Heterotic Line Bundle Models on Smooth Calabi-Yau Manifolds

Andrei Constantin (University of Oxford)

Joint work with: Evgeny Buchbinder, Andre Lukas and Challenger Mishra

String Phenomenology 2014, Trieste

The Generation Problem
Hints from Heterotic Line Bundle Models

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A Wish List for (Heterotic) String Pheno

Four-dimensional EFT with:

- $\mathcal{N} = 1$ SUSY
- SM gauge interactions
- massless spectrum containing 3 chiral generations of quarks and leptons and no extra fields charged under the SM gauge group; uncharged fields (moduli) allowed for the moment
- massless spectrum containing Higgs doublets throughout the moduli space
- stable proton
- hierarchy of holomorphic Yukawa couplings consistent with a heavy top
- stable moduli; broken SUSY
- compute physical Yukawa couplings

Why 3 Generations?

A Heterotic Setup

In 10d, the heterotic string is specified by a metric and a non-abelian gauge field. To compactify: (X, V).

Constraints:

$\mathrm{ch}_2(V) - \mathrm{ch}_2(TX) = [W]$
(Green-Schwarz anomaly cancellation)

$F_{ij} = F_{\bar{i}\bar{j}} = 0$ (V holomorphic)

$g^{i\bar{j}} F_{i\bar{j}} = 0$ DUY theorem guarantees the HYM equation is satisfied provided that V is polystable and has slope zero.

A Heterotic Setup - continued

The simplest and best understood situation: X Calabi-Yau three-fold.
(Complex, Kähler manifold with a no-where vanishing top form.
Non-Kähler compactifications covered in Eirik Svanes' talk.)

In this case, the possible bundles can be divided into two classes:

- $V = TX$
  standard embedding: corresponds to $(2, 2)$ worldsheet susy
- $V \neq TX$
  general embeddings: correspond to $(0, 2)$ worldsheet susy

In the following I will refer to the $E_8 \times E_8$ heterotic string.

In the standard embedding case ($V = TX$), upon compactification one obtains E_6 GUT models. The number of chiral families is given by:

$$N_{\text{gen}} = -\frac{1}{2}\chi(X)$$

For general embeddings involving bundles V with structure group $SU(5)$, $SU(4)$ or $SU(3)$ (leading to $SO(10)$, $SU(5)$ and E_6 GUTs):

$$N_{\text{gen}} = -\text{ind}(V)$$

Can we say more than this?

The Heterotic Line Bundle Setup

Simplest choice for V (for, e.g. stability checks and cohomology computations): sum of line bundles

$$V = \bigoplus_{a=1}^{\text{rk}(V)} \mathcal{L}_a = \bigoplus_{a=1}^{\text{rk}(V)} \mathcal{O}(\vec{k}_a)$$

where $\vec{k}_a = c_1(\mathcal{L}_a)$.

E_6-models are obtained for $\text{rk}(V) = 3$, $SO(10)$-models for $\text{rk}(V) = 4$ and $SU(5)$-models for $\text{rk}(V) = 5$.

The (intermediate) GUT group contains 2,3 and respectively 4 extra $U(1)$ symmetries. These are phono ok and can greatly constrain the superpotential.

The Heterotic Line Bundle Setup – continued

Topological constraints on V:

- $c_1(V) = 0$
- $c_2(TX) - c_2(V) \geq 0$
- $\text{ind}(V) = -3$

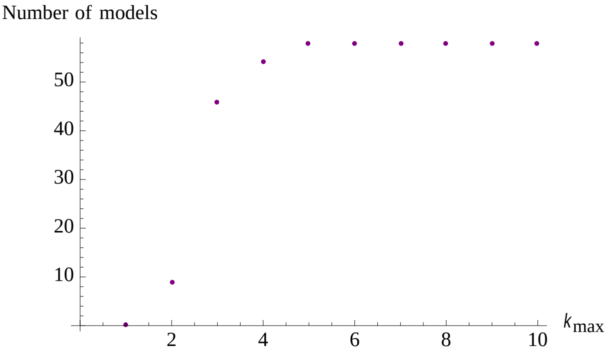
In addition, impose poly-stability and slope zero:

$$\mu(\mathcal{L}_a) = \int_X c_1(\mathcal{L}_a) \wedge J^2 = d_{ijk}\, \vec{k}_a^i\, t^j\, t^k = 0$$

simultaneously for all $a = 1, \ldots, \text{rk}(V)$

Result: intermediate GUT with a bunch of (effectively global) $U(1)$ symmetries, and 3 chiral families of matter.

FINITENESS



A Theoretical Bound

$$\sum_a \mathbf{k}_a^T \widetilde{G}\, \mathbf{k}_a \leq |c_{2i}(TX)|$$

$$\widetilde{G} = \kappa\, G/(6|\mathbf{t}|)$$

$$G_{ij} = \frac{1}{2\,\mathrm{Vol}(X)} \int_X J_i \wedge \star J_j = -3\left(\frac{\kappa_{ik}}{\kappa} - \frac{2\kappa_i \kappa_j}{3\kappa^2}\right)$$

where $\mathrm{Vol}(X) = \kappa/6$ is the Calabi-Yau volume with respect to the Ricci-flat metric, $\kappa = d_{ijk}\, t^i\, t^j\, t^k$, $\kappa_i = d_{ijk}\, t^j\, t^k$ and $\kappa_{ij} = d_{ijk}\, t^k$.

$$\boxed{\sum_a |\mathbf{k}_a|^2 \leq \frac{\text{num factor}}{\lambda_{\min}}}$$

To derive this, we used the slope-zero conditions and the bound on $c_2(V)$ from the anomaly cancellation. The bound is not sensitive to the number of line bundles involved in the sum, nor to the index of V. [Buchbinder, AC, Lukas]

Position in the Kähler Moduli Space

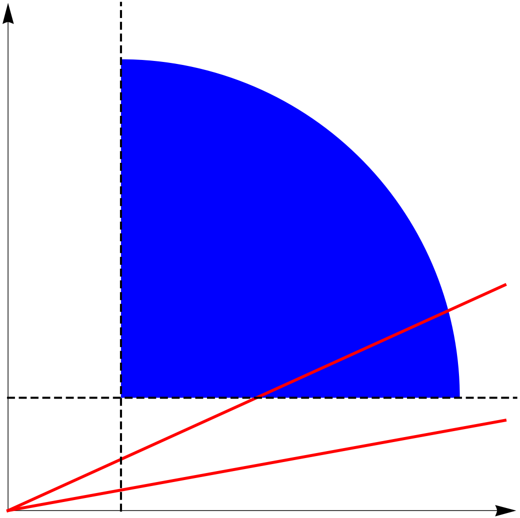
The constraints imposed by poly-stability $\mu(\mathcal{L}_1) = 0$ define a certain locus in the Kähler moduli space. The slope zero equations are homogeneous in the t^i. Thus $t^i \to \lambda t^i$ leaves this locus invariant.

There is a physically allowed region in the Kähler moduli space:

Supergravity limit: $t^i > 1$

finiteness of low-energy coupling constants: $\text{Vol}(X) \lesssim V_{\text{max}}$.

$$\text{Vol}(X) = \frac{1}{6} d_{ijk}\, t^i\, t^j\, t^k$$



$SO(10)$ Models on the Tetraquadric CY

Calabi-Yau data:

$$X = \begin{array}{c} \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \end{array} \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \end{array} \right]^{4,68}_{-128}$$

$$d_{ijk} = \int_X J_i \wedge J_j \wedge J_k = \begin{cases} 2 & \text{if } i \neq j, j \neq k \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Vol}(X) = 2\,(t_1 t_2 t_3 + t_1 t_2 t_4 + t_1 t_3 t_4 + t_2 t_3 t_4)$$

$$c_2(TX) = (24, 24, 24, 24)\ .$$

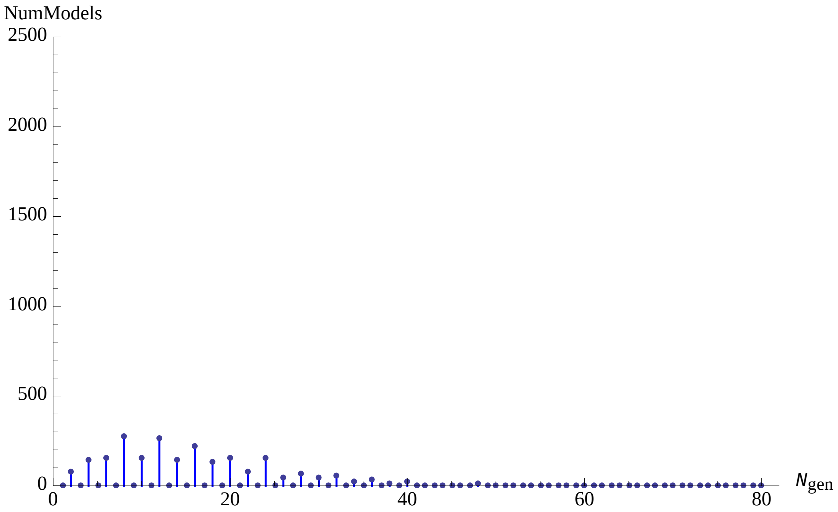
Scan

We scanned for line bundle sums that are poly-stable and have slope-zero in the 'allowed' region of the Kähler moduli space for different values of $V_{\max}$.

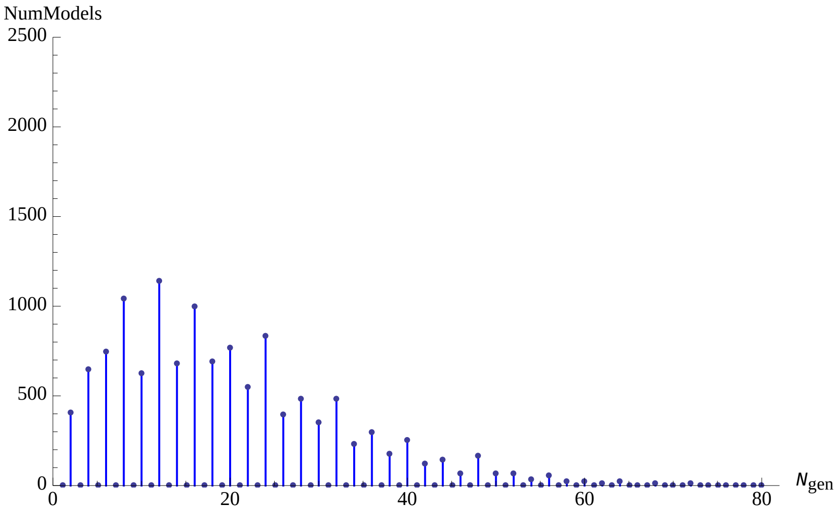
We increased the range of integers defining $c_1(\mathcal{L}_a)$ until no new models could be found. In all cases, the total number of models was finite.

We did this for many values of $\mathrm{ind}(V)$, corresponding to $1 \leq N_{gen} \leq 80$.

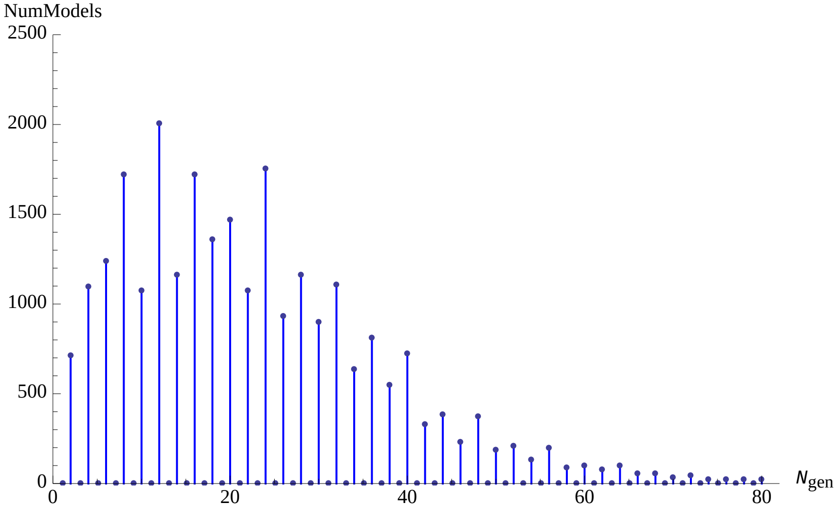
STATISTICS FOR VOL$(X) = 100$



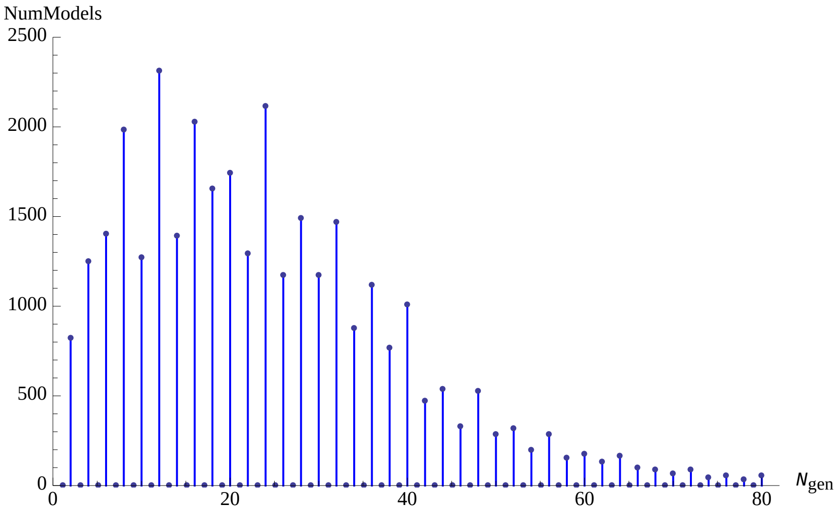
Statistics for $\mathrm{Vol}(X) = 1000$



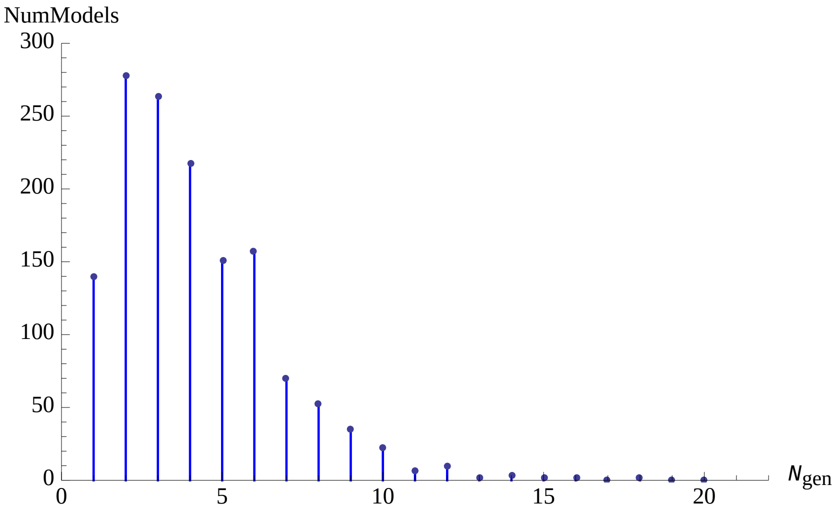
Statistics for Vol(X) = 10,000



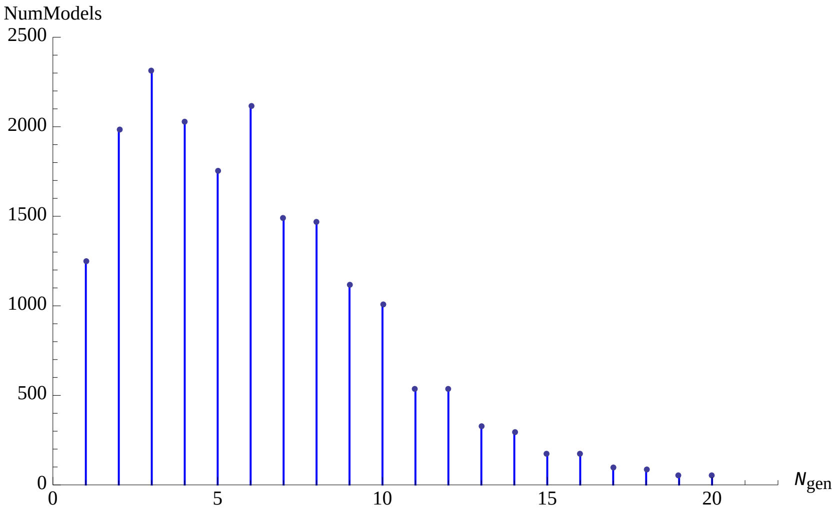
STATISTICS FOR $\mathrm{Vol}(X) = 100,000$



Statistics for Vol(X) = 100 on TQ/G_4



Statistics for $\mathrm{Vol}(X) = 100,000$ on TQ/G_4



CY volume and Physical Couplings

$$\alpha_{\mathrm{GUT}} = \frac{1}{2s} = \frac{1}{25}$$

$$M_{\mathrm{GUT}} = x\,\frac{1}{2\pi l}\,\frac{1}{s^6} \simeq 2\cdot 10^{16}\,GeV$$

$$G_N = \frac{l^2}{8\pi}\,\frac{1}{s^{2/3}}\mathrm{Vol}(X)^{-1/3} = \frac{1}{M_{\mathrm{Pl}}^2}$$

$$\mathrm{Vol}(X) = \left(\frac{1}{32\pi^3}\right)^3\ x^6\ \frac{(2\alpha)^3}{M_{\mathrm{GUT}}^6\,G_N^3} \simeq x^6 10^5$$

Conclusions

For a given manifold, there is an upper bound on the maximal number of generations that one can construct via a line bundle sum model.

This bound can be controlled by imposing restrictions on $\text{Vol}(X)$, which can be computed in terms of physical couplings.

This gives a new relation between the number of generations, the topology of the compactification and physical constants in 4d.

Thank you!