F-Theory

Gary Shiu

F-term Axion Monodromy Inflationary Theory

Gary Shiu

Based on: Marchesano, GS, Uranga, arXiv:1404.3040 [hep-th]

[See Uranga's talk on Wednesday]

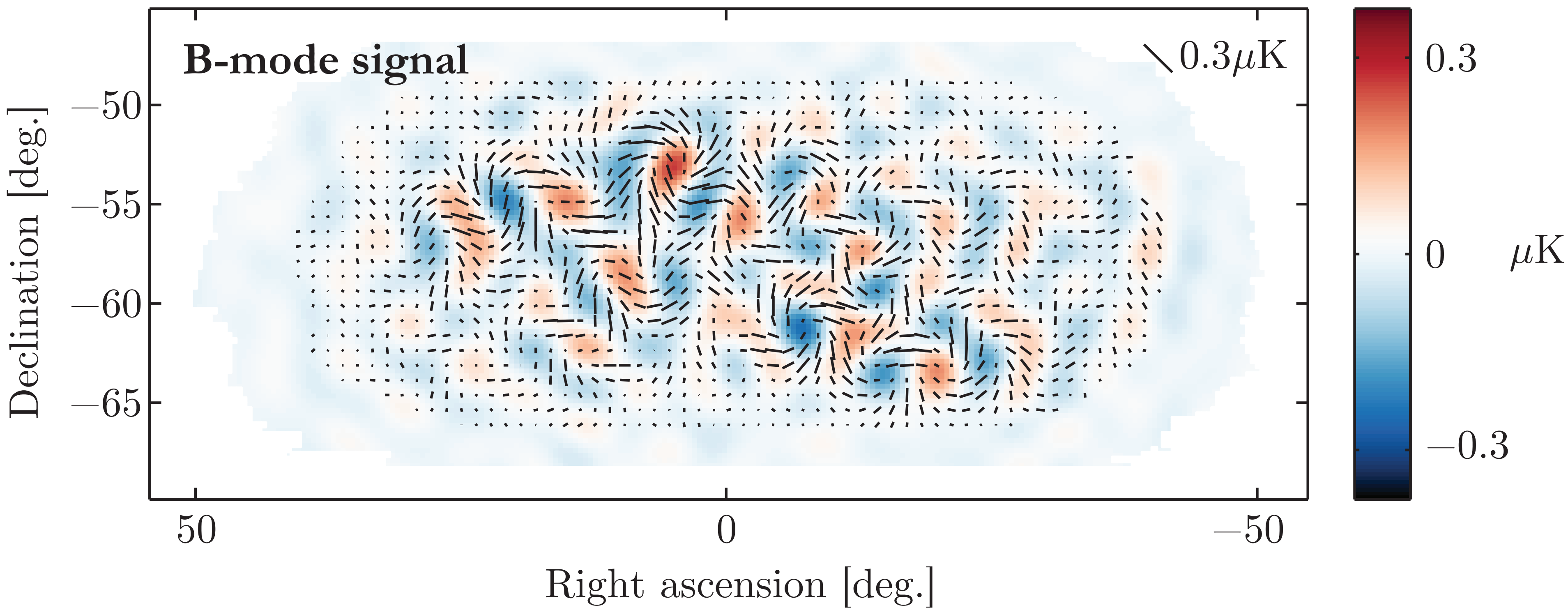
Brane Curvature Corrections to F-Theory EFT

Gary Shiu

Junghans, GS, arXiv:1407.0019 [hep-th]

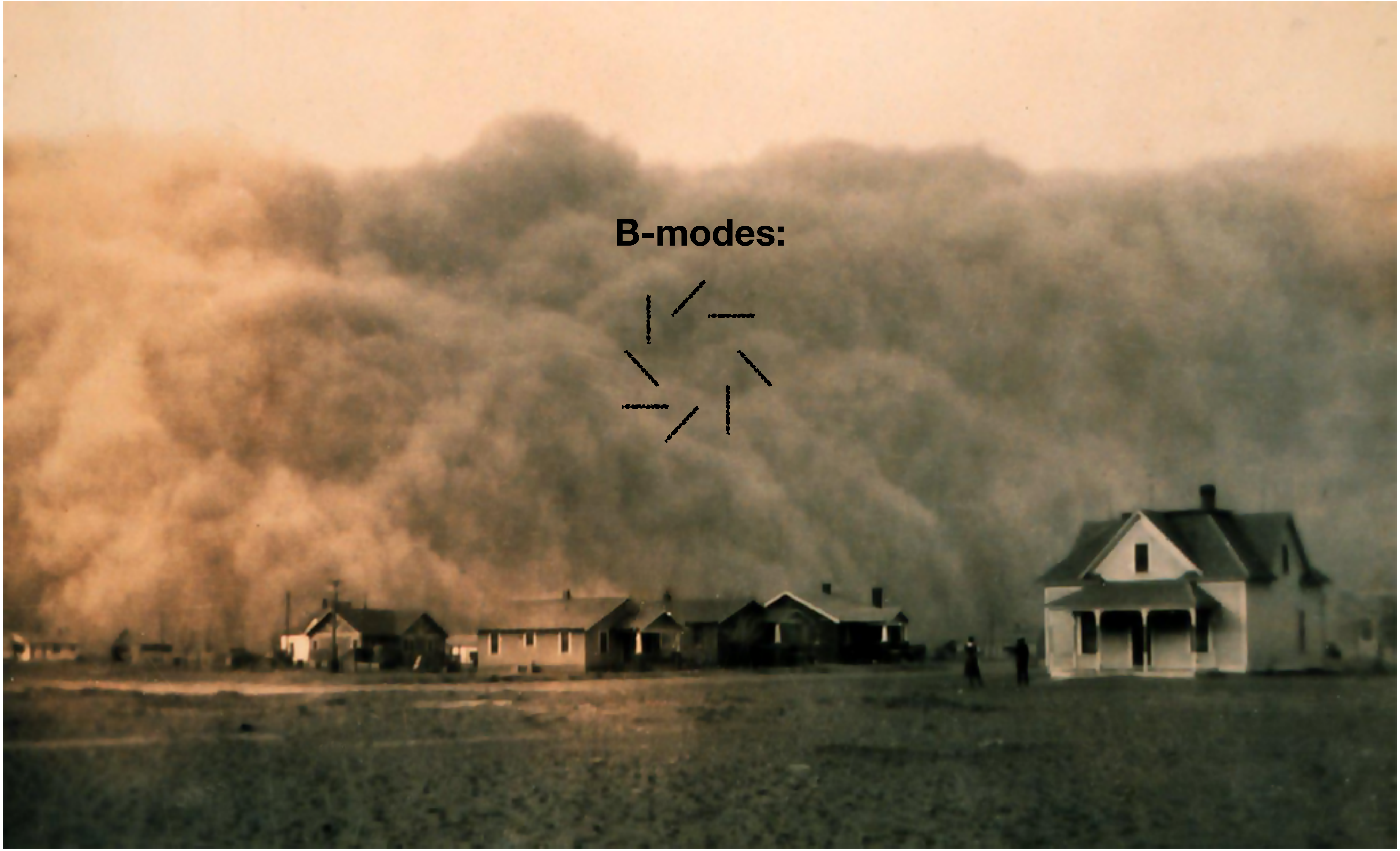
[See Junghans's talk on Wednesday]

Primordial B-mode?



BICEP2 Collaboration

Dust is not entirely settled …



[Mortonson & Seljak]
[Flauger, Hill & Spergel]

Possible "Tension" with PLANCK

BICEP2 and Inflation

If the BICEP2 results are confirmed to be primordial, natural interpretations:

- ✦ Inflation took place
- ✦ The energy scale of inflation is the GUT scale

$$E_{\rm inf} \simeq 0.75 \times \left(\frac{r}{0.1}\right)^{1/4} \times 10^{-2} M_{\rm Pl}$$

- ✦ The inflaton field excursion was super-Planckian

$$\Delta\phi \gtrsim \left(\frac{r}{0.01}\right)^{1/2} M_{\rm Pl}$$

Lyth '96

- ✦ Great news for string theory due to strong UV sensitivity!

Assumptions in the Lyth Bound

- single field
- slow-roll
- Bunch-Davies initial conditions
- vacuum fluctuations

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Particle production during inflation can be a source of GWs

Cook and Sorbo

Senatore, Silverstein, Zaldarriaga

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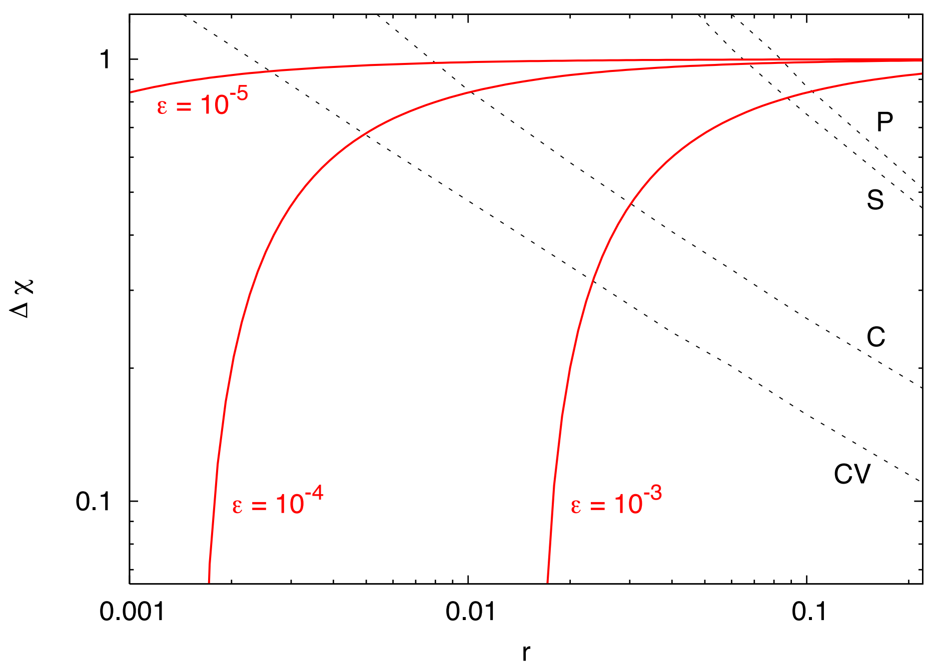


Only known model of particle production that:

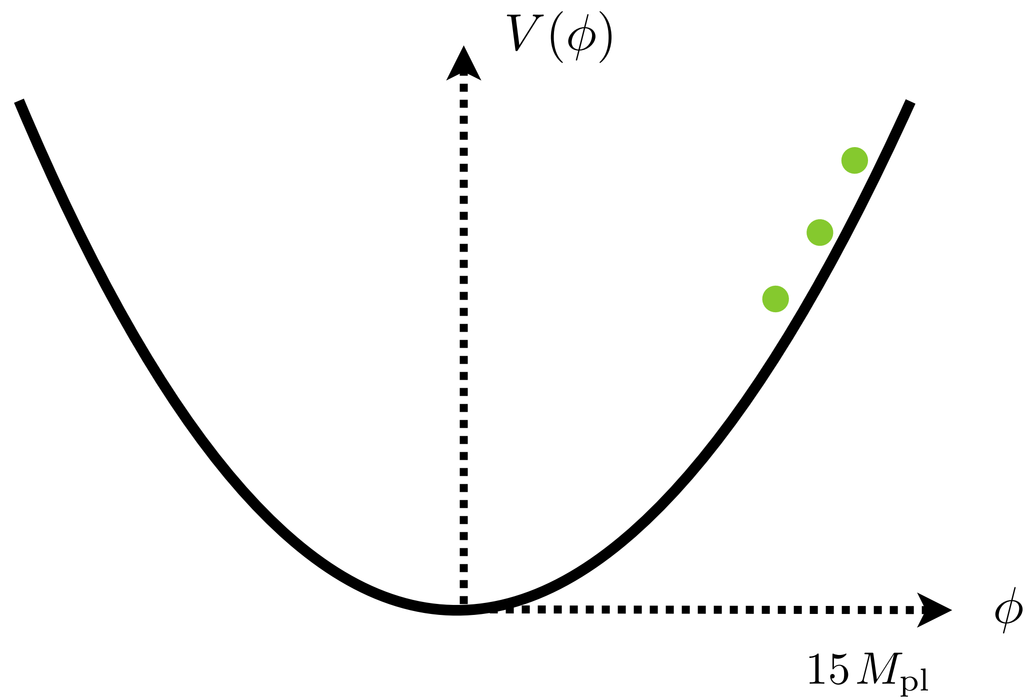
- Detectable tensors w/o too large non-Gaussianity
- ***Chiral***, ***non-Gaussian*** tensor spectrum
- Can accommodate a blue tensor tilt

due to an ***axionic*** a $F \wedge F$ coupling

Chaotic Inflation

Linde '86

- A poster child inflation model (also seems favored) is V = m²φ²:



  - Loop corrections involving inflaton and gravitons are small due to approximate shift symmetry

$$\phi \mapsto \phi + \text{const.}$$

  - Coupling to UV degrees of freedom in quantum gravity a priori breaks this shift symmetry and lead to corrections that spoil inflation, because of the large field excursions

$$\mathcal{L}_{\rm eff}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \sum_{i=1}^{\infty} c_i\, \phi^{2i}\Lambda^{4-2i}$$

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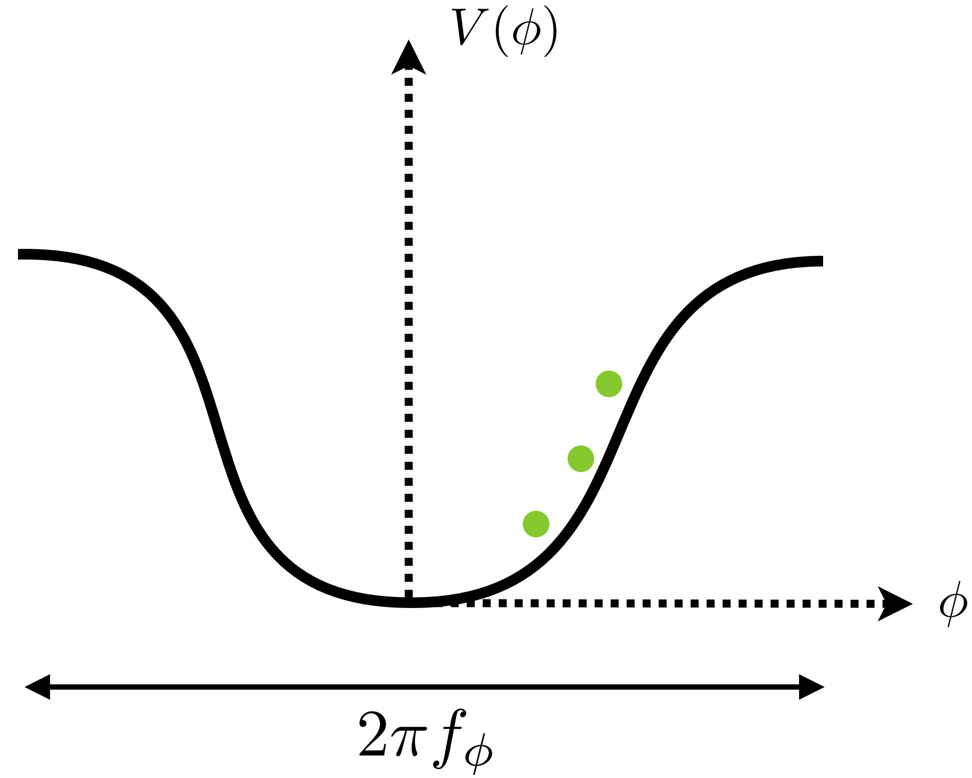
figure taken from Baumann & McAllister '14

Natural Inflation

Freese, Frieman, Olinto '90

- String models where the inflaton is an axion in principle can avoid this problem
  - Shift symmetry broken by non-perturbative effects+UV completion, but periodicity is exact
  - In string theory axions generically come from p-forms, so above the KK scale the shift symmetry becomes a gauge symmetry



$$\phi = \int_{\pi_p} C_p$$

$$F_{p+1} = dC_p$$
$$C_p \to C_p + d\Lambda_{p-1}$$

Dimopoulos et al.' 05

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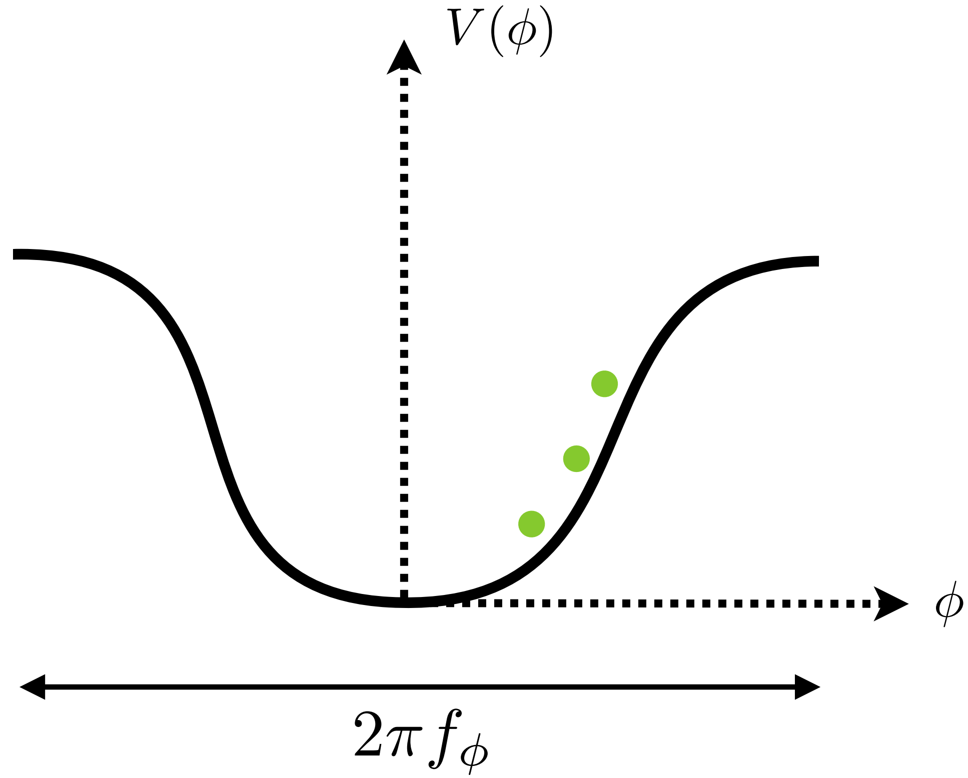
  - Shift symmetry broken by non-perturbative effects+UV completion, but periodicity is exact
  - In string theory axions generically come from p-forms, so above the KK scale the shift symmetry becomes a gauge symmetry
  - However, these axions have sub-Planckian decay constants



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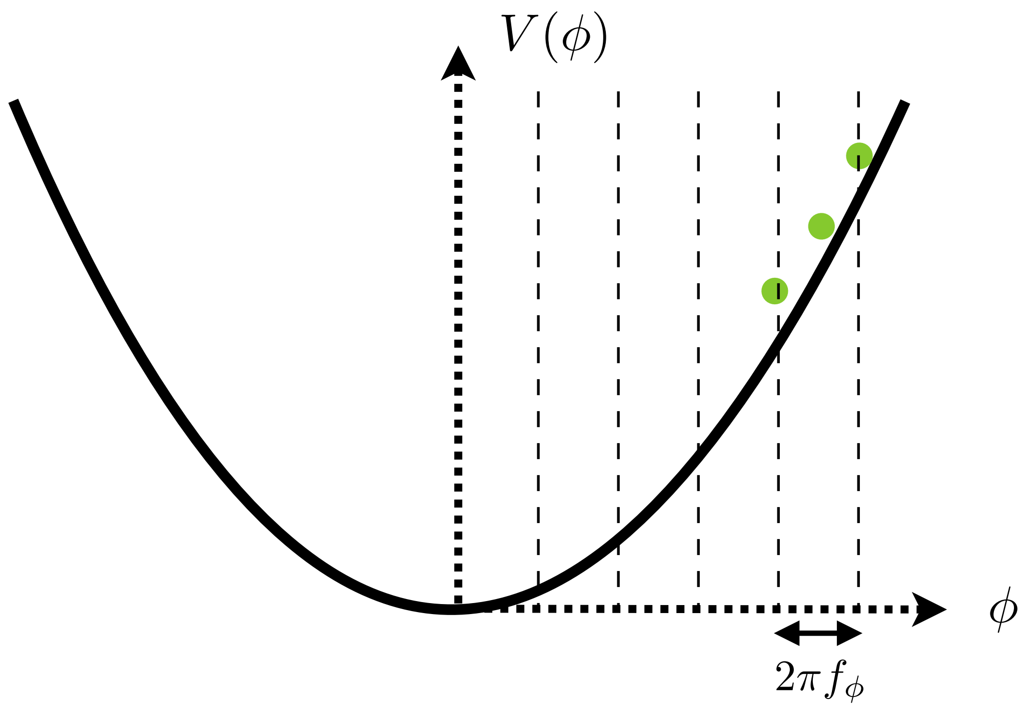
Banks et al. '03 *Svrcek & Witten '06*

Axion Monodromy Inflation

Siverstein & Westphal '08

Idea: Combine chaotic inflation and natural inflation



The axion periodicity is lifted, allowing for super-Planckian displacements. The UV corrections to the potential should still be constrained by the underlying symmetry.

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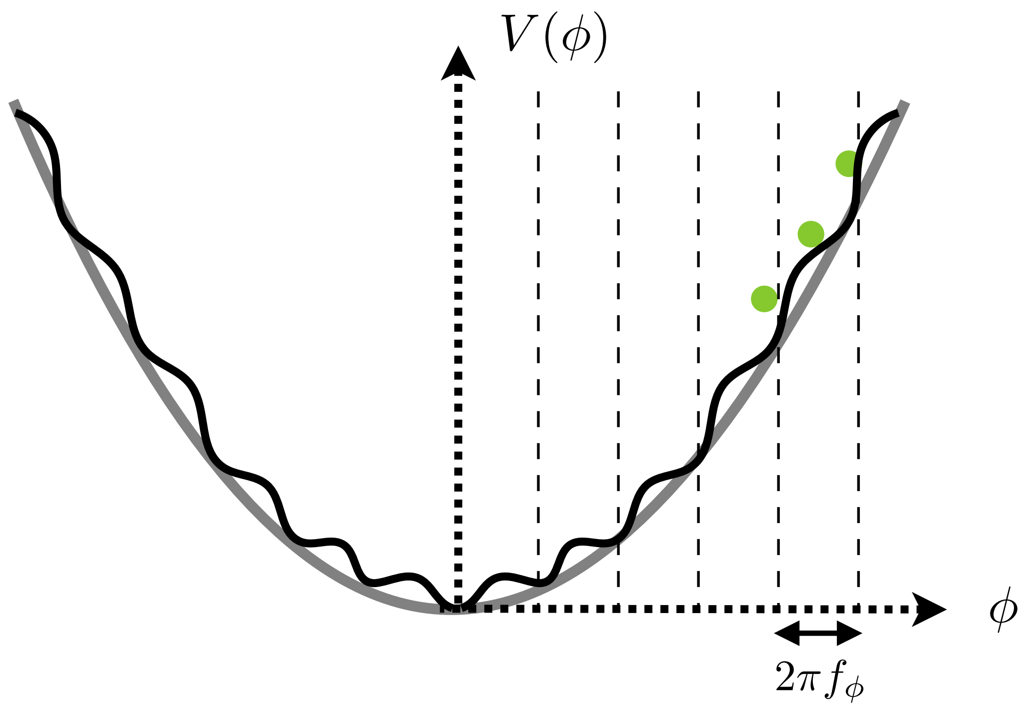

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- ✦ McAllister, Silverstein, Westphal → String scenarios

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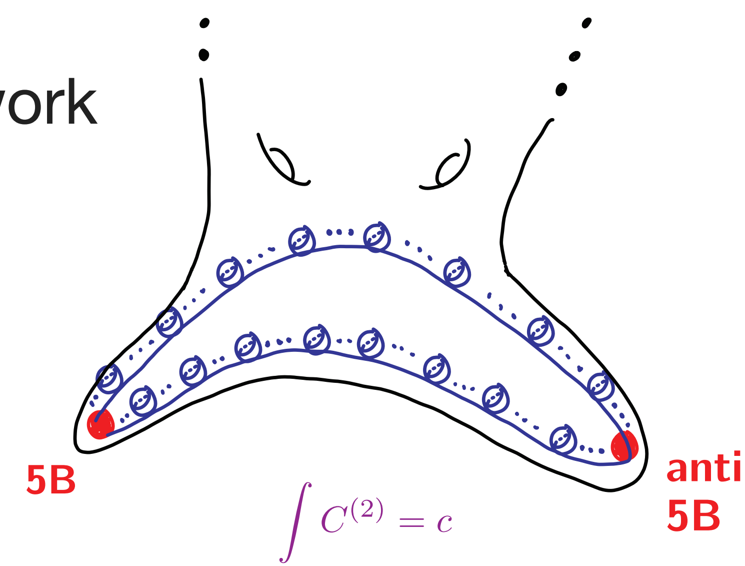

figure taken from McAllister, Silverstein, Westphal '08

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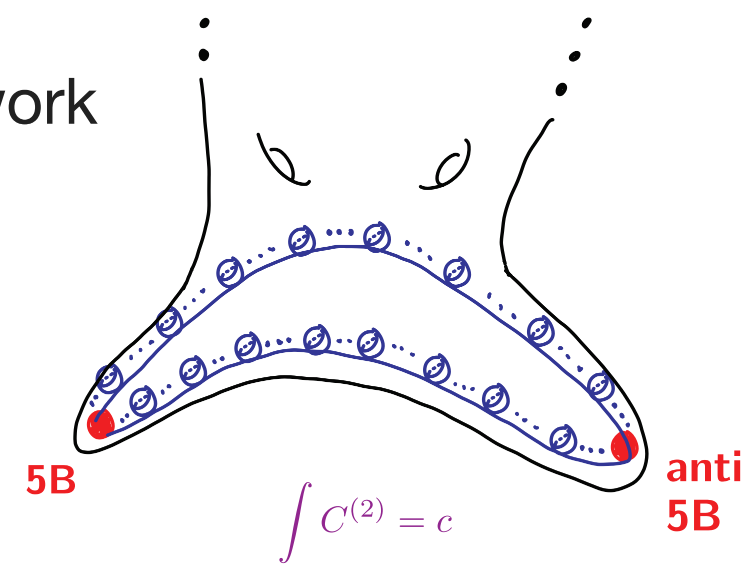

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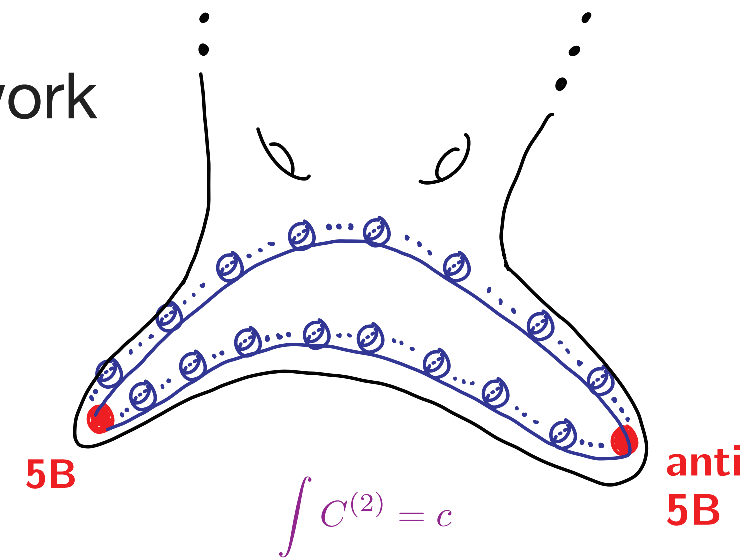

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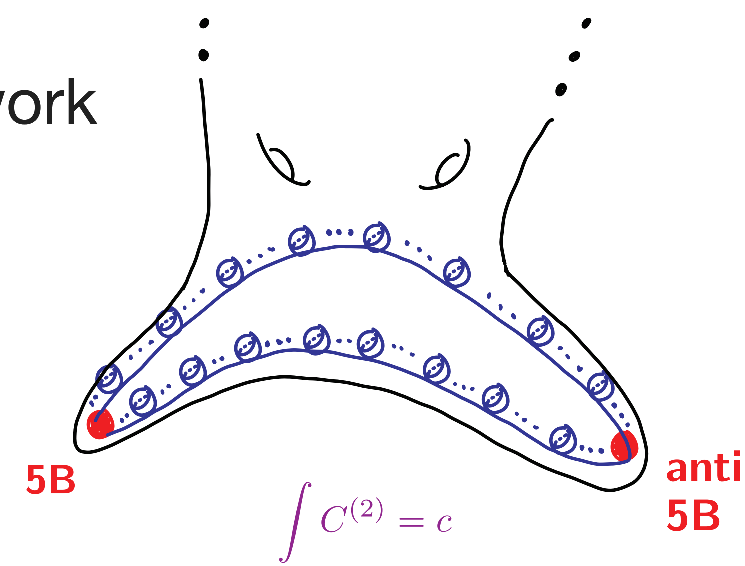
See also talks of Nilles, Hebecker, Witowski, Uranga, Retolaza, Valenzuela, and references therein.



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F-term Axion Monodromy Inflation

Obs: Axion Monodromy ~ Giving a mass to an axion

- Done in string theory within the moduli stabilization program: adding ingredients like background fluxes generate superpotentials in the effective 4d theory

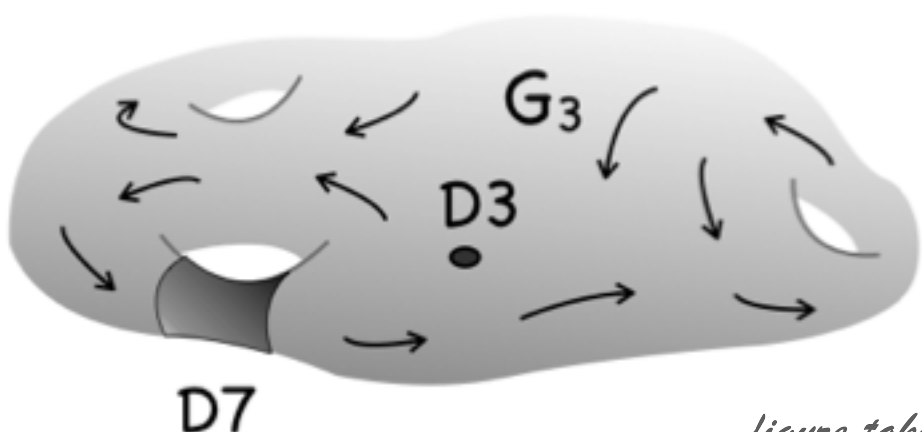

figure taken from Ibañez & Uranga '12

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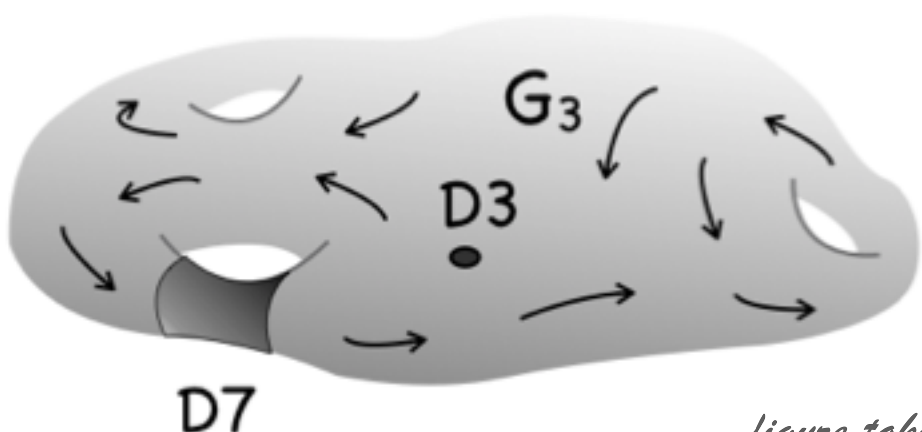

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Idea: Use same techniques to generate an inflation potential

- Simpler models, all sectors understood at weak coupling
- Spontaneous SUSY breaking, no need for brane-anti-brane
- Clear endpoint of inflation, allows to address reheating

Toy Example: Massive Wilson line

- Simple example of axion: (4+d)-dimensional gauge field integrated over a circle in a compact space Π_d

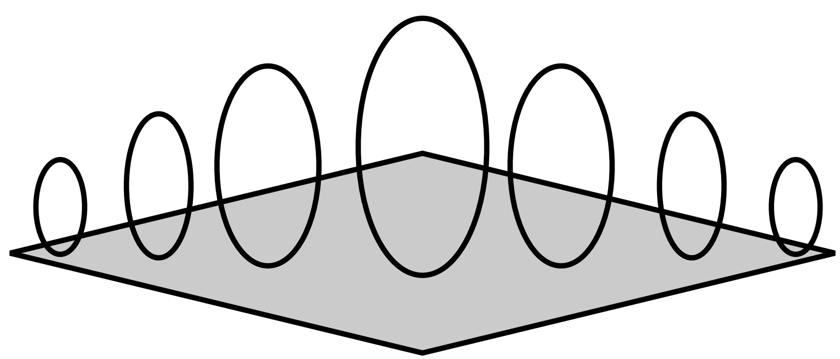
$$\phi = \int_{S^1} A_1 \qquad \text{or} \qquad A_1 = \phi(x)\,\eta_1(y)$$

  - ϕ massless if $\Delta\eta_1 = 0 \Rightarrow S^1$ is a non-trivial circle in Π_d

    exact periodicity and (pert.) shift symmetry

  - ϕ massive if $\Delta\eta_1 = -\mu^2\,\eta_1 \Rightarrow kS^1$ homologically trivial in Π_d

    (non-trivial fibration)



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$$F_2 = dA_1 = \phi\, d\eta_1 \sim \mu\phi\, \omega_2 \quad \Rightarrow$$ shifts in ϕ increase energy via the induced flux F_2

$\Rightarrow$ periodicity is broken and shift symmetry approximate

MWL and twisted tori

- Simple way to construct massive Wilson lines: consider compact extra dimensions Π_d with circles fibered over a base, like the twisted tori that appear in flux compactifications

- There are circles that are not contractible but do not correspond to any harmonic 1-form. Instead, they correspond to torsional elements in homology and cohomology groups

$$\mathrm{Tor}\, H_1(\Pi_d, \mathbb{Z}) = \mathrm{Tor}\, H^2(\Pi_d, \mathbb{Z}) = \mathbb{Z}_k$$

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- Simplest example: twisted 3-torus $\tilde{\mathbb{T}}^3$

$$H_1(\tilde{\mathbb{T}}^3, \mathbb{Z}) = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_k$$

two normal 1-cycles

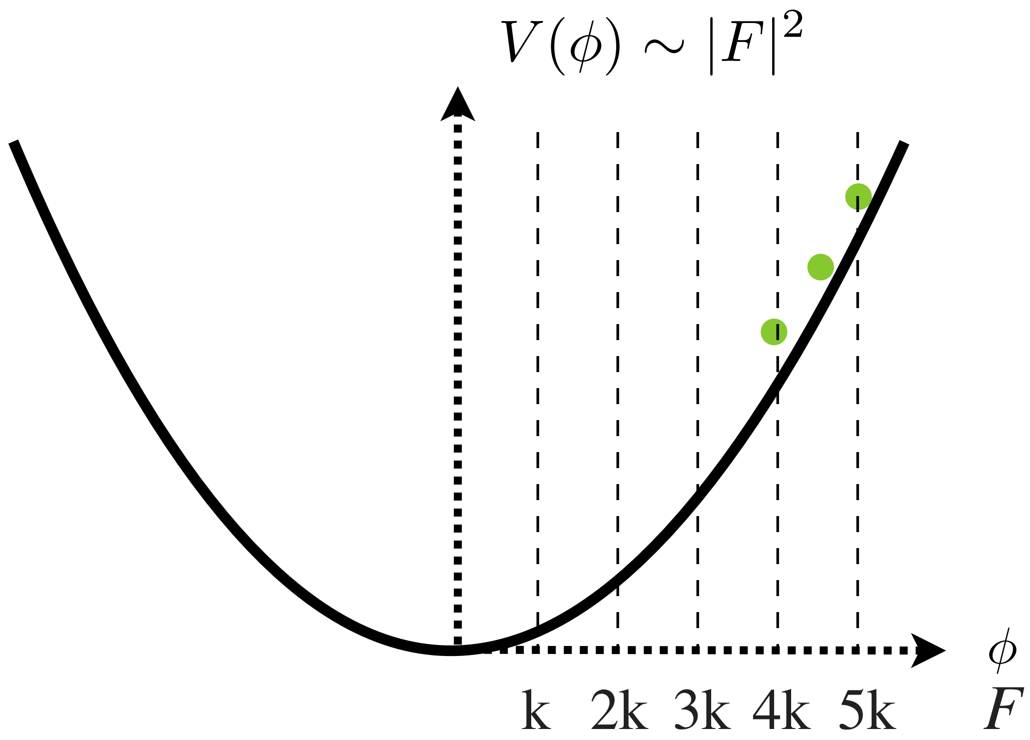
one torsional 1-cycle

$$d\eta_1 = k dx^2 \wedge dx^3 \longrightarrow F = \phi\, k\, dx^2 \wedge dx^3$$

$$\mu = \frac{kR_1}{R_2 R_3}$$

under a shift $\phi \to \phi + 1$
F_2 increases by k units

MWL and monodromy



Question:

How does monodromy and approximate shift symmetry help prevent wild UV corrections?

Torsion and gauge invariance

- Twisted tori torsional invariants are not just a fancy way of detecting non-harmonic forms, but are related to a hidden gauge invariance of these axion-monodromy models
- Let us again consider a 7d gauge theory on $M^{1,3}$ x $\tilde{\mathbb{T}}^3$
  - Instead of A_1 we consider its magnetic dual V_4

$$V_4 = C_3 \wedge \eta_1 + b_2 \wedge \sigma_2 \xrightarrow{d\eta_1 = k\,\sigma_2} dV_4 = dC_3 \wedge \eta_1 + (db_2 - kC_3) \wedge \sigma_2$$

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  - From dimensional reduction of the kinetic term:

$$\int d^7x\, |dV_4|^2 \longrightarrow \boxed{\int d^4x\, |dC_3|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2}$$

  - Gauge invariance $\quad C_3 \to C_3 + d\Lambda_2 \qquad b_2 \to b_2 + k\Lambda_2$
  - Generalization of the Stückelberg Lagrangian

Quevedo & Trugenberger '96

Effective 4d theory

- The effective 4d Lagrangian

$$\int d^4x\,|dC_3|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2$$

describes a massive axion, has been applied to QCD axion ⇒ generalized to arbitrary V(ϕ)

Kallosh et al. '95
Dvali, Jackiw, Pi '05
Dvali, Folkerts, Franca '13

- Reproduces the axion-four-form Lagrangian proposed by Kaloper and Sorbo as 4d model of axion-monodromy inflation with mild UV corrections

$$\int d^4x\,|F_4|^2 + |d\phi|^2 + \phi F_4 \qquad \begin{aligned} F_4 &= dC_3 \\ d\phi &= *_4 db_2 \end{aligned}$$

Kaloper & Sorbo '08

- It is related to an F-term generated mass term

Groh, Louis, Sommerfeld '12

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$$F_4 = dC_3$$
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- Gauge symmetry ⇒ UV corrections only depend on F_4

$$\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mu^2\phi^2 + \Lambda^4 \sum_{i=1}^{\infty} c_i \, \frac{\phi^{2i}}{\Lambda^{2i}}$$

$$\sum_n c_n \, \frac{F^{2n}}{\Lambda^{4n}} \longrightarrow \mu^2\phi^2 \sum_n c_n \left(\frac{\mu^2\phi^2}{\Lambda^4}\right)^n$$

⇒ suppressed corrections up to the scale where $V(\phi) \sim \Lambda^4$

⇒ effective scale for corrections $\Lambda \to \Lambda_{\text{eff}} = \Lambda^2/\mu$

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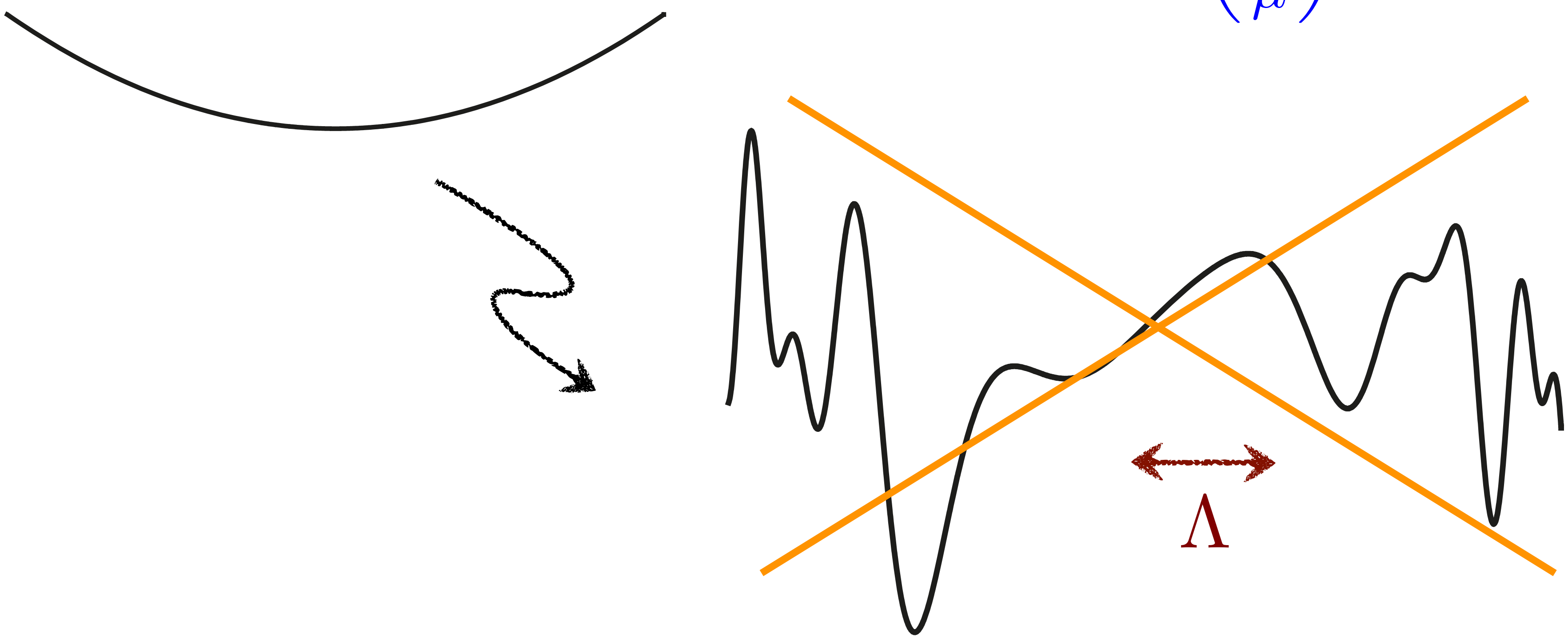
$$\Lambda \to \Lambda_{\text{eff}} = \Lambda \left(\frac{\Lambda}{\mu} \right)$$



Discrete symmetries and domain walls

- The integer k in the Lagrangian

$$\int d^4x \, |F_4|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$

corresponds to a discrete symmetry of the theory broken spontaneously once a choice of four-form flux is made. This amounts to choose a branch of the scalar potential

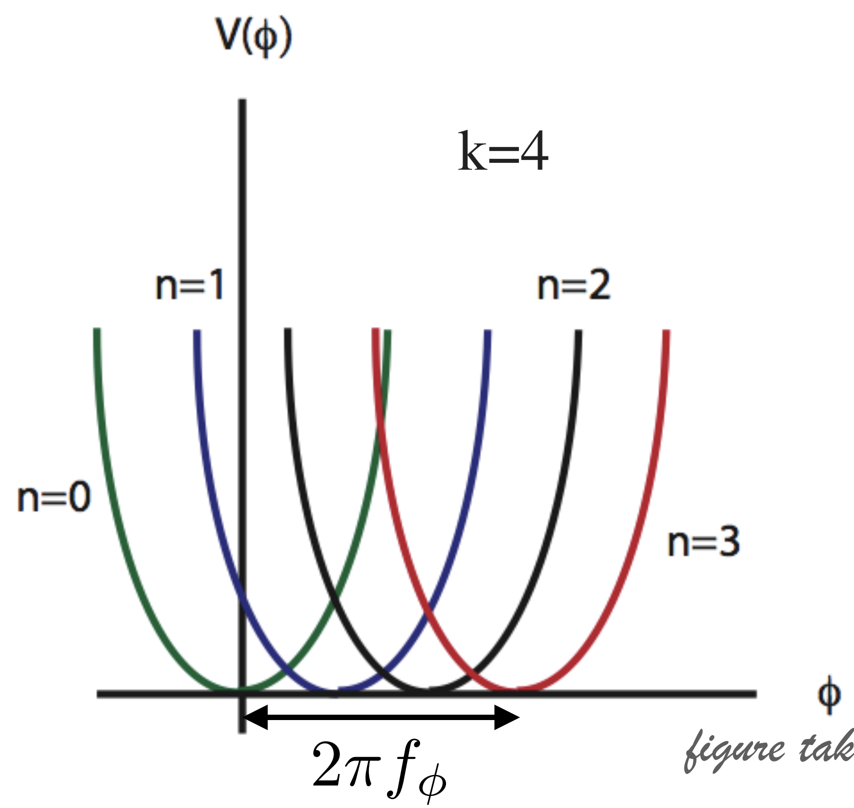


figure taken from Kaloper & Lawrence '14

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- Branch jumps are made via nucleation of domain walls that couple to C_3, and this puts a maximum to the inflaton range

- Domain walls analysed in string constructions [Uranga's talk]:

  Berasaluce-Gonzalez, Camara, Marchesano, Uranga '12

  - They correspond to discrete symmetries of the superpotential/ landscape of vacua, and appear whenever axions are stabilised
  - k domain walls decay in a cosmic string implementing $\phi \to \phi+1$

Massive Wilson lines in string theory

- Simple example of MWL in string theory: D6-brane on $M^{1,3} \times \tilde{\mathbb{T}}^3$
- An inflaton vev induces a non-trivial flux F_2 proportional to ϕ but now this flux enters the DBI action

$$\sqrt{\det\left(G + 2\pi\alpha' F_2\right)} \;=\; d\text{vol}_{M^{1,3}}\;\left(|F_2|^2 + \text{corrections}\right)$$

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- For small values of ϕ we recover chaotic inflation, but for large values the corrections are important and we have a potential of the form

$$V = \sqrt{L^4 + \langle\phi\rangle^2} - L^2$$

Similar to the D4-brane model of Silverstein and Westphal except for the inflation endpoint

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Massive Wilson lines and flattening

- The DBI modification

$$\langle\phi\rangle^2 \to \sqrt{L^4 + \langle\phi\rangle^2} - L^2$$

can be interpreted as corrections due to UV completion

- E.g., integrating out moduli such that H < m_{mod} < M_{GUT} will correct the potential, although not destabilise it

Kaloper, Lawrence, Sorbo '11

- In the DBI case the potential is flattened: argued general effect due to couplings to heavy fields

Dong, Horn, Silverstein, Westphal '10

- Large vev flattening also observed in examples of confining gauge theories whose gravity dual is known [Witten'98]

Dubovsky, Lawrence, Roberts '11

Other string examples

- We can integrate a bulk p-form potential C_p over a p-cycle to get an axion

$$F_{p+1} = dC_p, \quad C_p \to C_p + d\Lambda_{p-1} \qquad\qquad c = \int_{\pi_p} C_p$$

- If the p-cycle is torsional we will get the same effective action

$$\int d^{10}x |F_{9-p}|^2 \longrightarrow \int d^4x \, |dC_3|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$

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- The topological groups that detect this possibility are

$$\mathrm{Tor}\, H_p(\mathbf{X}_6, \mathbb{Z}) = \mathrm{Tor}\, H^{p+1}(\mathbf{X}_6, \mathbb{Z}) = \mathrm{Tor}\, H^{6-p}(\mathbf{X}_6, \mathbb{Z}) = \mathrm{Tor}\, H_{5-p}(\mathbf{X}_6, \mathbb{Z})$$

one should make sure that the corresponding axion mass is well below the compactification scale (e.g., using warping)

Franco, Galloni, Retolaza, Uranga '14

Other string examples

- Axions also obtain a mass with background fluxes
- Simplest example: $\phi = C_0$ in the presence of NSNS flux H_3

$$W = \int_{\mathbf{X}_6} (F_3 - \tau H_3) \wedge \Omega \qquad \tau = C_0 + i/g_s$$

- We also recover the axion-four-form potential

$$\int_{M^{1,3}\times\mathbf{X}_6} C_0 H_3 \wedge F_7 = \int_{M^{1,3}} C_0 F_4 \qquad F_4 = \int_{\mathrm{PD}[H_3]} F_7$$

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- M-theory version: *Beasley, Witten '02*
- A rich set of superpotentials obtained with type IIA fluxes

$$\int_{\mathbf{X}_6} e^{J_c} \wedge (F_0 + F_2 + F_4) \qquad J_c = J + iB$$

potentials higher than quadratic

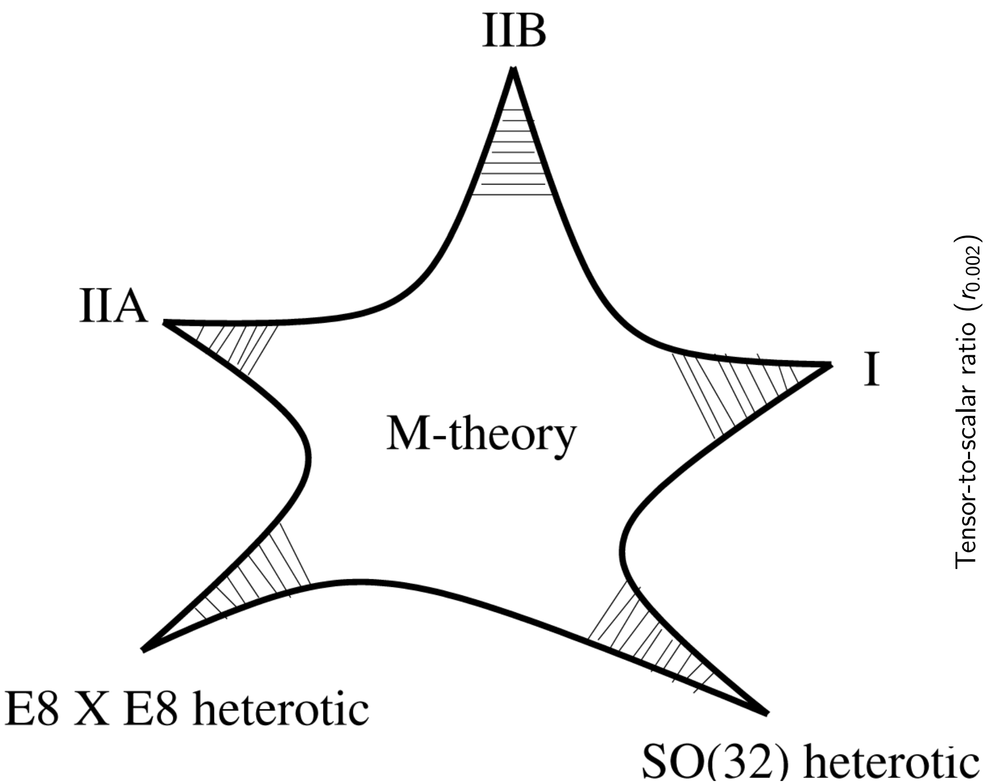
- Massive axions detected by torsion groups in K-theory

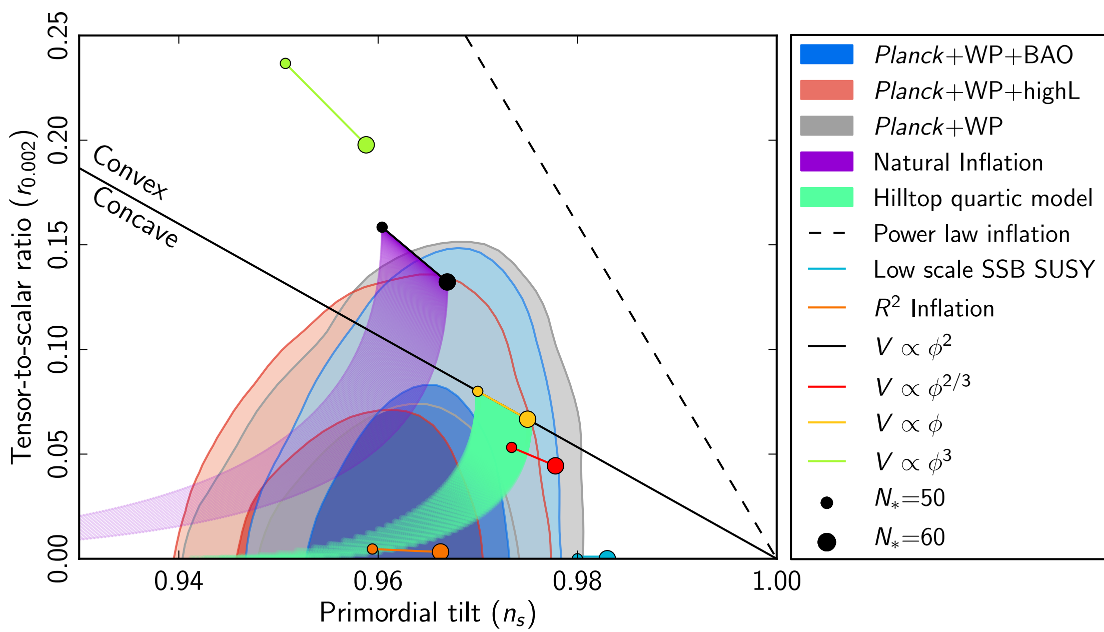
Conclusions

- Axion monodromy is an elegant idea that combines chaotic and natural inflation, aiming to prevent disastrous UV corrections to the inflaton potential.
- We have discussed its ***concrete*** implementation in a ***new*** framework, dubbed F-term axion monodromy inflation compatible with spontaneous supersymmetry breaking.
- In a simple set of models the inflaton is a massive Wilson line. They show the mild UV corrections for large inflaton vev.
- Effective action reproduces the axion-four-form action proposed by Kaloper and Sorbo. Discrete symmetries classified by K-theory torsion groups.
- α' corrections to EFT [See Junghans's talk and references therein] are important for inflation and moduli stabilization.

Conclusions

- A broad class of large field inflationary scenarios that can be implemented in any limit of string theory w/ rich pheno:





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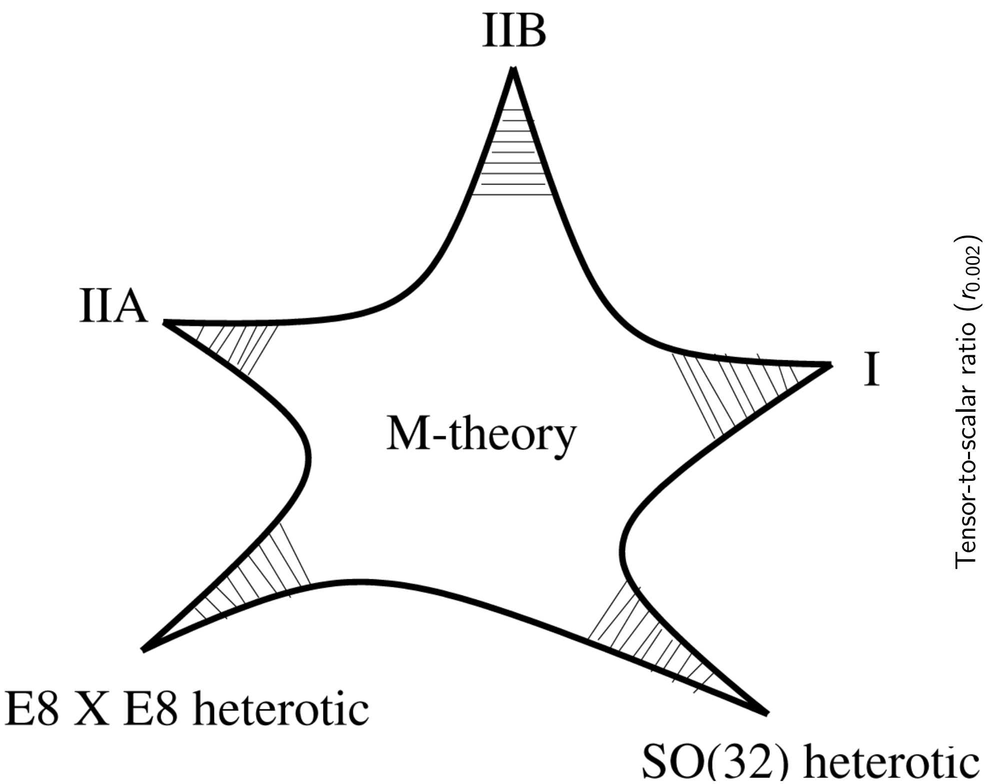

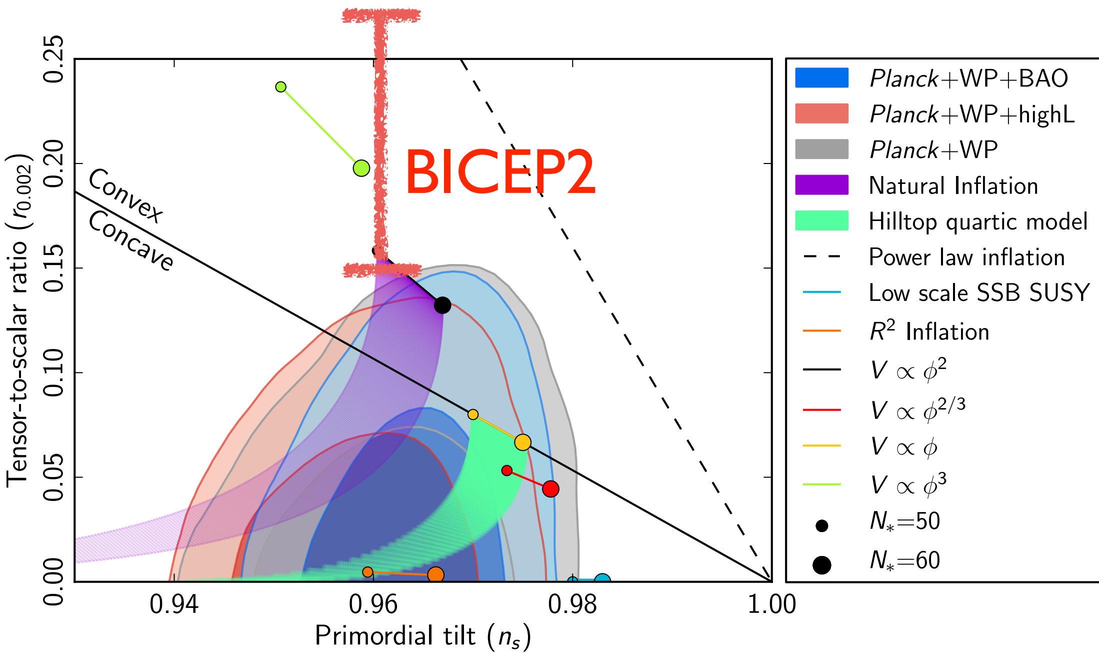

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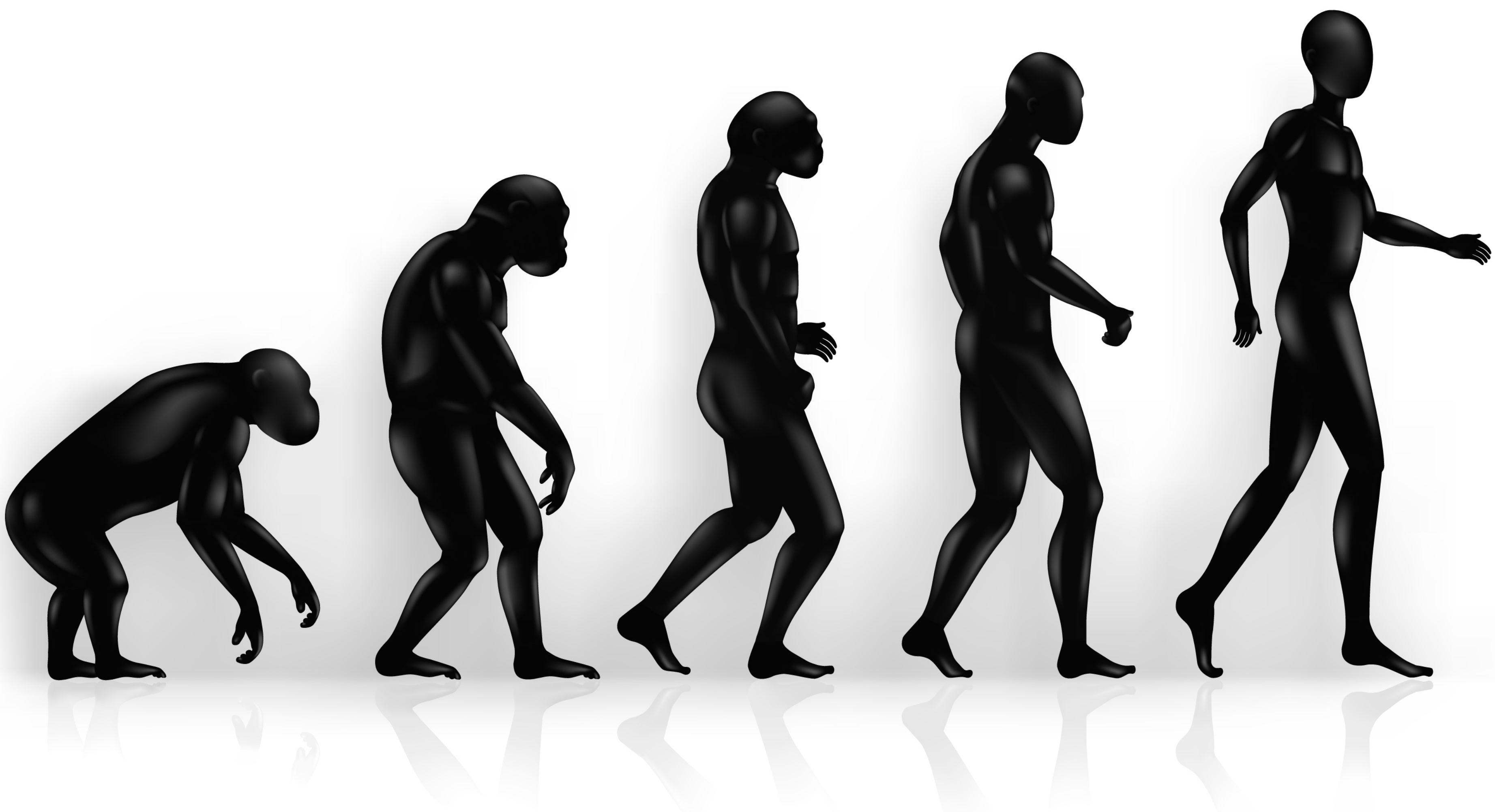
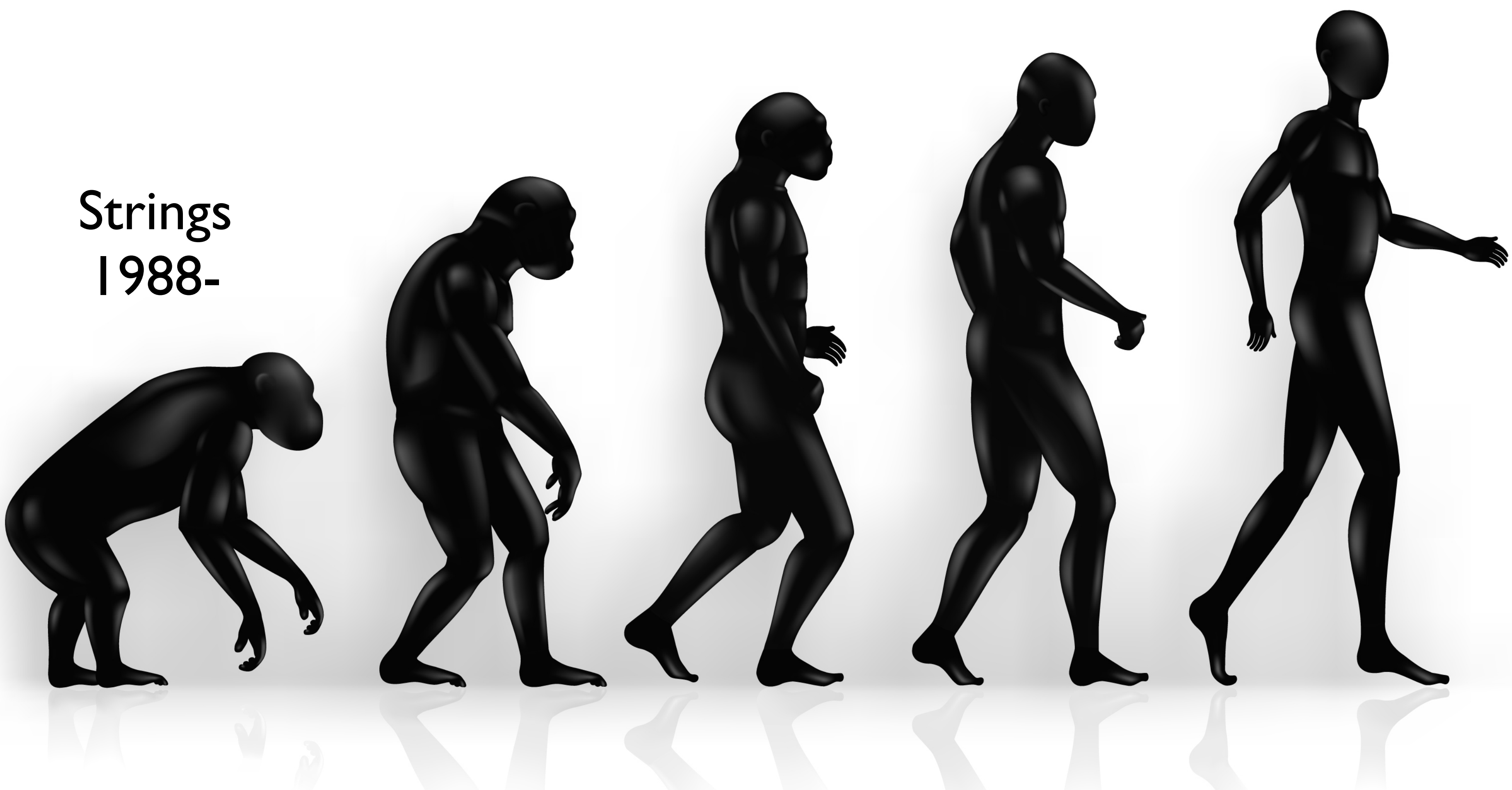
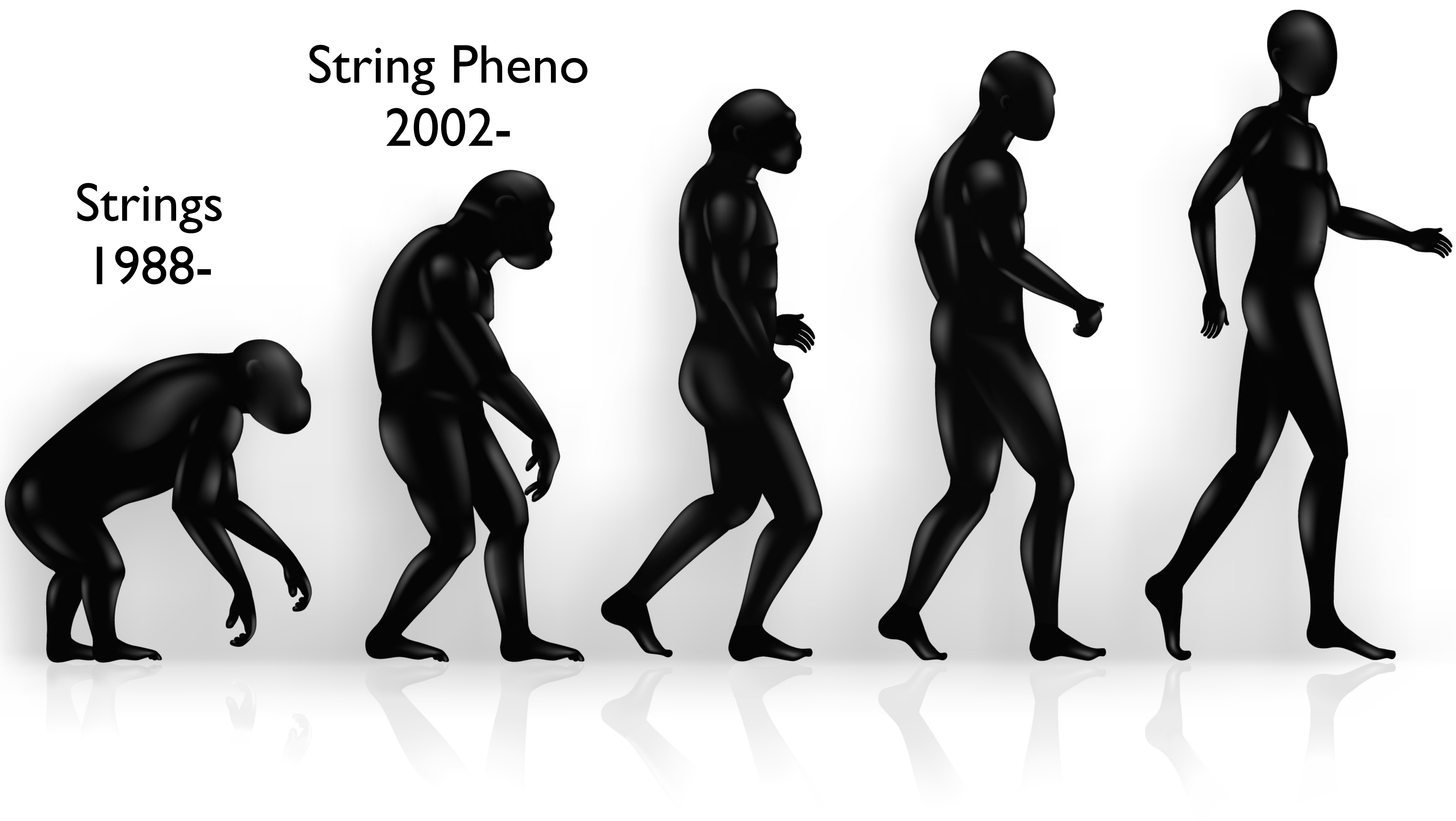
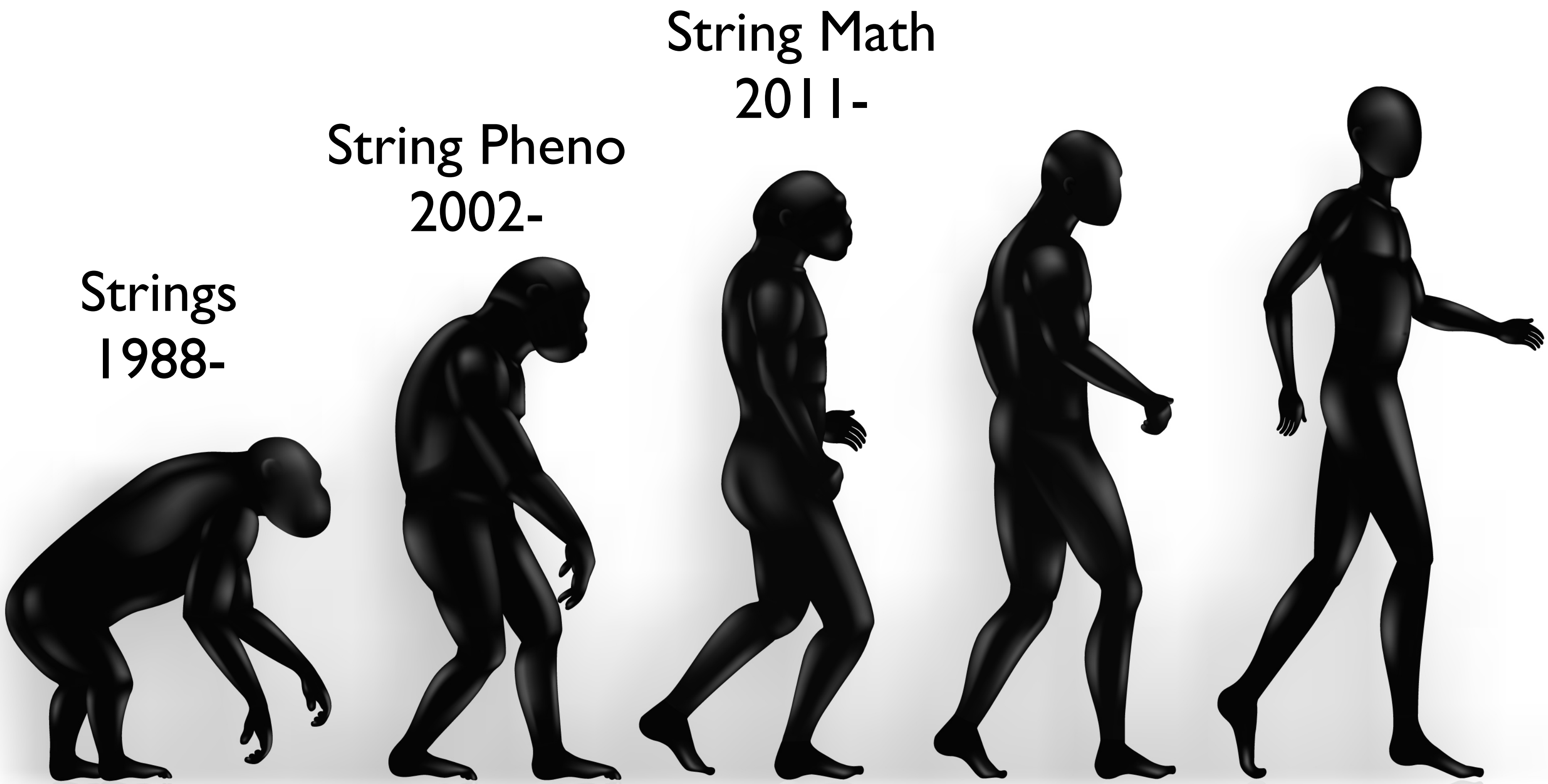


Strings
1988-




String Pheno
2002-
Strings
1988-

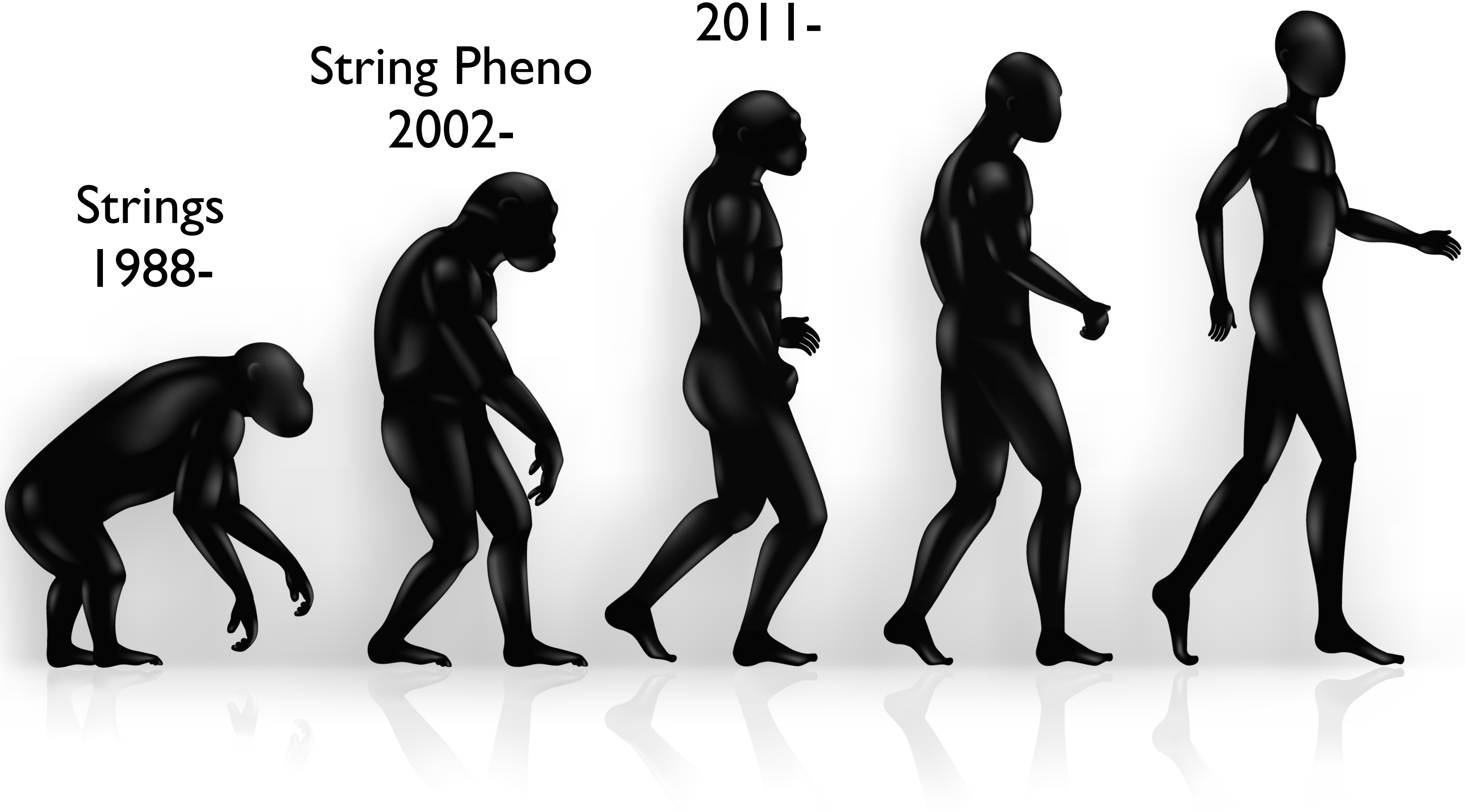

Strings
1988-
String Pheno
2002-
String Math
2011-

String Cosmo
2015-

String Math
2011-

String Pheno
2002-

Strings
1988-



Gordon Research Conferences

String Theory & Cosmology

New Ideas Meet New Experimental Data

May 31 - June 5, 2015
The Hong Kong University of Science and Technology
Hong Kong, China

Chair:
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Vice Chair:
Ulf Danielsson

Application Deadline

Applications for this meeting must be submitted by **May 3, 2015**. Please apply early, as some meetings become oversubscribed (full) before this deadline. If the meeting is oversubscribed, it will be stated here. *Note*: Applications for oversubscribed meetings will only be considered by the Conference Chair if more seats become available due to cancellations.

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