

U(1) mixing and F-theory GUTs

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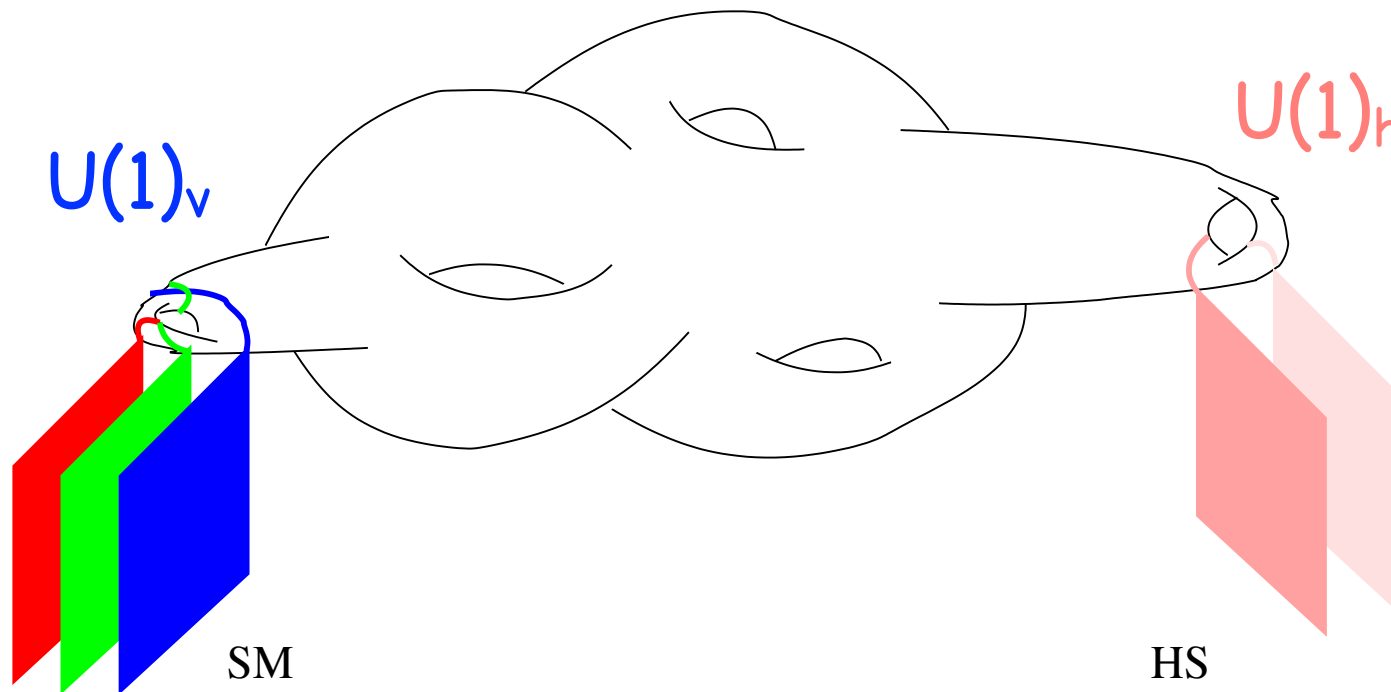
Based on:

**F.M., Regalado, Zoccarato
[1406.2729]**

Motivation

✿ Typical type II scenario

- ✦ Open string $U(1)$ from **visible** gauge sector
- ✦ Extra $U(1)$ from **hidden** gauge sector

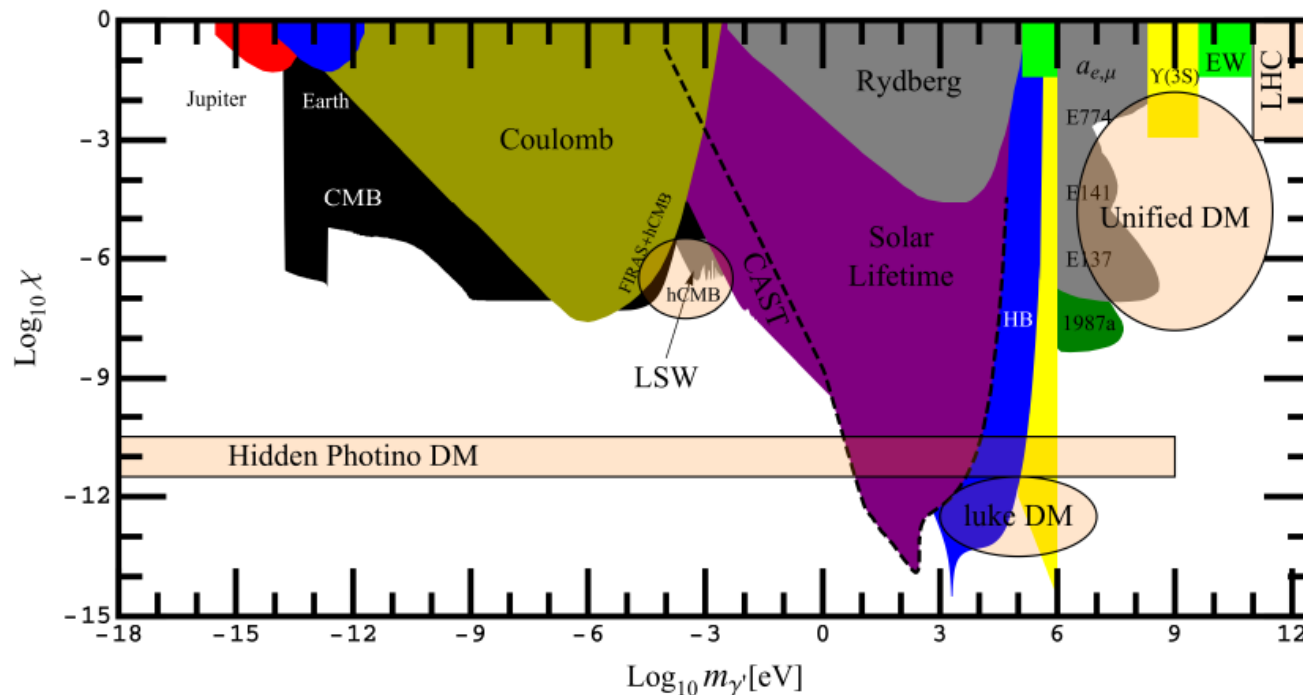


Motivation

❖ Typical type II scenario

- ◆ Open string U(1) from visible gauge sector
- ◆ Extra U(1) from hidden gauge sector, compatible with experiment as massless or very light hidden gauge symmetries

$$\mathcal{L}_{4d} \supset -\frac{1}{4} \sum_{i=v,h} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + \frac{1}{2} \chi_{vh} F_{\mu\nu}^{(v)} F^{(h)\mu\nu} + \frac{1}{2} m_{\gamma'}^2 A_{\mu}^{(h)} A^{(h)\mu}$$



taken from
Jaeckel and Ringwald '10

Motivation

❖ Typical type II scenario

- ◆ Open string $U(1)$ from visible gauge sector
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- ◆ Natural scenario: massless hidden $U(1)$ with charged light matter

non-trivial kinetic mixing χ_{vh}



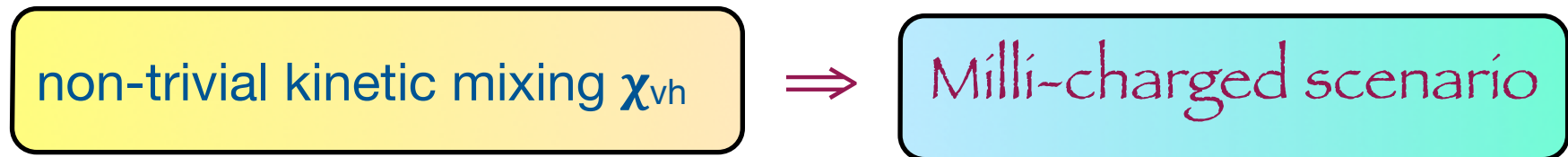
Milli-charged scenario

Holdom '86

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- ✦ Open-open $U(1)$ mixing arises at one loop *Holdom '86*

Abel², Cicoli, Goodsell⁴, Jaeckel⁴, Khoze², Redondo, Ringwald⁵ '06-11

Gmeiner, Honecker³, Ripka, Staessens '09-12

CFT computation

Williams, Burgess, Maharana, Zuevedo '11

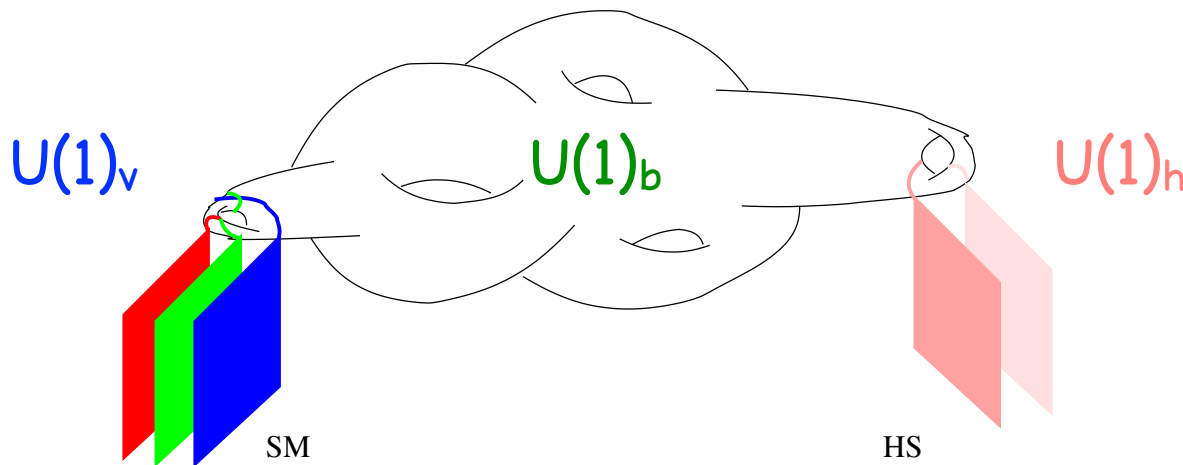
Bullimore, Conlon, Witowski '10

Shiu, Soler, Ye '13

Motivation

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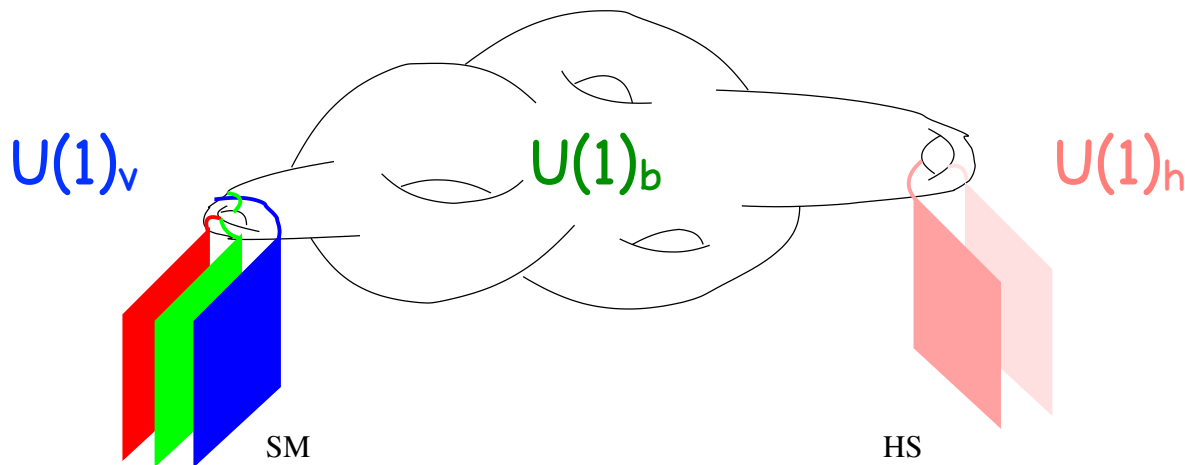
- ◆ Open string $U(1)$ from visible gauge sector
- ◆ Extra $U(1)$ from hidden gauge sector, compatible with experiment as massless or very light hidden gauge symmetries
- ◆ Natural scenario: massless hidden $U(1)$ with charged light matter
- ◆ Also natural to consider a massless hidden $U(1)$ arising from the closed string sector of the compactification



Motivation

❖ Closed string U(1)'s in type II

- ◆ Arise from dimensional reduction of **RR potential**: $C_p = A_1 \wedge \omega_{p-1}$
- ◆ **Mix** with open string U(1)'s **at tree level**
- ◆ Do **not** have any **light state charged** under them



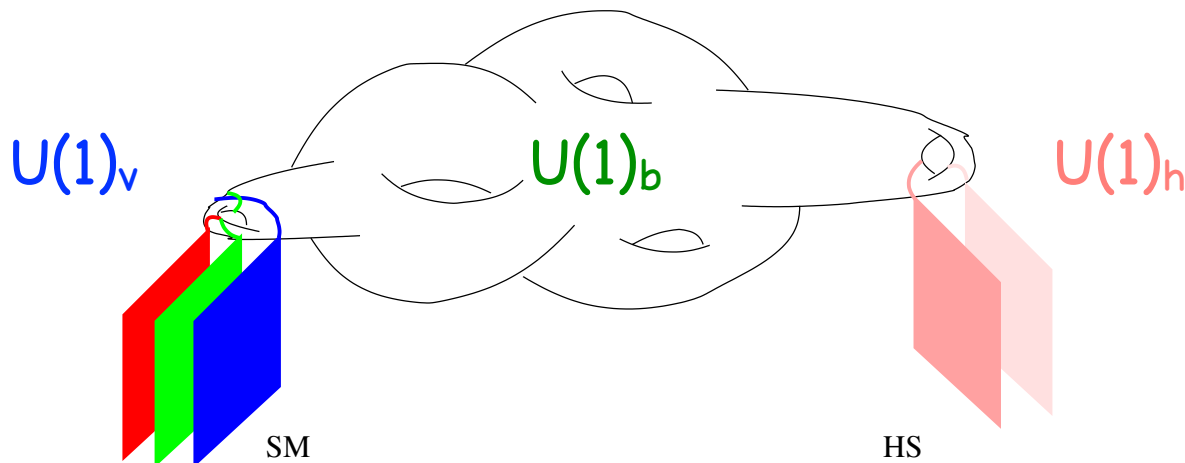
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But then, why bother?



Reason I: Millicharges

- ✿ $U(1)_b$ can mix with $U(1)_v$ and $U(1)_h$ at the same time

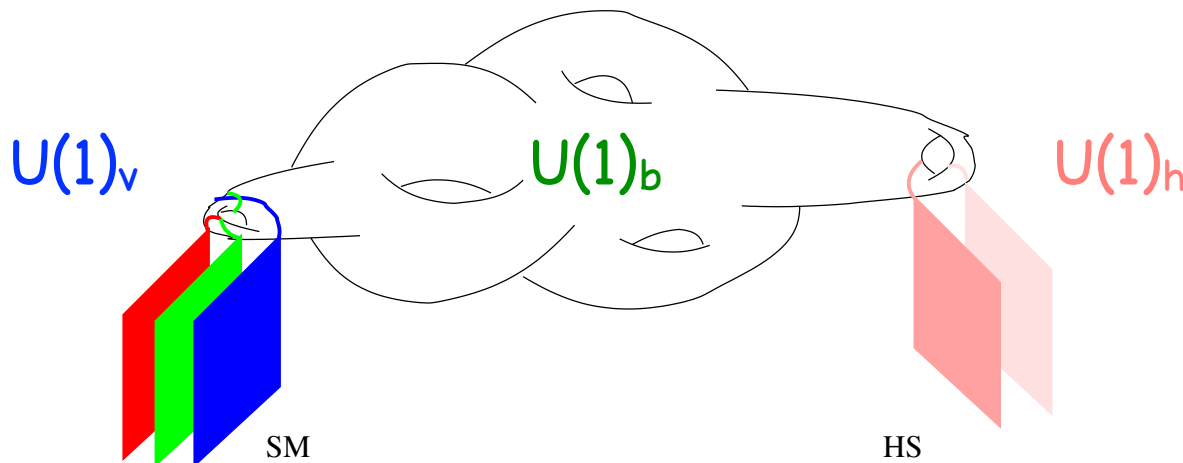
$$\mathcal{L}_{4d} \supset -\frac{1}{4} \sum_{i=v,h,b} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + \frac{1}{2} \left(\chi_{vb} F_{\mu\nu}^{(v)} F^{(b)\mu\nu} + \chi_{hb} F_{\mu\nu}^{(h)} F^{(b)\mu\nu} \right)$$

- ✦ Removing the χ 's by a change of basis induces a hypercharge on matter charged under $U(1)_h$

$$\delta_{vh}^{\text{eff}} = \frac{\chi_{vb}\chi_{hb}}{\sqrt{1 - \chi_{vb}^2 - \chi_{hb}^2}}$$

compared to

$$\delta_{vh}^{\text{one-loop}} = \frac{\chi_{vh}}{\sqrt{1 - \chi_{vh}^2}}$$



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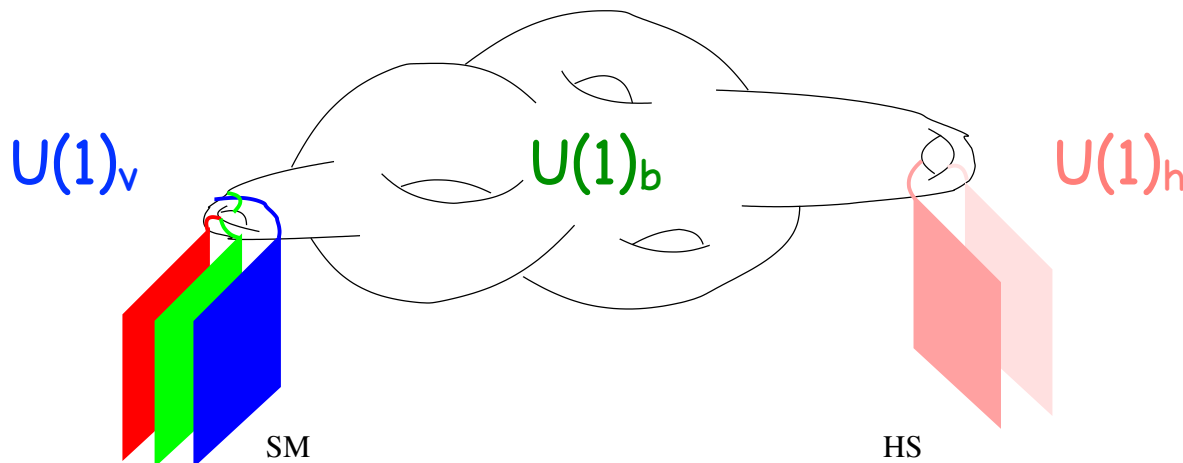
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- Tree level
- Rel. position indep.
- One-loop
- Rel. position dep.



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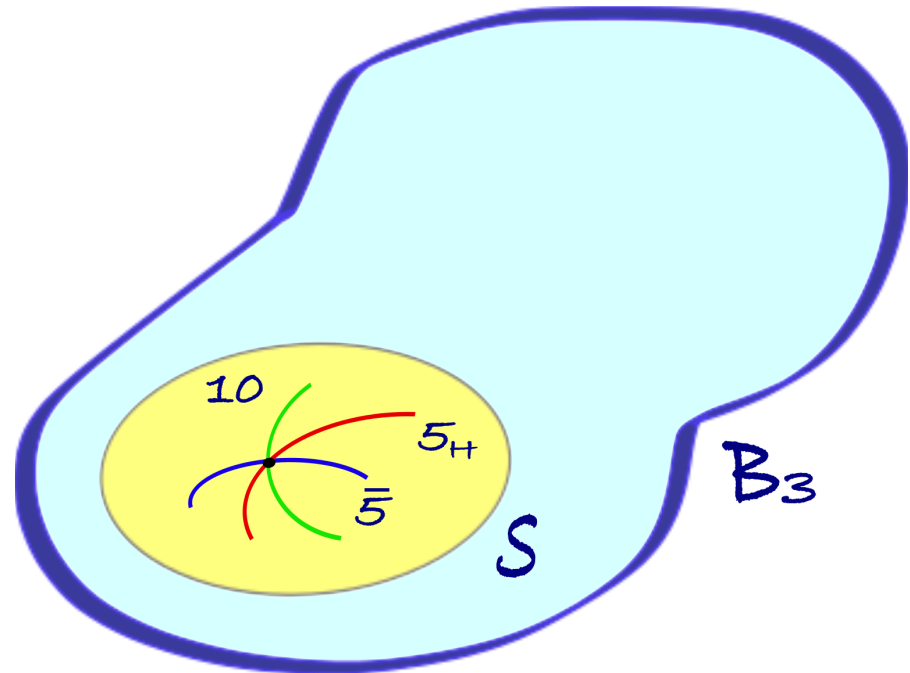
Both contributions
could be comparable

- Tree level
- Rel. position indep.
- One-loop
- Rel. position dep.

Reason II: Unification

- ✿ If $U(1)_V = U(1)_Y \subset SU(5)$ or G_{GUT} , then **mixing** with $U(1)_b$ **changes** the **GUT relations**

$$g_1^{\text{measured}} = \frac{g_1^{\text{GUT}}}{\sqrt{1 - \chi_{vb}^2}}$$



Reason II: Unification

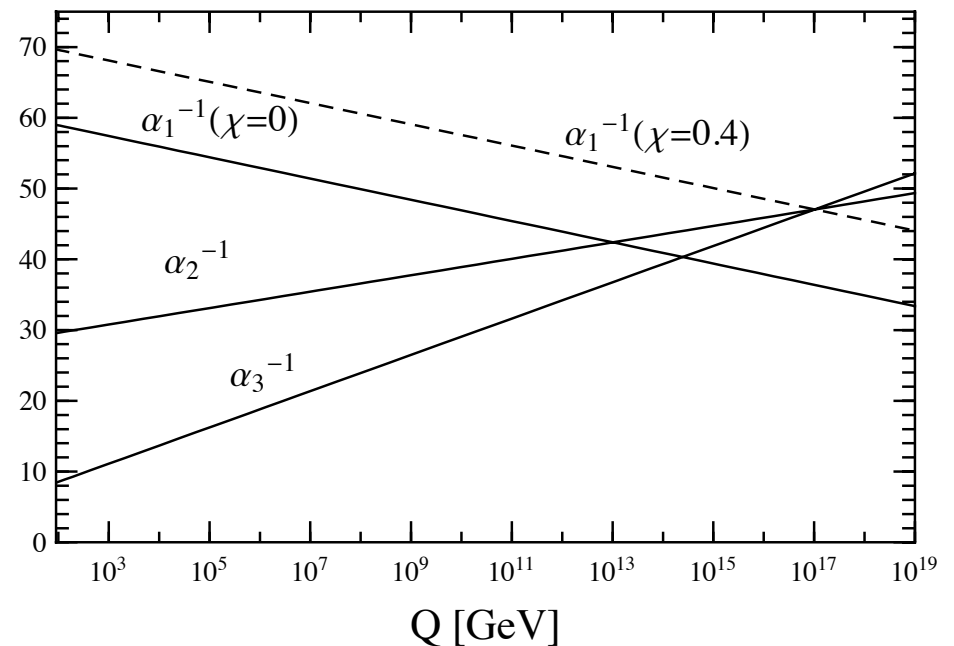
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Redondo '08

SM

Could explain deviations
from unification in SM



Reason II: Unification

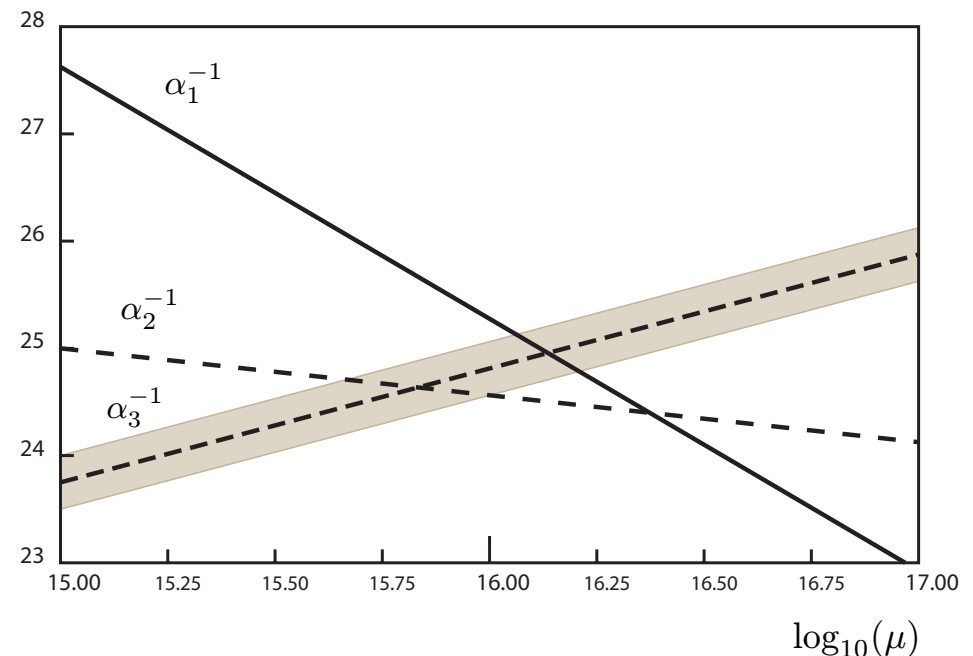
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Redondo '08

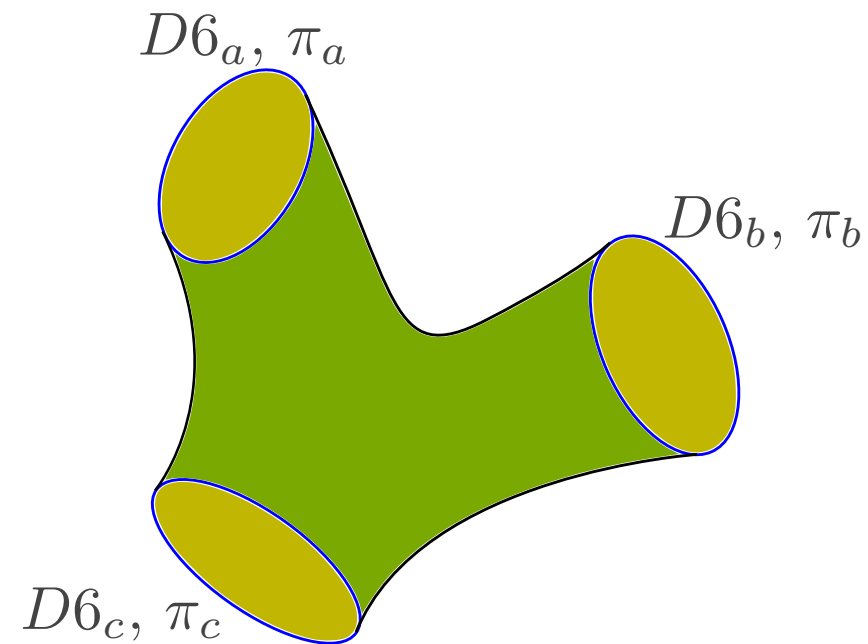
MSSM

Gives further corrections
to $U(1)_Y$ coupling constant
in F-theory GUTs



see also Hebecker's talk

Computing Open-Closed U(1) mixing



Mixing from DBI

- ✿ **Mixing** between open and closed string $U(1)$'s can already be seen at the **level of the DBI action**

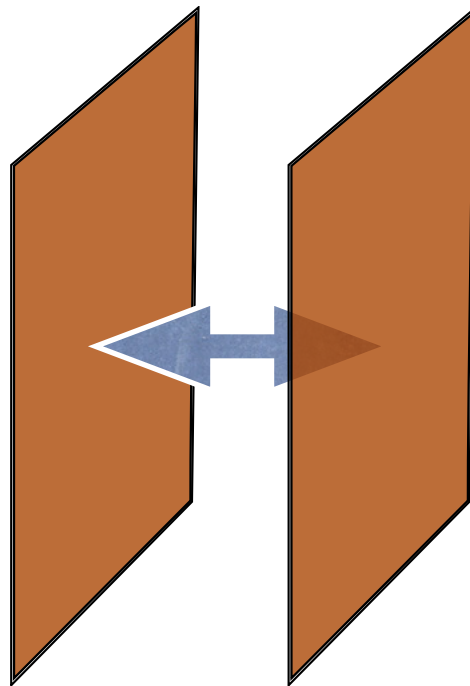
Jockers & Louis '04

Grimm & Lopes '11

Kerstan & Weigand '11

- ✦ Simple setup: **separating two D-branes**

Cámara, Ibáñez, F.M. '11



$$\pi_a \sim \pi_b \sim \pi$$

Adjoint Higgsing

$$SU(2) \xrightarrow{\phi} U(1)$$

$$U(1) = \frac{1}{2}[U(1)_a - U(1)_b]$$

mixing depends on the vev of ϕ and some topological conditions

Mixing from DBI

✿ Type IIA with Higgsed D6-branes

✦ Closed string U(1)'s

$$C_3 = A_1^i \wedge \omega_i \quad \omega_i \in \mathcal{H}_+^{1,1}$$

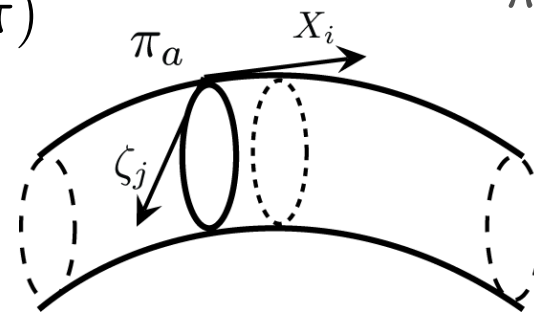
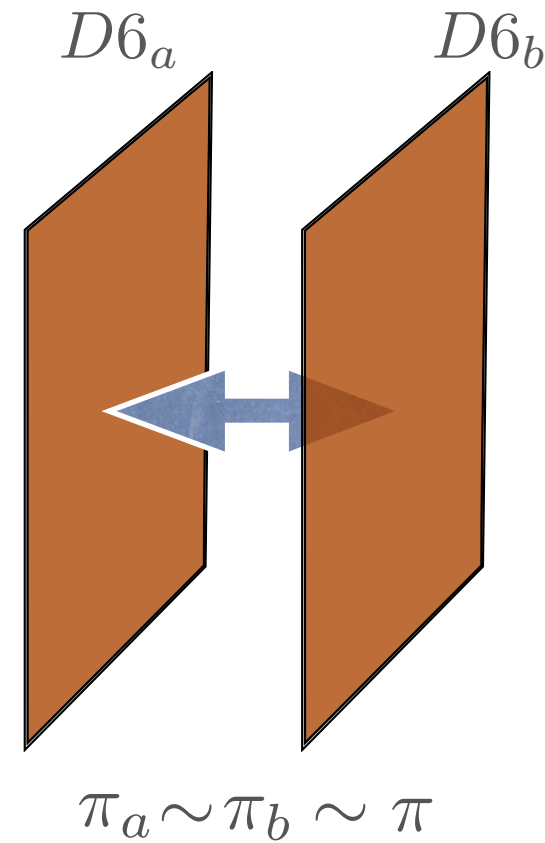
✦ D6-brane moduli

$$\Phi_a = \Phi_a^j \zeta_j \quad \zeta_j \in \mathcal{H}^1(\pi)$$

$$\Phi_a^j = \theta_a^j + \lambda_i^j \phi_a^i$$

✦ Open-closed mixing

$$f_{i(a-b)} = -\frac{i}{4l_s^3} (\Phi_a^j - \Phi_b^j) \int_{\pi} \zeta_j \wedge \omega_i$$



Mixing from DBI

❖ Type IIA with Higgsed D6-branes

◆ Open-closed mixing

$$f_{i(a-b)} = -\frac{i}{4l_s^3} (\Phi_a^j - \Phi_b^j) \int_{\pi} \zeta_j \wedge \omega_i$$

Vanishes whenever

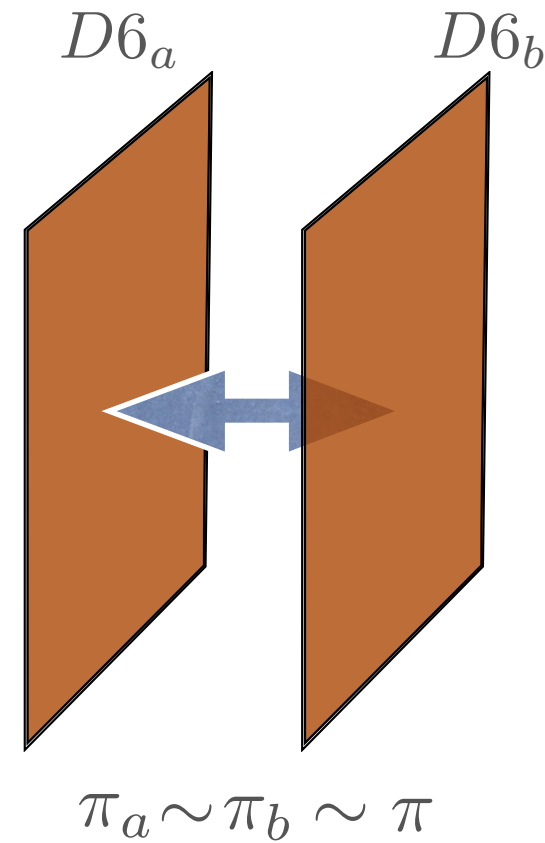
$$\Phi_a^j = \Phi_b^j \rightarrow SU(2)$$

or

$$\int_{\pi} \omega_i \wedge \zeta_j = \int_{\rho_j} \omega_i = 0$$



the 2-cycles ρ_j
of π are trivial in
ambient space



Mixing from the Witten effect

- ❖ DBI dim. reduction only sees D-brane moduli dependence, but typically we aim for models without open string moduli
- ❖ More powerful method → use of the Witten effect

Witten '79

Gauge theory that breaks CP



Magnetic monopoles
have electric charge

Simplest case: $\theta F \wedge F$



$$Q^E = -\frac{\theta}{2\pi}$$

Mixing from the Witten effect

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Magnetic **monopoles** have **electric charge**

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Multiple U(1)'s



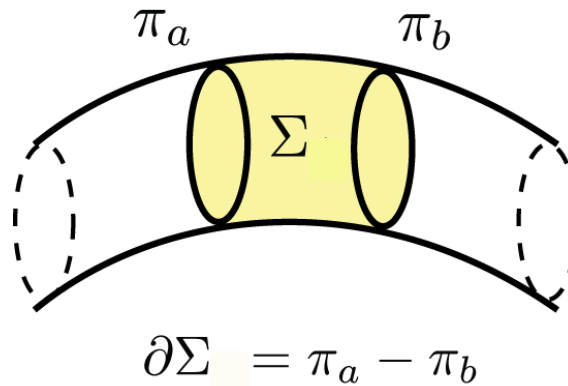
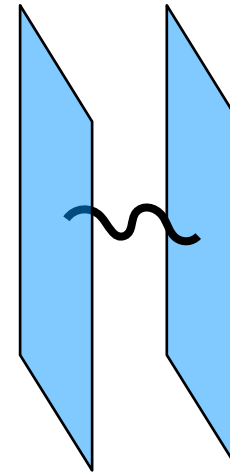
$$\begin{cases} Q_I^E = n_I^e - \text{Im} f_{IJ} n_J^m \\ Q_I^M = n_I^m \end{cases}$$

$$S_{4d} \supset - \int_{\mathbb{R}^{1,3}} \text{Re} f_{pq} F_p \wedge *F_q + \text{Im} f_{pq} F_p \wedge F_q$$

U(1)'s and Monopoles

✿ Upon **adjoint Higgsing** $SU(2) \rightarrow U(1)$ we obtain the following massive states

- ✦ W-bosons (fund. open string)
- ✦ Magnetic **monopoles**
(Dp-brane on p-chain Σ)

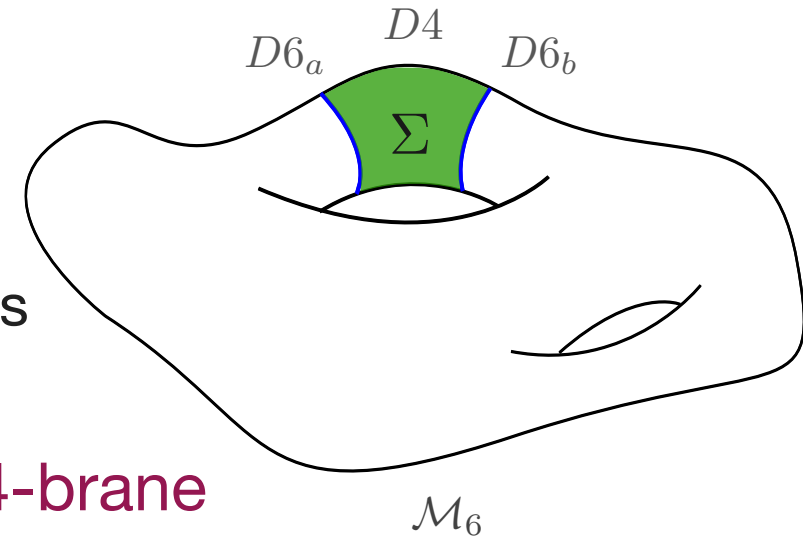


Monopoles and Mixing

❖ In our type IIA example these **monopoles** are **D4-branes** interpolating **between the D6-branes**

◆ **4-chain Σ** interpolates positions

◆ **Flux F^{D4}** on Σ interpolates Wilson lines



❖ From dimensional reduction of the **D4-brane CS action** one can obtain the **monopole charges** under the closed string $U(1)$, and then obtain the **open-closed mixing via the Witten effect**

$$f_{i(a-b)} = \frac{1}{2} \int_{\Sigma} (J - i\mathcal{F}^{D4}) \wedge \omega_i$$

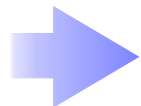
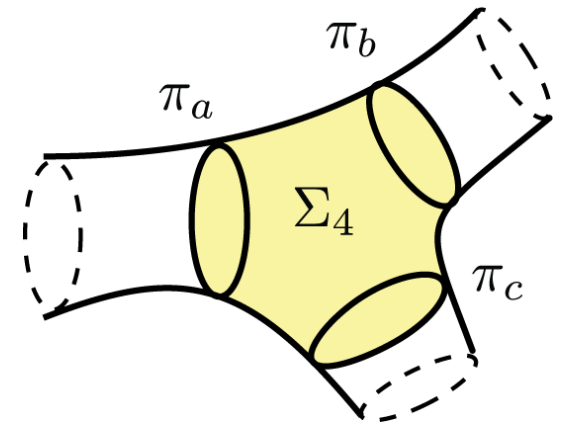
Monopoles and Mixing

- ✿ This **method is general** and does not rely on
 - ✦ D6-branes having moduli
 - ✦ U(1) coming from a Higgsed U(1) [homotopic 3-cycles]
- ✿ It can be applied to **any open string U(1)_x**

- ✦ Massless condition

$$\pi_X = \sum_{\alpha} n_{X\alpha} \pi_{\alpha} = \partial \Sigma_4 \quad n_{X\alpha} \in \mathbb{Z}$$

- ✦ Σ_4 is wrapped by the open string monopole



$$f_{iX} = \frac{1}{2} \int_{\Sigma_X} (J - i\mathcal{F}^{D4}) \wedge \omega_i$$

Mixing and M-theory

✿ The expression

$$f_{iX} = \frac{1}{2} \int_{\Sigma_X} (J - i\mathcal{F}^{D4}) \wedge \omega_i$$

matches previous results motivated by M-theory

Cámara, Ibáñez, F.M. '11

✿ We can establish the following dictionary:

U(1)'s

Monopoles

M-theory

$$\omega \in \mathcal{H}^{1,1}(\mathcal{M}_7)$$

$$\Lambda_5 \in H_5(\mathcal{M}_7)$$

type IIA

$$\omega \in \mathcal{H}_+^{1,1}(\mathcal{M}_6)$$

$$\pi_X - \pi_X^* = \partial \Sigma_4$$

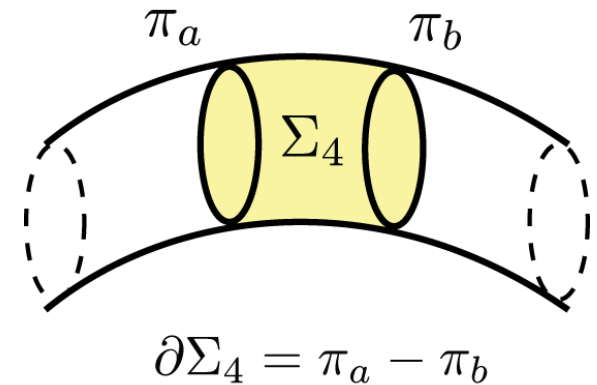
$$\pi_4 \in H_4^-(\mathcal{M}_6, \pi_{D6}) \begin{matrix} \nearrow H_4^-(\mathcal{M}_6) \\ \searrow \{\Sigma_4\} \end{matrix}$$

relative homology group

Mixing and Linear Equivalence

- ✿ The vanishing kinetic mixing condition

$$\text{Re } f_{i(a-b)} = \frac{1}{2} \int_{\Sigma_4} J \wedge \omega_i = 0$$



is similar to asking that the 3-cycles π_a and π_b are linearly equivalent

- ✿ **Linear equivalence**: criterion to compare **p-cycles in the same homology class** [harmonic forms on (p+1)-chains vanish]

- ✦ Typically used for divisors but can be applied to more general cycles wrapped by BPS D-branes

Hitchin '99

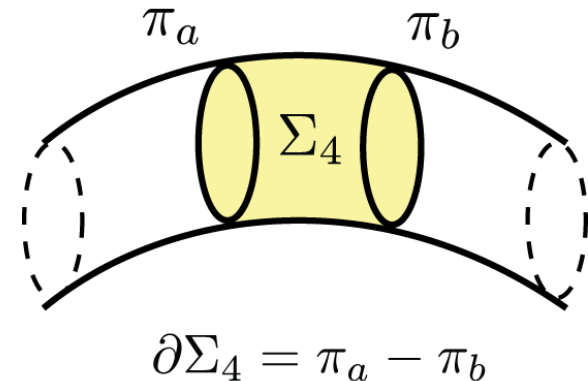
- ✦ Allows to write the kinetic mixing as

$$\text{Re } f_{i(a-b)} = \frac{1}{2} \int_{\mathcal{M}_6} J \wedge \omega_i \wedge \varpi_2 \qquad d\varpi_2 = \delta_3(\pi_3^a) - \delta_3(\pi_3^b)$$

Recap of type IIA

- ❖ Open-closed U(1) mixing is a holomorphic quantity of the 4d effective theory that can be computed via a chain integral

$$f_{iX} = \frac{1}{2} \int_{\Sigma_X} (J - i\mathcal{F}^{D4}) \wedge \omega_i$$

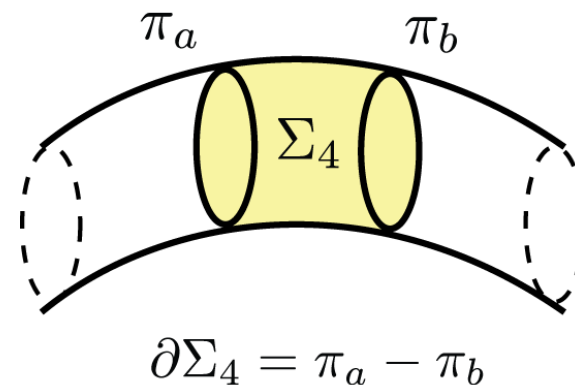


- ❖ The physical meaning of this chain is the internal worldvolume of the open string U(1) monopole
- ❖ The mathematical meaning is the measurement of linear equivalence between submanifolds, or generalised version that include D-brane Wilson lines

Recap of type IIA

- ❖ Open-closed U(1) mixing is a holomorphic quantity of the 4d effective theory that can be computed via a chain integral

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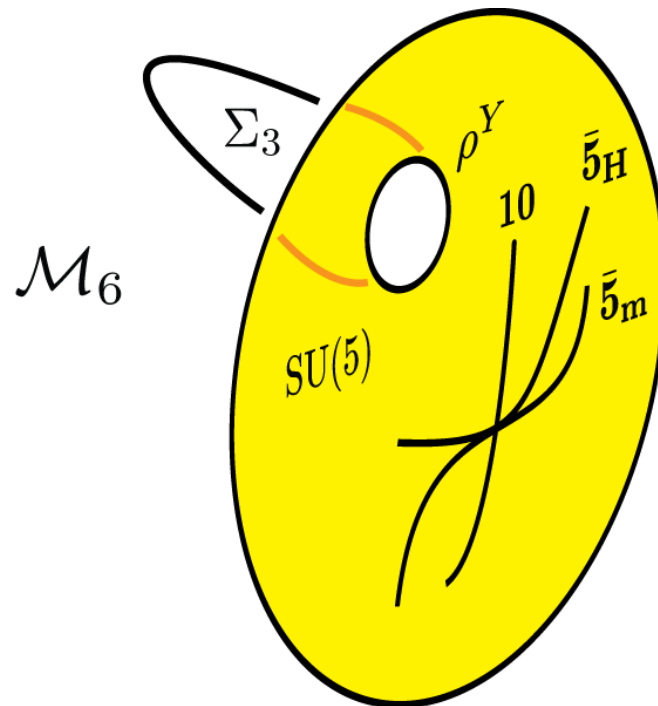
- ❖ Another holomorphic quantity computed via a chain integral is the D6-brane superpotential

Martucci '06

$$W_{D6} \sim \int_{\mathcal{M}_6} (J - i\mathcal{F})^2$$

both quantities related in N=2 (unorientifolded CY geometry)

Mixing in type IIB and F-theory GUTs

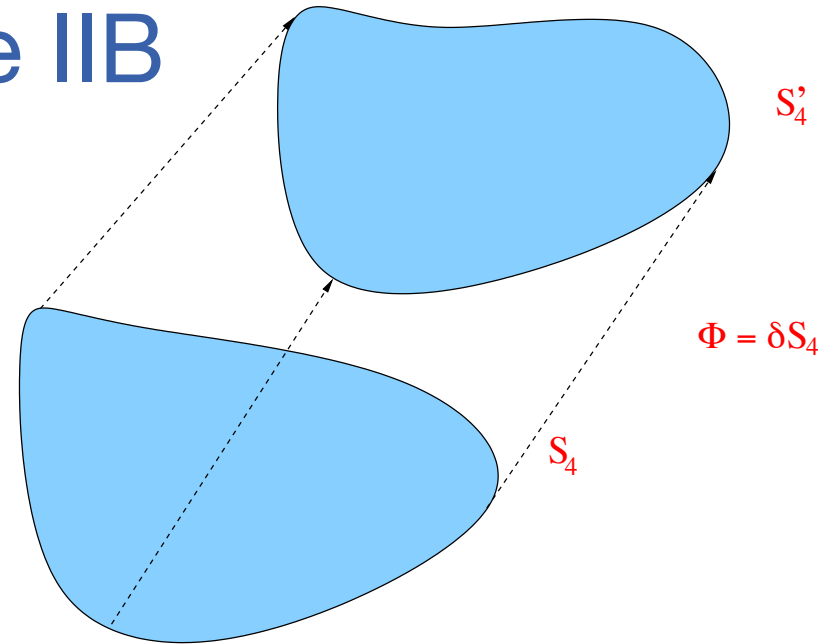


Mixing in type IIB

❖ Type IIB with Higgsed D7-branes

◆ Closed string U(1)'s

$$C_4 = A_1^i \wedge \text{Re } \gamma_i \quad \gamma_i \in \mathcal{H}_+^{2,1}$$



- ◆ Two D7-branes in the same homology class of a CY are always linearly equivalent to each other [no harmonic 5-form]
- ◆ However, magnetised D7-branes carry charge of D5-brane, for which linear equivalence is non-trivial
- ◆ Open-closed mixing from DBI analysis with moduli

Jockers & Louis '04

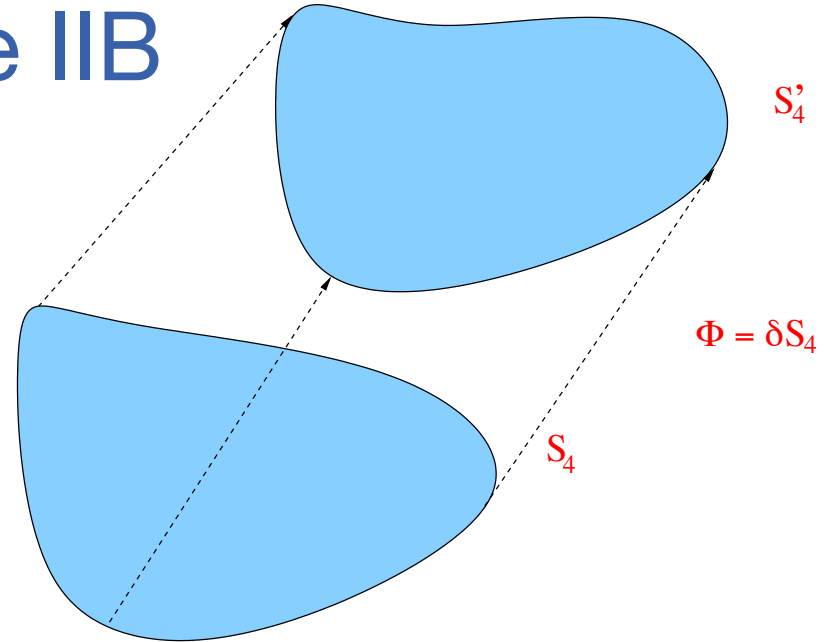
$$f_{i(a-b)} = -\frac{i}{4}(a_a^j - a_b^j) \int_S A_j \wedge \gamma_i - \frac{i}{4}(\Phi_a^m - \Phi_b^m) \int_S \iota_{X_m} \gamma_i \wedge F_2$$

Mixing in type IIB

✿ Type IIB with Higgsed D7-branes

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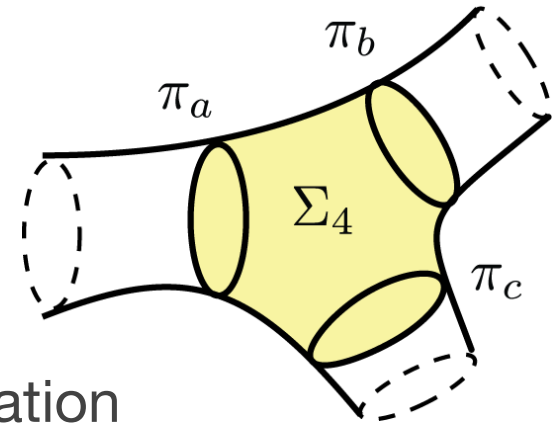
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- ◆ However, magnetised D7-branes carry charge of D5-brane, for which linear equivalence is non-trivial
- ◆ Open-closed mixing from Witten effect [D5-brane monopole]

$$f_{i(a-b)} = -\frac{i}{2} \int_{\Gamma} \gamma_i \wedge \tilde{\mathcal{F}}$$

Mixing in type IIB

✿ General case

- ✦ Fluxes contribute to the Stückelberg mass.
Massless open string U(1) for a linear combination such that all induced D-brane charges also vanish
- ✦ Appropriate framework: generalised homology
Monopoles are described by D-brane networks on generalised chains
- ✦ Mixing can still be extracted from the Witten effect on these open string magnetic monopoles



Euslin and Martucci '07

$$f_{iX} = -\frac{i}{2} j_{(\mathfrak{S}, \mathfrak{F})_X}(\gamma_i),$$

Hypercharge mixing in F-theory GUTs

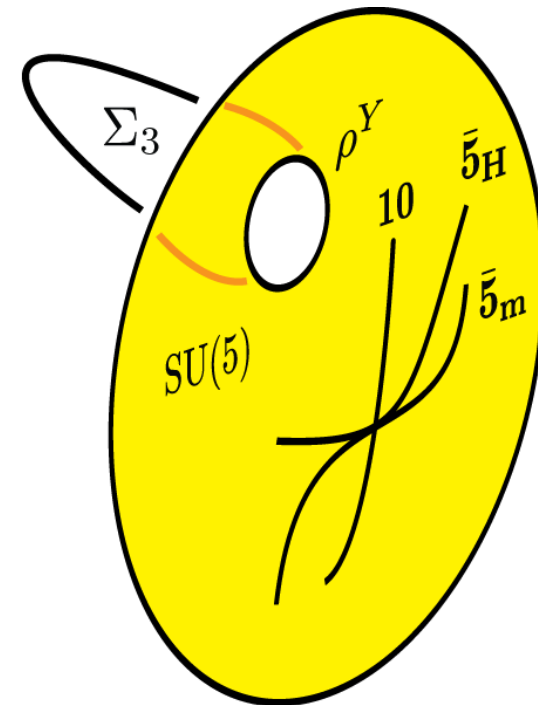
✿ Consider F-theory SU(5) with hypercharge breaking

✦ Flux F_Y is non-trivial in $H^2(S)$
but trivial in ambient space

Buican et al. '06

Donaghi & Winkholt '08

Beasley et al. '08



Hypercharge mixing in F-theory GUTs

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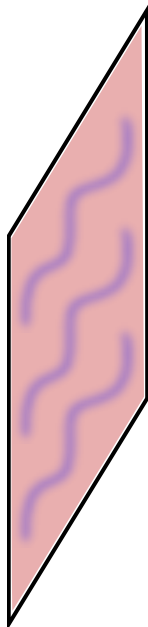
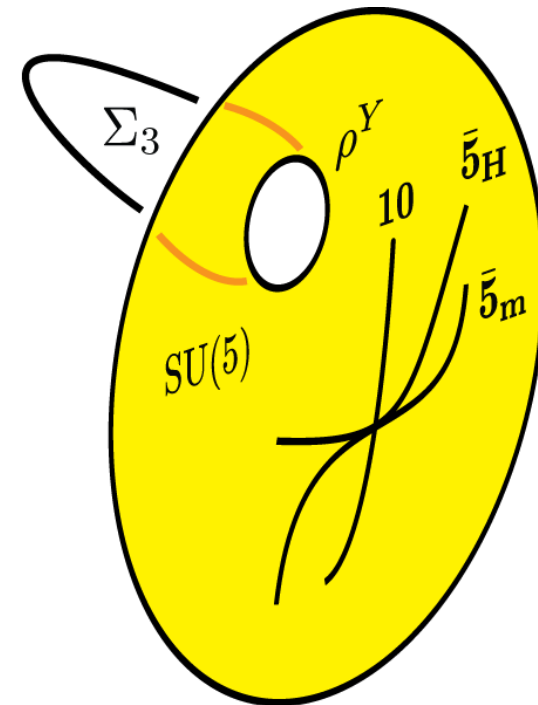
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✦ Monopole is subtle: genuine D-brane network in a generalised chain

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$D7_a$
 (S_a, F_a)

Hypercharge mixing in F-theory GUTs

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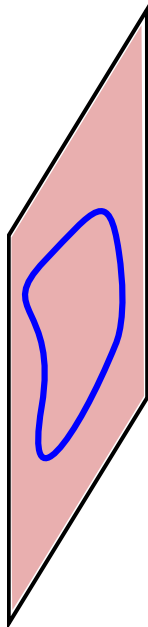
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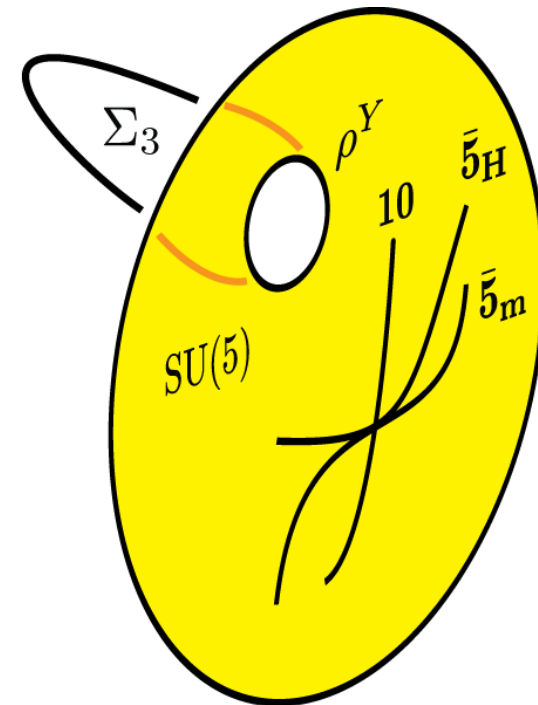
Beasley et al. '08



$D7_a + D5_a$

$$f_{iY} = -\frac{i}{2} \int_{S_a} \gamma_i \wedge A_{\Pi_a}$$

$$dA_a = F_a - \delta(\Pi_a)$$



Hypercharge mixing in F-theory GUTs

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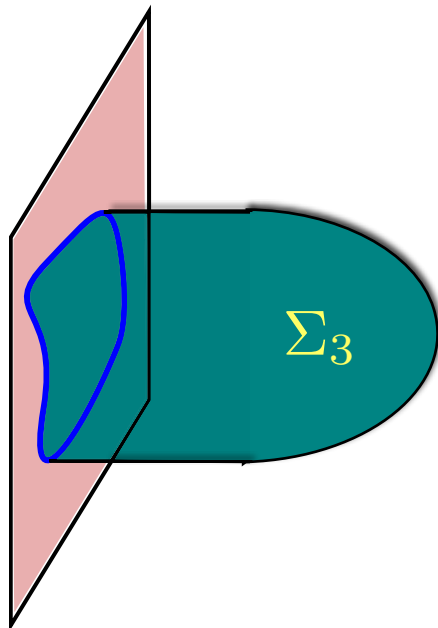
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Buican et al. '06

Donaghi & Wijnholt '08

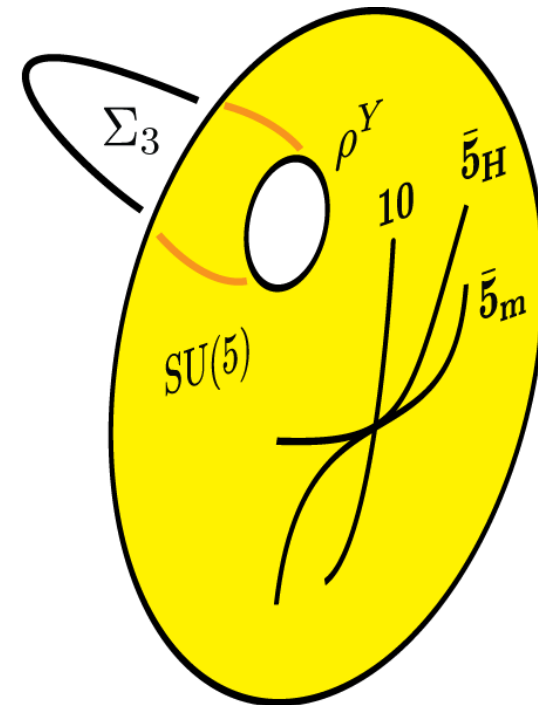
Beasley et al. '08

- ✦ Monopole is subtle: genuine D-brane network in a generalised chain



$$f_{iY} = -\frac{i}{2} \left[\int_{S_a} \gamma_i \wedge A_{\Pi_a} + \int_{\Sigma_3} \gamma_i \right]$$

$$dA_a = F_a - \delta(\Pi_a)$$



Conclusions

- ❖ Open-closed U(1) kinetic mixing is phenomenologically relevant as a source of millicharged particles and inducing corrections to gauge coupling unification
- ❖ In general it can be computed via a chain integral.
Physical meaning: U(1) magnetic monopoles and Witten effect
- ❖ Mathematical meaning: linear equivalence of submanifolds and generalised version for D-branes (generalised geometry)
- ❖ Particularly interesting case: F-theory GUT hypercharge mixing with bulk U(1)'s. Monopole is subtle and so is the expression for the mixing

The String Theory Universe

& 20th European Workshop on String Theory 2nd COST MP1210 Meeting

22–26 September 2014
Philosophicum, JGU Mainz

www.strings2014.uni-mainz.de



The conference is dedicated to all aspects of superstring, supergravity and supersymmetric theories and is embedded in the MITP programme String Theory and its Applications.

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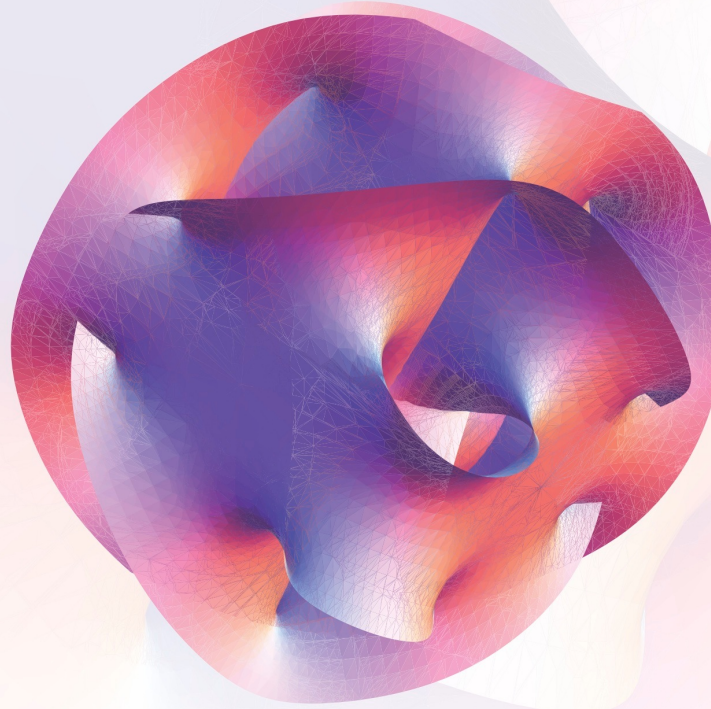
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Strings and the Cosmic
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MITP Public Lecture

Dieter Lust | Munich
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Mainzer Wissenschaftsmarkt
Saturday, 13 September 2014 at 6pm.

Working Groups

Gauge/Gravity Duality
String Phenomenology
Cosmology and Quantum Gravity