Patrick K. S. Vaudrevange

Universe Cluster Munich

July 7th, 2014

String Pheno 2014, Trieste

Based on:
- H. P. Nilles and P. V.: 1403.1597
Motivation

- Connect string theory to particle physics
- Framework: $E_8 \times E_8$ heterotic string
- Compactify from 10d to 4d on orbifold

Questions in this talk:
- How many 6d orbifold geometries are there?
- What are their common properties for model-building?
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Orbifolds

Lattice Λ
Orbifolds

Lattice $\Lambda$ spanned by $e_1$ and $e_2$
Orbifolds

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Lattice $\Lambda$
Orbifolds

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Orbifolds

Lattice $\Lambda \Rightarrow$ torus $T^2$

$T^2$ defined by $x \sim x + e_i$ for $x \in \mathbb{R}^2$
Orbifolds

Lattice $\Lambda \Rightarrow$ torus $T^2$

$P$ rotational symmetry of $\Lambda$

$P = \{1, \theta, \theta^2\} = \mathbb{Z}_3$

$\theta \equiv 120^\circ$
Orbifolds

Lattice $\Lambda \Rightarrow$ torus $T^2$

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Orbifold $T^2/P$

$x \sim \theta x$
Orbifolds

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Orbifold $T^2/P$ with 3 fixed points
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Heterotic string on orbifolds

Patrick Vaudrevange  
String Pheno in the Heterotic Orbifold Landscape
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Heterotic string on orbifolds

untwisted string from $E_8 \times E_8$
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Heterotic string on orbifolds

untwisted string $\bigcirc$ from $E_8 \times E_8$

twisted string $\bigcirc$ at fixed point
Classification of Orbifolds in 6D

1) Lattice $\Lambda$

2) $P$ rotational symmetry of $\Lambda$

$$P \iff \mathcal{N} = 1 \text{ SUSY}$$

3) Include roto-translations $x \mapsto \theta x + t$

$\Rightarrow$ Crystallography
Classification of Orbifolds

in 6D
1) Lattice $\Lambda$
2) $P$ rotational symmetry of $\Lambda$
   \[ P \iff N = 1 \text{ SUSY} \]
3) Include roto-translations $x \mapsto \theta x + t$
   \[ \Rightarrow \text{Crystallography} \]

Example in 2d:
1) 17 different tilings
2) $P = \mathbb{Z}_N$ with $N = 2, 3, 4, 6$
   E.g. no $\mathbb{Z}_5$ in 2d:

$e_1$
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Classification of Orbifolds in 6D

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Example in 2d:

1) 17 different tilings

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Classification of Orbifolds

Results in 6d:
1) 60 point groups $P$ with $\mathcal{N} \geq 1$ SUSY
2) 186 lattices $\Lambda$
   include roto-translations:
3) 520 inequivalent orbifold geometries in 6d
   162 with $P$ abelian
   358 with $P$ non-abelian
Classification of Orbifolds

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$h^{2,1}$ vs. $h^{1,1}$ for abelian orbifold geometries
Number of generations for abelian orbifold geometries

M. Fischer, M. Ratz, J. Torrado and P. V. 2012

String Pheno in the Heterotic Orbifold Landscape
Number of generations for abelian orbifold geometries

- $h^{1,1} - h^{2,1}$ always divisible by six
- Only exception: $(h^{1,1}, h^{2,1}) = (20, 0)$
- No geometry with three generations
  $\Rightarrow$ discrete Wilson lines needed for three generations
- computer program “orbifolder” to create and analyse orbifold models with abelian $P$

H.P. Nilles, S. Ramos-Sanchez, P. V. and A. Wingerter 2012
Non-abelian Orbifolds: untwisted Moduli \((h_{U}^{(1,1)}, h_{U}^{(2,1)})\)

<table>
<thead>
<tr>
<th>untwisted moduli ((h_{U}^{(1,1)}, h_{U}^{(2,1)}))</th>
<th>non-abelian point groups</th>
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<tr>
<td>(2,2)</td>
<td>(S_3, D_4, D_6)</td>
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<td>(2,0)</td>
<td>(\mathbb{Z}_8 \times \mathbb{Z}_2, \mathbb{Z}_3 \times S_3, \mathbb{Z}_3 \times \mathbb{Z}_8, SL(2, 3) - I, \mathbb{Z}_3 \times D_4, \mathbb{Z}_3 \times Q_8, (\mathbb{Z}_4 \times \mathbb{Z}_4) \times \mathbb{Z}_2, \mathbb{Z}_3 \times (\mathbb{Z}_3 \times \mathbb{Z}_4), \mathbb{Z}_6 \times S_3, \mathbb{Z}_3 \times SL(2, 3), \mathbb{Z}_3 \times ((\mathbb{Z}_6 \times \mathbb{Z}_2) \times \mathbb{Z}_2), SL(2, 3) \times \mathbb{Z}_4)</td>
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<tr>
<td>(1,1)</td>
<td>(A_4, S_4)</td>
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<tr>
<td>(1,0)</td>
<td>(T_7, \Delta(27), \mathbb{Z}_3 \times A_4, \Delta(48), \Delta(54), \mathbb{Z}_3 \times S_4, \Delta(96), \Sigma(36\phi), \Delta(108), PSL(3, 2), \Sigma(72\phi), \Delta(216))</td>
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</tbody>
</table>
$h^{2,1}$ vs. $h^{1,1}$ for non-abelian orbifold geometries
Number of generations for all orbifold geometries

- 65 orbifold geometries with $h^{1,1} = h^{2,1}$
  \[ \Rightarrow \text{always non–chiral using standard heterotic CFT} \]
- Magnetized orbifolds to create chirality in blow–up

S. GrootNibbelink and P. V. 2012
The Orbifold Landscape

1) all $\mathbb{Z}_N$ and some $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds
2) use “orbifolder” for MSSM models
   we find $\approx 12000$ MSSM-like models
3) lessons from this Orbifold Landscape?
   - location of matter
   - location of Higgs
   - flavor symmetries
   - gaugino condensation and SUSY breaking
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<thead>
<tr>
<th>orbifold</th>
<th># MSSM</th>
<th>max. # of indep. WLs</th>
<th># models with indep. vanishing WLs</th>
<th># MSSM without $U(1)_{\text{anom}}$</th>
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<td>(4,1)</td>
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## The OrbifoldLandscape

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<tr>
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<th># models with 1 indep. vanishing WLs</th>
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<th># models with 3 indep. vanishing WLs</th>
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<td>2</td>
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Local $SO(10)$ GUTs

- Matter in complete GUT representation: $16$ of $SO(10)$
- Origin? Twisted matter at fixed point with enhanced $SO(10)$ symmetry
Local $SU(5)$ GUTs

H.P. Nilles and P.V. 2014
Any local GUTs

H.P. Nilles and P. V. 2014

String Pheno in the Heterotic Orbifold Landscape
# Locations with split multiplets for Higgs

- Higgs in split GUT representation
- Origin? Location in higher dim. where GUT group is broken, i.e. 10d bulk

H.P. Nilles and P. V. 2014

Patrick Vaudrevange  String Pheno in the Heterotic Orbifold Landscape
Flavor symmetries

- (Discrete) flavor symmetry from symmetries of orbifold geometry
- Broken by Wilson lines
- If Wilson line vanishes $\Rightarrow$ larger flavor symmetry

H.P. Nilles and P. V. 2014
Scale of Gaugino Condensation

- Hidden sector gauge group from hidden $E_8$
- "Intermediate" size due to modular invariance
- $\beta$-function $\Rightarrow$ strong coupling at intermediate scale $\Lambda$
- SUSY breaking by dilaton $F$ term
- Gravity mediation: $m_{3/2} \sim \frac{\Lambda^3}{M_{Pl}^2}$
Conclusion

- Complete classification of orbifold geometries with $N \geq 1$ SUSY (Abelian and non-Abelian)
  - $\Rightarrow 520$ orbifold geometries
- Useful tool for abelian $P$: “orbifolder”
- The OrbifoldLandscape: $\approx 12000$ MSSM-like models
- Lessons:
  - Location of matter: local GUTs
  - Location of Higgs: 10d bulk
  - Discrete flavor symmetries
  - Low energy SUSY breaking from hidden sector
- Lessons valid outside heterotic context?
Conclusion

- Complete classification of orbifold geometries with $\mathcal{N} \geq 1$ SUSY (Abelian and non-Abelian)
- $\Rightarrow$ 520 orbifold geometries
- Useful tool for abelian $P$: “orbifolder”
- The OrbifoldLandscape: $\approx 12000$ MSSM-like models
- Lessons:
  - Location of matter: local GUTs
  - Location of Higgs: 10d bulk
  - Discrete flavor symmetries
  - Low energy SUSY breaking from hidden sector
- Lessons valid outside heterotic context?