline{\partial}_\nabla$ are not physical. Correspond to field [Sen 1986; de la Ossa, Svanes 2014].

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Conditions from Bianchi Identity

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We now want to include the heterotic Bianchi Identity (from heterotic anomaly)

$$\mathrm{d}H = 2\overline{\partial\partial}^{\dagger}\rho = \frac{\alpha'}{4}(\mathrm{tr}F^2 - \mathrm{tr}R^2) \quad (*)\ ,$$

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Natural to add T^*X, and consider a holomorphic structure

$$\overline{D} \;=\; \overline{\partial} + \mathcal{F} + \mathcal{R} + h$$

on $\mathcal{Q} = T^*X \oplus \mathrm{End}(TX) \oplus \mathrm{End}(V) \oplus TX$. Here $h \in \Omega^{(0,1)}(T^*X \otimes T^*X)$.

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Hence, heterotic supergravity corresponds to a holomorphic structure $\overline{D}$ on $\mathcal{Q}$.

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$H^{(0,1)}_{\overline{D}}(\mathcal{Q})$ computes the first order heterotic moduli space $T\mathcal{M}$, at least when $H^{(0,2)}(X) = 0$. Technicallity to ensure that deformations of $\overline{D}$, $H^{(0,1)}_{\overline{D}}(\mathcal{Q})$, and deformations of anomaly cancellation agree [Anderson et al 2014, de la Ossa et al 2014].

Conditions from Bianchi Identity

Long exact sequence in cohomology:

$$\ldots \rightarrow H^0(\mathcal{Q}_2) \xrightarrow{\mathcal{H}} H^{(0,1)}(T^*X) \rightarrow H^{(0,1)}(\mathcal{Q}) \rightarrow$$
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$$T\mathcal{M} = H^{(0,1)}(\mathcal{Q}) \cong \left(H^{(0,1)}(T^*X) \big/ \mathrm{Im}(\mathcal{H}) \right) \oplus \ker(\mathcal{H})$$

$$\ker(\mathcal{H}) \subseteq H^{(0,1)}(\mathcal{Q}_2)$$
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$$\ker(\mathcal{F}) \cap \ker(\mathcal{R}) \subseteq H^{(0,1)}(TX) \ ,$$

$H^{(0,1)}(\text{End}(TX))$ are unphysical, corresponding to field redefinitions, and $H^{(0,1)}(T^*X)$ are hermitian moduli.

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$H^{(0,1)}(\mathrm{End}(TX))$ are unphysical, corresponding to field redefinitions, and $H^{(0,1)}(T^*X)$ are hermitian moduli.

For the hermitian moduli, we also mod out by $\mathrm{Im}(\mathcal{H}) \cong \mathrm{Im}(\mathcal{F}) = \{\mathcal{F}(\alpha) \mid \alpha \in H^0(\mathrm{End}(V))\}$, which automatically incorporates the Yang-Mills condition. In the case polystable bundles, this set is non-trivial.

Long exact sequence in cohomology:

$$\ldots \rightarrow H^0(\mathcal{Q}_2) \xrightarrow{\mathcal{H}} H^{(0,1)}(T^*X) \rightarrow H^{(0,1)}(\mathcal{Q}) \rightarrow$$
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$$T\mathcal{M} = H^{(0,1)}(\mathcal{Q}) \cong \left(H^{(0,1)}(T^*X)\Big/\text{Im}(\mathcal{H})\right) \oplus \ker(\mathcal{H})$$

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Note that the Bianchi Identity imposes the constraint that the moduli are in $\ker(\mathcal{H})$.

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- $\overline{D}$ gives a nice venue to study the moduli space $\mathcal{M}$. Obstructions live in $H^{(0,2)}(\mathcal{Q})$. Comes with a natural Kähler potential.

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- Higher orders in α'? Does this structure survive? Work in progress with Xenia de la Ossa.

Thank you!

Thank you for your attention!

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