Mordell-Weil meets Tate

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Work in collaboration with Moritz Küntzler 1406.5174

Work in progress with Craig Lawrie and Damiano Sacco

Work in progress with Jenny Wong

Synopsis of Talk

String Pheno as dual carriage way....

- From Model Building to Math...
  $\Rightarrow$ Lots of inspiration from F-theory model building for the mathematics of elliptic fibrations.

- ...and Math to Model Building
  Phenomenological requirement: $U(1)$ Symmetries
  $\Rightarrow$ Geometry constrains charges that sare possible in global models
  $\Rightarrow$ Use tool from number theory (Tate's algorithm) to survey all possibilities

Main theme for geometric engineering in F-theory:

Geometric constraints $\Rightarrow$ Phenomenological constraints

1. Geometry from F-theory

F-theory

- Non-perturbative IIB with $SL_2\mathbb{Z}$-compatible geometrization of the axio-dilaton $\tau = C_0 + ie^{-\phi}$
  $\Rightarrow$ τ= complex structure of elliptic curve
  $\Rightarrow$ elliptically fibered CY compactifications

- Duality to M-theory: useful for effective theory [see: Grimm's talk]

$$M \text{ on } S^1_A \times S^1_B \quad \xrightarrow{R_A \to 0} \quad IIA \text{ on } S^1_B \quad \xrightarrow{R_B \to 0} \quad IIB$$

$$R_A, R_B \to 0\,, \qquad g_s = R_A/R_B = \text{fixed}$$

More generally: F-theory from M-theory on $\mathbb{E}_\tau$

$$\text{Elliptic curve} \qquad \mathbb{E}_\tau \sim S^1_A \times S^1_B : \quad \begin{cases} \text{Im}(\tau) = g_s = \text{fixed} \\ \text{Vol}(\mathbb{E}_\tau) \to 0 \end{cases}$$

7-branes in F-theory

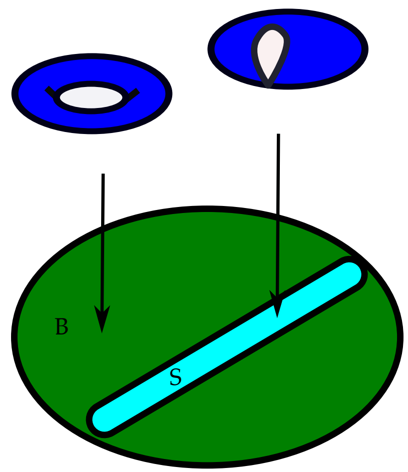
7-branes are singularities of the elliptic curve, with complex structure τ

- 7-branes in IIB sources F_9, dual to C_0:

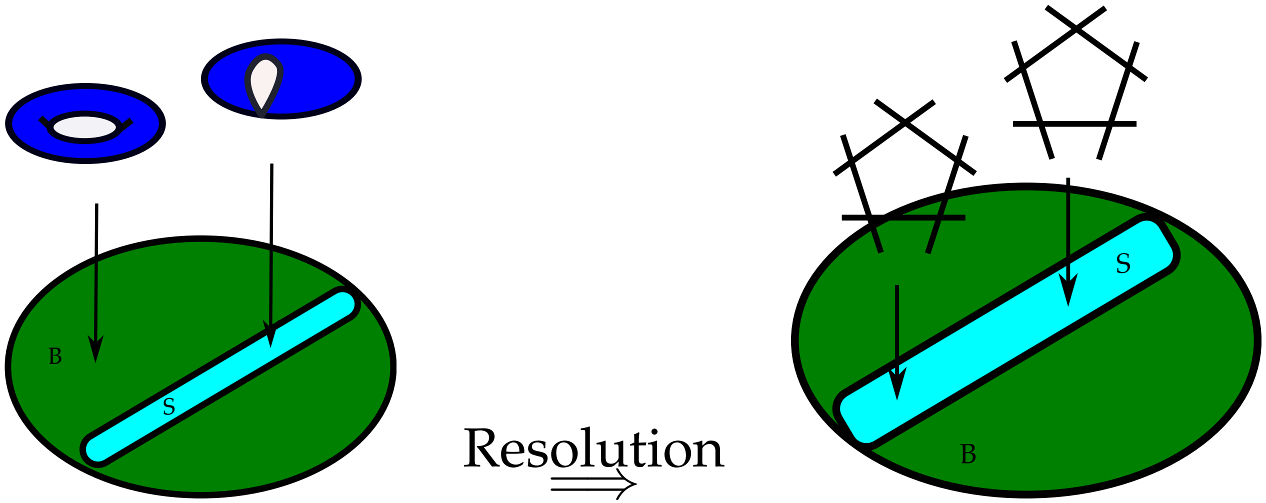
$$\tau(z) = \tau(z_0) + \frac{1}{2\pi i}\log(z - z_0) + \cdots$$

$\Rightarrow$ Monodromy: $\tau \to \tau + 1$

- (p, q) 7-branes with $SL_2\mathbb{Z}$ monodromy
- τ diverges at location of 7-brane



Gauge degrees of freedom from Singular Fibers



Geometrically:

resolved singular fibers are trees of $\mathbb{P}^1$s, intersecting in (extended) ADE Dynkin diagrams

Effective field theory:

$C_3 = A_i \wedge \omega_i^{(1,1)}$ and M2 wrapping modes give rise to gauge degrees of freedom.

Elliptic Fibrations

- Elliptic fibration Y_4: $\mathbb{E}_\tau \longrightarrow B_3$, with a section, has Weierstrass realization

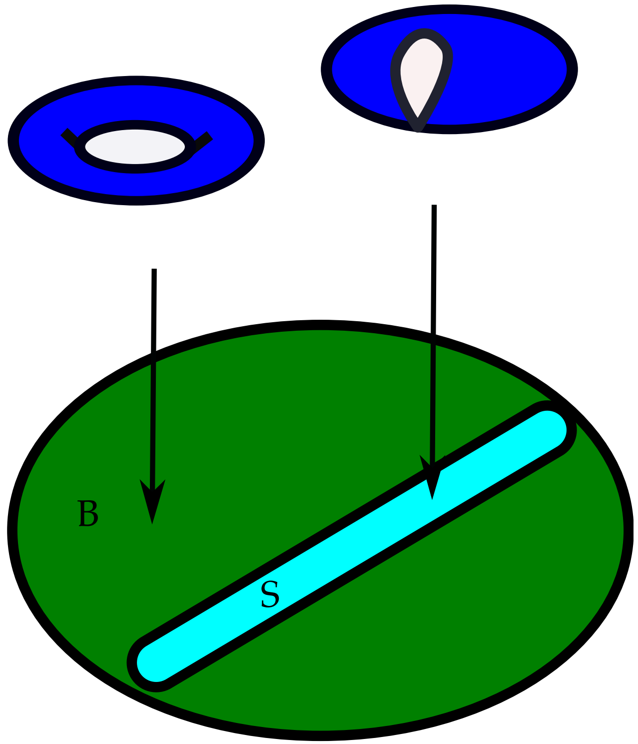
$$y^2 = x^3 + fx + g\,, \qquad\qquad f, g \in H^0(K_{B_3}^{-4/6})\,.$$

- Discriminant:
  on B_3 let $z = 0$ be a surface S
  Singular fiber above $z = 0$:

$$\Delta = 4f^3 + 27g^2 = O(z^n)$$

  Assume B, S smooth.

- Kodaira fiber above $z = 0$, i.e. in codim 1.



Classification of Singular Fibers

Elliptic fibration $Y_n : \mathbb{E}_\tau \longrightarrow B_{n-1}$. Kodaira-Néron classified singular fibers for $n = 2$, believed to hold in codim 1 for any n

Singular fibers $\mathrm{ord}(f, g, \Delta)$	$\longleftrightarrow$	(Decorated) ADE affine Dynkin diagram

ON ANALYTIC SURFACES: II 565

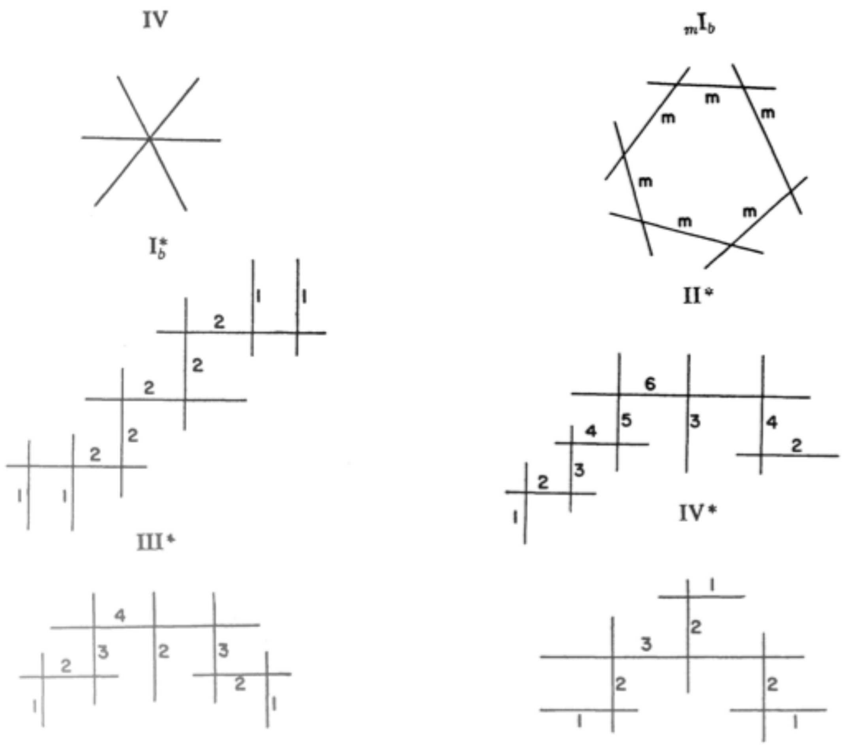


FIGURE 1. Each line represents $\Theta_{\rho s}$; the integer attached to the line gives $n_{\rho s}$.

	$\mathrm{ord}_S(f)$	$\mathrm{ord}_S(g)$	$\mathrm{ord}_S(\Delta)$	singularity	local gauge group factor
I_0	≥ 0	≥ 0	0	none	–
I_1	0	0	1	none	–
I_2	0	0	2	A_1	$SU(2)$
$I_m, m \geq 1$	0	0	m	A_{m-1}	$Sp([\frac{m}{2}])$ or $SU(m)$
II	≥ 1	1	2	none	–
III	1	≥ 2	3	A_1	$SU(2)$
IV	≥ 2	2	4	A_2	$Sp(1)$ or $SU(3)$
I_0^*	≥ 2	≥ 3	6	D_4	G_2 or $SO(7)$ or $SO(8)$
$I_m^*, m \geq 1$	2	3	$m+6$	D_{m+4}	$SO(2m+7)$ or $SO(2m+8)$
IV^*	≥ 3	4	8	E_6	F_4 or E_6
III^*	3	≥ 5	9	E_7	E_7
II^*	≥ 4	5	10	E_8	E_8
non-minimal	≥ 4	≥ 6	≥ 12	non-canonical	–

Kodaira's classification of singular fibers and gauge groups

Higher codimension singular fibers

Inspired from GUT model building: $SU(5)$ discriminant has expansion

$$\Delta = z^5 \delta_5 + z^6 \delta_6 + O(z^7)$$

Higher order term:

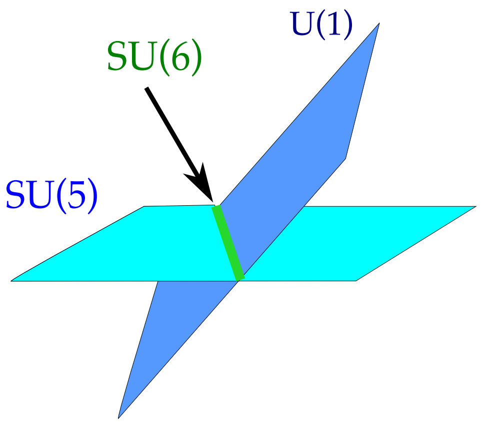
$$\begin{array}{rll} \text{Gauge} & \text{codim } 1: & z = 0 \\ \text{Matter} & \text{codim } 2: & z = \delta_5 = 0 \\ \text{Yukawa} & \text{codim } 3: & z = \delta_5 = \delta_6 = 0 \end{array}$$

[Beasley, Heckman, Vafa], [Donagi, Wijnholt]

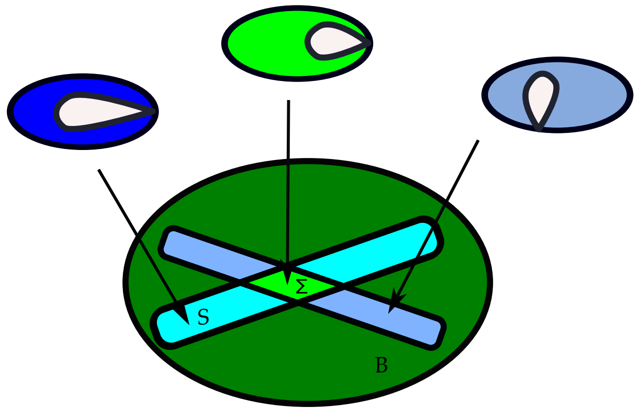
$\Rightarrow$ Singularity type changes along higher codim loci
$\Rightarrow$ Mathematical spinoff:
Beautiful classification results à la Kodaira and Néron for codim 2 and 3

[Hayashi, Lawrie, SSN][Hayashi, Lawrie, Morrison, SSN][Lawrie, SSN]

Codim 2: Matter



⇒



⇒ Bifundamental matter is localized along codimension 2 loci: curves Σ given by $z = \delta_5 = 0$

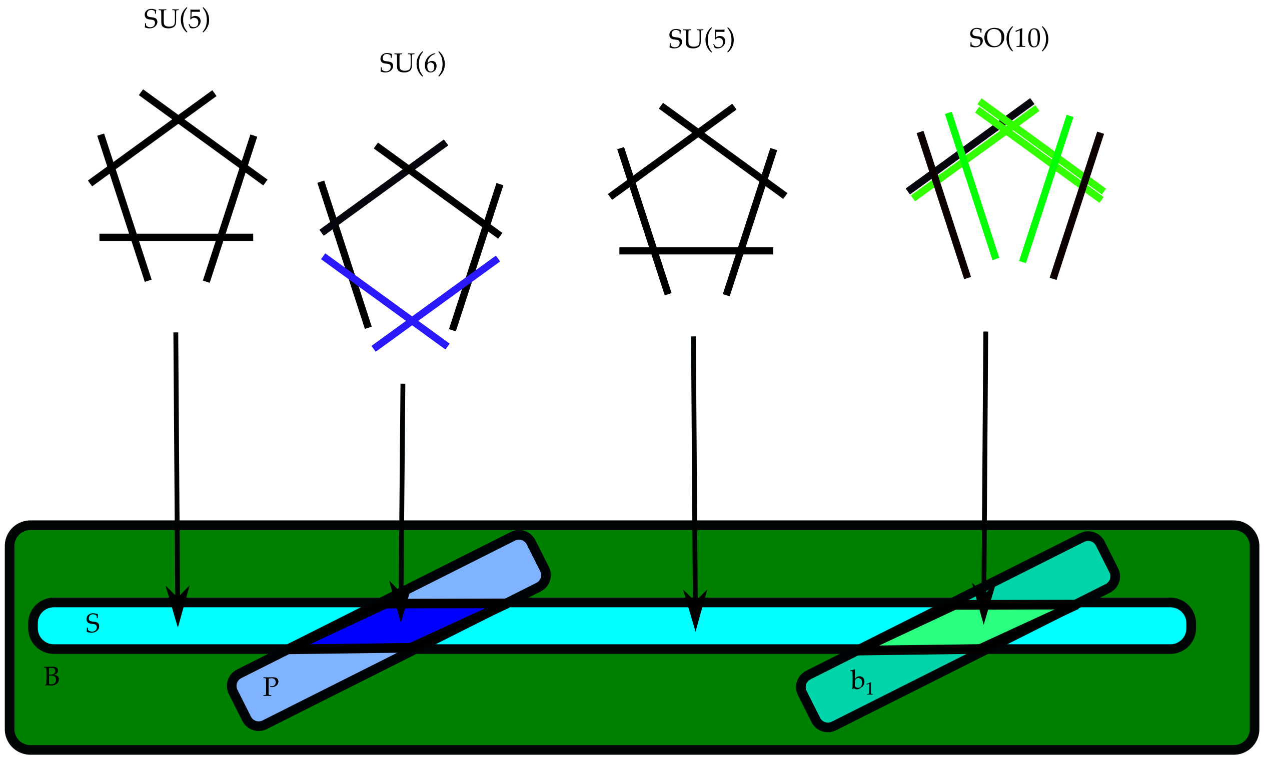
⇒ Matter type determined by singular fiber above Σ

$$G_\Sigma = SO(10) \text{ or } SU(6) \quad \longrightarrow \quad SU(5) \times U(1)$$

Codim 2: Matter

What is fiber structure in higher codim? Recall: $\Delta \sim z^n \longrightarrow z^{n+i}$

Number of fiber components increases: matter dof's from wrapped M2



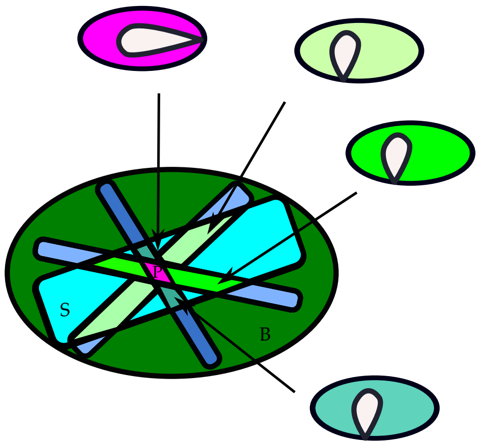
Along codim 2 curves:

$SU(5)$ fibers become reducible and enhance to $SO(10)$ or $SU(6)$ fibers.

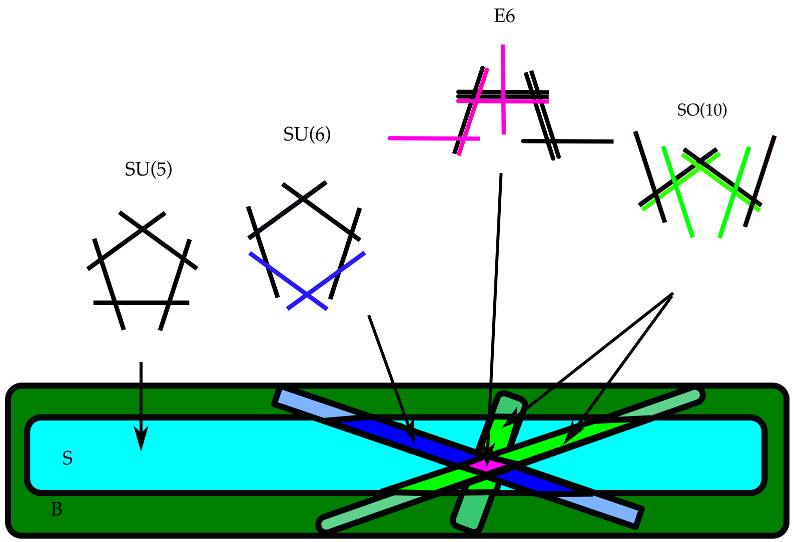
Codim 3: Yukawas

$\Rightarrow$ Yukawa couplings from codimension 3 points p in B_3 $z = \delta_5 = \delta_6 = 0$

$\Rightarrow$ Wrapped M2s above matter loci become homologically equivalent.



Resolution $\Longrightarrow$



Classification of Singular Fibers

Elliptic fibration $Y_n : \mathbb{E}_\tau \longrightarrow B_{n-1}$: Singular fibers characterized by

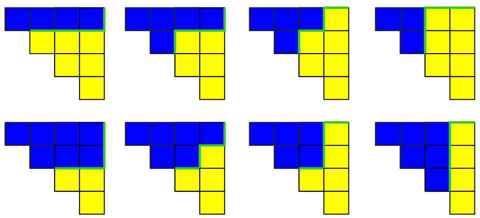
- Codim 1: Kodaira-Néron for given Lie algebria $\mathfrak{g}$

Singular Fiber Codim 1	$\longleftrightarrow$	(Decorated) affine Dynkin diagram of $\mathfrak{g}$

- Codim 2: **R**= representation of $\mathfrak{g}$

Singular Fiber Codim 2	$\longleftrightarrow$	Box Graph = Decorated rep graph of **R**

E.g. **10** of $SU(5)$



[Hayashi, Lawrie, SSN][Hayashi, Lawrie, Morrison, SSN]

$\Rightarrow$ Classification result for three-folds [Lawrie, SSN]

Kodaira, Weierstrass and Tate

Back to model building issues:

Problem with Weierstrass form of a Kodaira fiber:

$$\text{Fiber type} \quad \leftrightarrow \quad \big(\text{ord}_z(f), \text{ord}_z(g), \text{ord}_z(\Delta)\big)$$

Example: $SU(5)$

$$\big(\text{ord}_z(f), \text{ord}_z(g), \text{ord}_z(\Delta)\big) = (0, 0, 5)$$

i.e.

$$f = f_0 + f_1 z + \cdots\,, \quad g = g_0 + g_1 z + \cdots\,, \qquad g_i, f_i \neq 0$$

But

$$\Delta = 4f^3 + 27g^2 = O(z^5)$$

$\Rightarrow$ Requires tuning of f, g.

Tate form

$$y^2 = x^3 + a_1 xy + a_2 x^2 + a_3 y + a_4 x + a_6$$

Fiber type encoded in $a_n = z^{i_n} b_n$, $\qquad b_n = O(1)$

Dictionary between gauge symmetry and geometry:

Gauge degrees of freedom $\Leftrightarrow$ Singular elliptic fibrations

$\Updownarrow$

Tate form

Theorem:

$N = 1$ 4d gauge theory, with gauge group G

Then any singular elliptic fibration that engineers this can be put into Tate form [Bershadskya et al],[Katz, Morrison, SSN, Sully]

Outliers: $SU(n)$ $n = 6, 7, 8, 9$ and $Sp(n)$.

Type	Group	a_1	a_2	a_3	a_4	a_6	Δ
I_1	—	0	0	1	1	1	1
I_2	$SU(2)$	0	0	1	1	2	2
I_3^{ns}	$Sp(1)$	0	0	2	2	3	3
I_3^{s}	$SU(3)$	0	1	1	2	3	3
I_{2n}^{ns}	$Sp(n)$	0	0	n	n	$2n$	$2n$
I_{2n}^{s}	$SU(2n)$	0	1	n	n	$2n$	$2n$
I_{2n+1}^{ns}	$Sp(n)$	0	0	$n+1$	$n+1$	$2n+1$	$2n+1$
I_{2n+1}^{s}	$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
III	$SU(2)$	1	1	1	1	2	3
IV^{ns}	$Sp(1)$	1	1	1	2	2	4
IV^{s}	$SU(3)$	1	1	1	2	3	4
$I_0^{*\,ns}$	G_2	1	1	2	2	3	6
$I_0^{*\,ss}$	$SO(7)$	1	1	2	2	4	6
$I_0^{*\,s}$	$SO(8)^*$	1	1	2	2	4	6
$I_1^{*\,ns}$	$SO(9)$	1	1	2	3	4	7
$I_1^{*\,s}$	$SO(10)$	1	1	2	3	5	7
$I_2^{*\,ns}$	$SO(11)$	1	1	3	3	5	8
$I_2^{*\,s}$	$SO(12)^*$	1	1	3	3	5	8
$I_{2n-3}^{*\,ns}$	$SO(4n+1)$	1	1	n	$n+1$	$2n$	$2n+3$
$I_{2n-3}^{*\,s}$	$SO(4n+2)$	1	1	n	$n+1$	$2n+1$	$2n+3$
$I_{2n-2}^{*\,ns}$	$SO(4n+3)$	1	1	$n+1$	$n+1$	$2n+1$	$2n+4$
$I_{2n-2}^{*\,s}$	$SO(4n+4)^*$	1	1	$n+1$	$n+1$	$2n+1$	$2n+4$
$IV^{*\,ns}$	F_4	1	2	2	3	4	8
$IV^{*\,s}$	E_6	1	2	2	3	5	8
III^*	E_7	1	2	3	3	5	9
II^*	E_8	1	2	3	4	5	10
non-min	—	1	2	3	4	6	12

Tate's Algorithm

Tate's Algorithm gives rise to standard forms for elliptic fibrations with a given Kodaira singular fiber.

Expand: $f = \sum_i f_i z^i$ and $g = \sum_i g_i z^i$

$$\Rightarrow \qquad \Delta = 4f^3 + 27g^2 = \left(4f_0^3 + 27g_0^2\right) + \left(12f_1f_0^2 + 54g_0g_1\right)z + O\left(z^2\right)$$

f_i, g_i are elements in local ring of functions, which is a unique factorization domain (UFD)

$\Rightarrow$ Solve $\Delta = 0$ over UFD

Tate's Algorithm: First two steps

- If z does not divide $\Delta \Rightarrow$ smooth fiber
- If $z|\Delta$:
  $4f_0^3 + 27g_0^2 = 0$. Over UFD: exists u_0 such that

$$f_0 = -\frac{1}{3}u_0^2 + O(z)\,, \qquad g_0 = \frac{2}{27}u_0^3 + O(z)$$

  Shifting $(x, y) \mapsto (x + \frac{1}{3}u_0, y)$:

$$y^2 = x^3 + u_0 x^2 + (f_1 z + f_2 z^2 + \cdots)x + (g_1 + \frac{1}{3}u_0 f_1)z + (g_2 + \frac{1}{3}u_0 f_2)z^2 + \cdots$$

  which is the Tate form for an I_1 fiber (U(1) or single 7-brane)
- If $z^2|\Delta$....

Obstructions to Tate forms

E.g. $SU(5)$ Tate model has discriminant

$$\Delta = b_1^4 \left(b_3^2 b_2 - b_3 b_1 b_4 + b_1^2 b_6\right) z^5 + O(z^6)$$

The second term enhances to $SU(6)$ and has general solution

$$b_1 = \sigma_1\sigma_3,\ b_3 = \sigma_1\sigma_2,\ b_6 = \sigma_2\sigma_5,\ b_2 = \sigma_3\sigma_4,\ b_4 = \sigma_3\sigma_5 + \sigma_2\sigma_4$$

However there is no shift to bring it back into Tate-type form

$$y^2 + \sigma_1\sigma_2 xy + \sigma_2\sigma_3 z^2 y = x^3 + \sigma_2\sigma_5 z x^2 + (\sigma_2\sigma_4 + \sigma_3\sigma_5) z^3 x + \sigma_3\sigma_4 z^5$$

$\Rightarrow$ "non-canonical form".

What's the point?

$\Rightarrow$ changes possible codimemsion 2/matter enhancements.

Luckily, however, this is not of phenomenological relevance for generic $SU(5)$, as our result shows that this can always be brought into canonical form. However.... this is not true for models with additional $U(1)$ symmetries.

2. Geometric constaints on F-theory models with $U(1)$s

$U(1)$s in F-theory

$U(1)$ selection rules have been haunting F-theory models for some time now: we need them but we can't seem to get them to work in quite the way we would like:

- Proton decay and μ-term protection
- Clash between GUT breaking with hypercharge and $U(1)$
  $\Rightarrow$ Anomalies, exotics,... [Dudas, Palti][Marsano][Dolan, Marsano,SSN][Palti]

Alternatives:

- Alternative GUT breaking: Wilson lines always have chiral exotics [Marsano, Clemens, Pantev, Raby]
- Discrete symmetries instead of gauged $U(1)$s: case by case dependent.

Instead: study global models with $U(1)$s systematically (global consistency will have to eventualy lead to automatic cancellation of anomaly).

Constraints on the $U(1)$s

Compatibility with Yukawas.

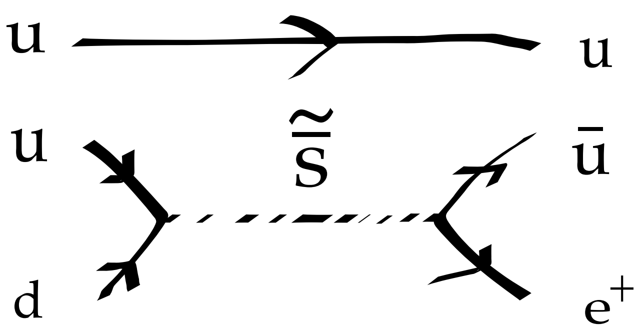
Protect model from Proton Decay: half-life $> 10^{36}$ years:

Dimension 4:

Models generically contain B/L-violating operators (R-parity violating)

$$W \supset \lambda^0_{ijk} L_i L_j \bar{e}_k + \lambda^1_{ijk} \bar{d}_i L_j Q_k + \lambda^2_{ijk} \bar{d}_i \bar{d}_j \bar{u}_k$$

Proton decay rate $\sim \lambda^1_{11k} \lambda^2_{11k}$ leading to $p^+ \to \pi^0 + e^+$



Bound on coupling: $W \supset \lambda_{ijk} \mathbf{\overline{5}}_i \mathbf{\overline{5}}_j \mathbf{10}_k \,,\qquad \lambda_{111} \leq \left(\frac{M_{SUSY}}{\text{TeV}} \right) 10^{-12}$

Fix: $U(1)_{B-L}$ or R-parity.

Dimension 5:

Coupling

$$\mathcal{L} \supset w_{ijkl} \mathbf{10}_i \mathbf{10}_j \mathbf{10}_k \overline{\mathbf{5}}_l$$

which gives rise to

$$w^1_{ijkl} Q_i Q_j Q_k L_l + w^2_{ijkl} \overline{u}_i \overline{u}_j \overline{e}_k \overline{d}_l + w^3_{ijkl} Q_i \overline{u}_j \overline{e}_k L_l$$

Bounds on couplings:

$$w_{112l} \leq 16\pi^2 \left(\frac{M_{SUSY}}{M^2_{GUT}} \right) \qquad l = 1, 2$$

Fix: $U(1)_{PQ}$

Characterize these by: absence of μ-term

$$q_{PQ}(H_u) + q_{PQ}(H_d) \neq 0$$

Elliptic Fibrations with $U(1)$s

Geometric engineering of $U(1)$ $\Leftrightarrow$ "Extra section" of elliptic fiberation

Section: Map $\sigma : B \longrightarrow \mathbb{E}_\tau$.

Why? $\sigma = 0$ divisor (copy of B), dual to $(1,1)$ form in fiber:

$$C_3 = \sum_i \omega^{(1,1)} \wedge A \qquad \Rightarrow \qquad U(1) \text{ gauge boson}$$

Mordell-Weil group: group of sections of an elliptic curve/fibration.

In practice: rational solutions to the elliptic curve equation. For Weierstrass in $\mathbb{P}^{123}[w,x,y]$ zero section

$$y^2 = x^3 + fxw^4 + gw^6 \qquad \sigma_0 : \quad w = 0,\ x = y = 1\,.$$

Construction of Extra Sections

Analog of Weierstrass models:

- $U(1)$: Embedding into $\mathbb{P}^{112}[w, x, y]$: [Morrison, Park]

$$y\left(y + b_0 x^2\right) = w\left(c_0 w^3 + c_1 w^2 x + c_2 w x^2 + c_3 x^3\right)$$

Sections: $y = w = 0$ and $w = y + b_0 x^2 = 0$.

Construction of Extra Sections

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Sections: $y=w=0$ and $w=y+b_0x^2=0$.

- $U(1)^2$: Embedding into $dP_2[w,x,y;l_1,l_2]$:

[Mayrhofer, Palti, Weigand][Cvetic, Klever, Piragua]

$$s_1l_1^2l_2^2w^3+s_2l_1^2l_2w^2x+s_3l_1^2wx^2+s_5l_1l_2^2w^2y+s_6l_1l_2wxy+s_7l_1x^2y+s_8l_2^2wy^2+s_9l_2xy^2$$

Sections: $l_1=0;\ l_2=0;\ x=s_9\ ,\ \ y=-s_7$

GUT models with extra $U(1)$s: Toric Models

- Toric Tops: Using toric geometry methods, one can generate models e.g. in $\mathbb{P}^{112}$ that have realize along $z = 0$ an $SU(5)$. The extra section guarantees $U(1)$.

  [Braun, Grimm, Keitel][Mayrhofer, Palti, Weigand][Cvetic, Klever, Piragua]

- Only one **10** matter locus.
- Phenomenological implications studied in

  [Krippendorf, Pena, Oehlmann, Ruehle]

However, these are not a complete class of elliptic fibrations with extra section.

$\Rightarrow$ Systematic approach – Tate-like forms for models with extra sections

$$y^2 + b_0(z)x^2y = c_0(z)w^4 + c_1(z)w^3x + c_2(z)w^2x^2 + c_3(z)wx^3$$

[Kuentzler, SSN][Lawrie, Sacco, SSN]

Tate's algorithm for $SU(5) \times U(1)$

[Kuentzler, SSN]

Any model with one $U(1)$ has embedding into $\mathbb{P}^{112}$

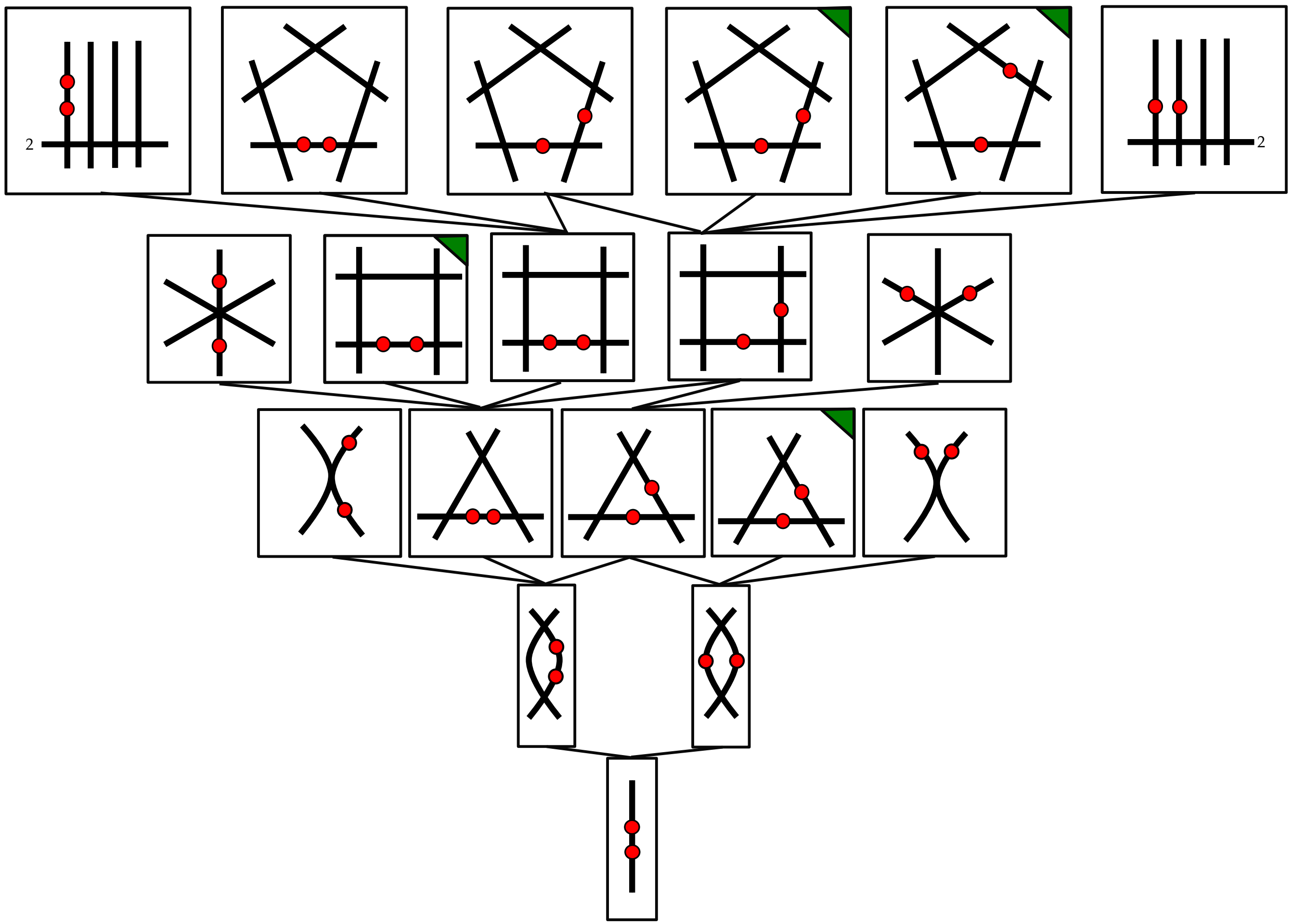
$$Q\,(i_1, i_2, i_3, i_4, i_5, i_6, i_7):$$
$$y^2 + b_0(z)x^2y = c_0(z)w^4 + c_1(z)w^3x + c_2(z)w^2x^2 + c_3(z)wx^3$$

where i_j determine vanishing order of the coefficients.

- Tate's algorithm $\Delta(z) = 0$: "solving polynomial eq over UFD"
- Kodaira fiber type + section information.
- Canonical and non-canonical models:

  Unlike Weierstrass: for $SU(5) \times U(1)$ in $\mathbb{P}^{112}$ not all model can be put into canonical Tate form. Characterized not only by vanishing order.

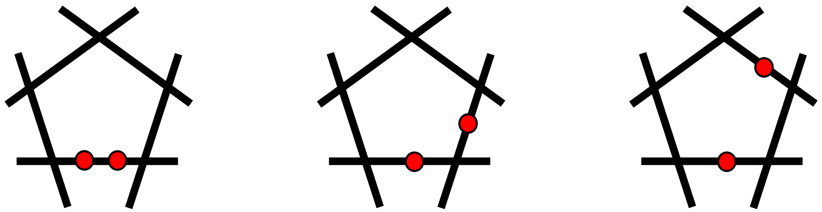
Tate Tree for $SU(5) \times U(1)$



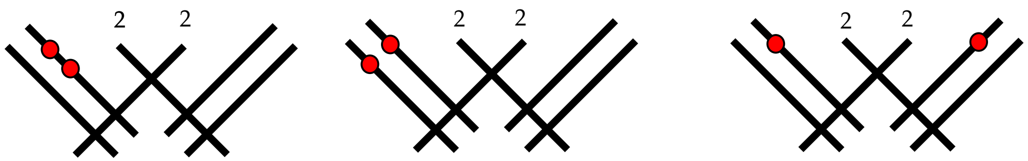
Kodaira-like Charactrization of Fibers

Fibers are characterized by

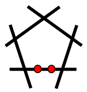
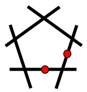
- Kodaira fibers and location of sections.
  For instance for $SU(5)$:



- Number and type of Codimension 2 fibers with location of sections:
  This differentiates between canonical and non-canonical fibers. For example: for $SO(10)$

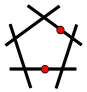
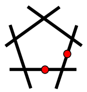


$SU(5) \times U(1)$: Canonical Models

Fiber	Model	Codim 2 locus	Representation	Codim 2 fiber
$I_5^{(01)}$ 	$\mathcal{Q}\,(5,3,1,0,0,0,2)$	$b_{1,0}$	$\mathbf{10}_0 + \overline{\mathbf{10}}_0$	$I_1^{*(01)}$
		$c_{3,0}$	$\mathbf{5}_{-1} + \overline{\mathbf{5}}_1$	$I_6^{(0\|1)}$
		$c_{3,0} + b_{0,0}b_{1,0}$	$\mathbf{5}_1 + \overline{\mathbf{5}}_{-1}$	$I_6^{(0\|1)}$
		$b_{1,0}^2 c_{0,5} - b_{1,0}b_{2,2}c_{1,3} + b_{2,2}^2 c_{2,1}$	$\mathbf{5}_0 + \overline{\mathbf{5}}_0$	$I_6^{(0\|1)}$
$I_5^{(0\|1)}$ 	$\mathcal{Q}\,(4,2,1,1,0,0,2)$	$b_{1,0}$	$\mathbf{10}_2 + \overline{\mathbf{10}}_{-2}$	$I_1^{*(0\|1)}$
		$b_{0,0}$	$\mathbf{5}_6 + \overline{\mathbf{5}}_{-6}$	$I_6^{(01)}$
		$b_{0,0}c_{2,1} - b_{1,0}c_{3,1}$	$\mathbf{5}_{-4} + \overline{\mathbf{5}}_4$	$I_6^{(0\|\|1)}$
		$b_{1,0}^2 c_{0,4} - b_{1,0}b_{2,2}c_{1,2} - c_{1,2}^2$	$\mathbf{5}_1 + \overline{\mathbf{5}}_{-1}$	$I_6^{(0\|1)}$

These correspond to top models from the toric construction.

Non-canonical $SU(5)$ Models

Fiber	Model	Codim 2	Representation	Codim 2 fiber
$I_{5,nc}^{(0\|\|1)}$	$Q(3,2,1,1,0,0,1)\|_{P_1=0}$	σ_3	$\mathbf{10}_1 + \overline{\mathbf{10}}_{-1}$	$I_1^{*(0\|\|1)}$
		σ_1	$\mathbf{10}_{-4} + \overline{\mathbf{10}}_4$	$I_1^{*(01)}$
		σ_2	$\mathbf{5}_{-7} + \overline{\mathbf{5}}_7$	$I_6^{(01)}$
		P_2	$\mathbf{5}_{-2} + \overline{\mathbf{5}}_2$	$I_6^{(0\|\|1)}$
		P_3	$\mathbf{5}_3 + \overline{\mathbf{5}}_{-3}$	$I_6^{(0\|\|\|1)}$
$I_{5,nc}^{(0\|1)}$	$Q(3,2,1,1,0,0,1)\|_{P_1=0}$	σ_1	$\mathbf{10}_2 + \overline{\mathbf{10}}_{-2}$	$I_1^{*(0\|1)}$
		σ_2	$\mathbf{10}_{-3} + \overline{\mathbf{10}}_3$	$I_1^{*(0\|\|1)}$
		$b_{0,0}$	$\mathbf{5}_6 + \overline{\mathbf{5}}_{-6}$	$I_6^{(01)}$
		P_2	$\mathbf{5}_{-4} + \overline{\mathbf{5}}_4$	$I_6^{(0\|\|1)}$
		P_3	$\mathbf{5}_1 + \overline{\mathbf{5}}_{-1}$	$I_6^{(0\|1)}$

where $P_1 = b_1^2 c_0 - b_1 b_2 c_1 + b_2^2 c_2 = 0$, implies in particular $b_1 = \sigma_i \sigma_j \Rightarrow$ two **10** curves Generalize top models.

Pheno (in progress)

- Single $U(1)$ models with one **10**: equivalent to toric models
  $\Rightarrow$ can realize PQ but problems with hypercharge
- Single $U(1)$ models with multiple **10**: new models
  $\Rightarrow$ PQ symmetries and more freedom in assigning matter.
- Multiple $U(1)$ models: (in progress) contain canonical toric models, but many non-canonical models, with multiple (up to 3) **10** curves.

Unlike local models: these are globally consistent, and the Tate-like models form a comprehensive list of all $G \times U(1)^n$ gauge groups with matter and charges in F-theory.

Summary and outlook

Global questions:

- Lifting local models:
  realization of GUT dof's, matter, Yukawas, chirality (G-flux)
  $\Rightarrow$ Resolution of singular elliptic CY4

- $U(1)$ symmetries:
  symmetries to protect from dim 5 proton decay, μ-term.
  $\Rightarrow$ Global $U(1)$ requires elliptic fibrations with extra sections
  ("ensures additional $\omega^{1,1}$ form")
  $\Rightarrow$ New Tate forms for elliptic fibrations with extra sections

- Global hypercharge flux:
  Open issue: resolve tension between trivial F_Y class in CY4, and required non-trivial restriction on Higgses.

Current F-theory slogan:
"to address pheno questions comprehensively, requires answering their reformulation in algebraic geometry"