Mordell-Weil meets Standard Model

- arXiv: 1406.6071 with L. **Lin**
- arXiv: 1303.5054 and 1307.2902
  with J. **Borchmann**, E. **Palti** and C. **Mayrhofer**
- arXiv: 1405.3656 with O. **Till**, C. **Mayrhofer**, D. **Morrison**
- arXiv: 1402.5144 with M. **Bies**, C. **Mayrhofer** and C. **Pehle**

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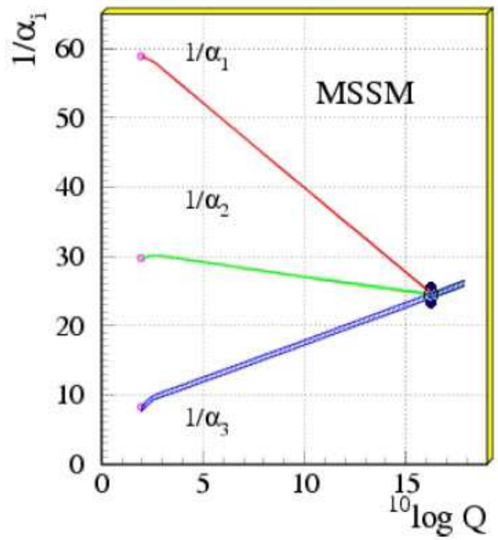
Motivation

Why are we interested in GUTs?

1) As a particle phenomenologist:

- Gauge coupling unification
  (especially in TeV scale SUSY context)
- Economy of spectrum:
  $\mathbf{10} \rightarrow (\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, 1) + (1, 1)$
  $\bar{\mathbf{5}} \rightarrow (\bar{\mathbf{3}}, 1) + (1, \mathbf{2})$
- ....



pic: Kazakov, 0012288

2) As a string phenomenologist:

- All of the above
- GUTs follow so beautifully from string theory
  (originally the heterotic string)
  $\Longrightarrow$ efficient path to Standard Model from string theory

Motivation

Why are we interested in F-theory?

1) As a particle phenomenologist:

- Because they also give rise to beautiful GUTs

[Beasley,Heckman,Vafa; Donagi,Wijnholt'08]

2) As a string phenomenologist:

- All of the above
- Because F-theory is a universal approach to 7-brane compactifications.

As such F-theory is not tied exclusively to GUT idea.

Alternative approach to particle phenomenology based on
direct implementation of $SU(3) \times SU(2) \times U(1)_Y$:

- The traditional approach in perturbative intersecting branes (IIA/B)
  Antoniadis,Blumenhagen,Cvetič,Honecker,Ibanez,Lüst,Marchesano,Shiu,Staessens, Uranga,...
- branes at singularities Aldazabal,Ibanez,Quevedo,Uranga,Krippendorf,...

Motivation

GUTs work particularly well with TeV scale SUSY, but a priori
SUSY <u>might</u> be broken at intermediate scales:

- eventually an experimental question
- motivation from 126 GeV Higgs within string theory

  [Hebecker,Knochel,TW'12/3][Ibanez,Marchesano,Regalado,Valenzuela][Ibanez,Valenzuela]'13
- or from axions [Chatzistavrakidis,Erfani,Nilles,Zavala]'12

Direct approach to SM particularly attractive for high scale SUSY:

- On-the-nose 'precision' gauge coupling unification is lost
  $\Longrightarrow$ accept it as an 'accident'?
  (especially if high corrections from fluxes are needed to achieve it)

- Interesting relation of high scale SUSY in GUTs and proton decay constraints
  [Ibanez,Marchesano,Regalado,Valenzuela'12][Hebecker,Unwin'14]; Hebecker's talk

Revisit non-GUT approach to SM with open mind towards ~~SUSY~~

Motivation

Direct SM realizations in F-theory extend the set of perturbative vacua

U(1) selection rules can differ in F-theory compared to perturbative IIB regime $\leftrightarrow$ crucial for phenomenology of couplings

Example:

- perturbative Intersecting Branes with $U(3)_a \times U(2)_b \times U(1)_c \times \ldots$
- Suppose that $U(1)_Y$ is compatible with
  $Q = (\mathbf{3}, \mathbf{2})_{1_a, -1_b}, \quad H^u = (\mathbf{1}, \mathbf{2})_{1_b, -1_c}, \quad u^c_R = (\overline{\mathbf{3}}, \mathbf{1})_{-2_a, 0}$
  (antisymmetric)
- $\Longrightarrow$ ~~$Q H^u u^c_R$~~ is specific property of 'diagonal U(1)' in Type IIB

F-theory includes more generic configurations without a 'diagonal U(1)'
Example: $\mathbf{10}\,\mathbf{10}\,\mathbf{5}$ in SU(5) GUTs

$\longrightarrow$ range of configurations is larger due to non-pert. branes:
more possibilities can exist in F-theory

Motivation

Study possibilities of direct SM in brane theories, i.e. in F-theory:

- Minimal requirement: elliptic fibration with $SU(3) \times SU(2) \times U(1)_Y$
- Extra U(1) symmetries welcome to add more structure for detailed phenomenology, e.g.
  - as family symmetry
  - as selection rules for unwanted couplings

In F-GUT context e.g. [Dudas,Palti'09][Marsano,Saulina,S.-Nameki'09], . . . ,
[Krippendorf,Mayorga,Oehlmann,Ruehle'14],. . .

Starting point:

- Elliptic fibrations with Mordell-Weil group of rank 2 $\leftrightarrow U(1) \times U(1)$
- extra $SU(3) \times SU(2)$ on two independent divisors

see however [Choi,Kobayahi'10][Choi'13]. . .

Outline

I.) Motivation

II.) Brief recap on Mordell-Weil group in F-theory

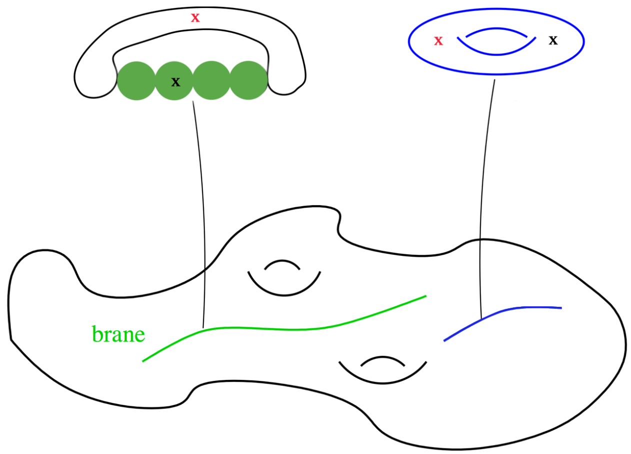
III.) $SU(3) \times SU(2) \times U(1) \times U(1)$ - fibrations

IV.) Preliminary Phenomenology

Mordell-Weil group

Elliptic fibration: $\pi : Y_4 \to B_3$ described as Weierstrass model:

$$y^2 = x^3 + f(b)\, xz^4 + g(b)\, z^6, \qquad [x : y : z] \in \mathbb{P}^2_{2,3,1} \qquad b \in B_3$$



Rational section σ:

- $\mathcal{B} \ni b \mapsto \sigma(b) = [x(b) : y(b) : z(b)]$
- $\sigma(b)$ is a rational point in fiber

Mordell-Weil group $E(K)$= group of rational sections

- zero element: $\sigma_0(b) = [1 : 1 : 0]$
- group law: fiberwise addition of points

Mordell-Weil theorem:

$$E(K) \simeq \mathbb{Z}^r \oplus \mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}, \qquad r = \text{rank of } E(K)$$

Birch & Swinnerton-Dyer conjecture $\in$ {Clay Institute Millennium problems}

Mordell-Weil group

Mordell-Weil group $E(K)$= group of rational sections

$$E(K) = \underbrace{\mathbb{Z}^r}_{\text{free part}} \oplus \underbrace{\mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}}_{\text{torsion part}}$$

Physical significance:

- Free part $\leftrightarrow$ $U(1)$ gauge symmetries [Morrison,Vafa'96],[Klemm,Mayr,Vafa'98],...
- Torsion part $\leftrightarrow$ Global structure of non-ab. gauge groups $(\pi_1(G))$

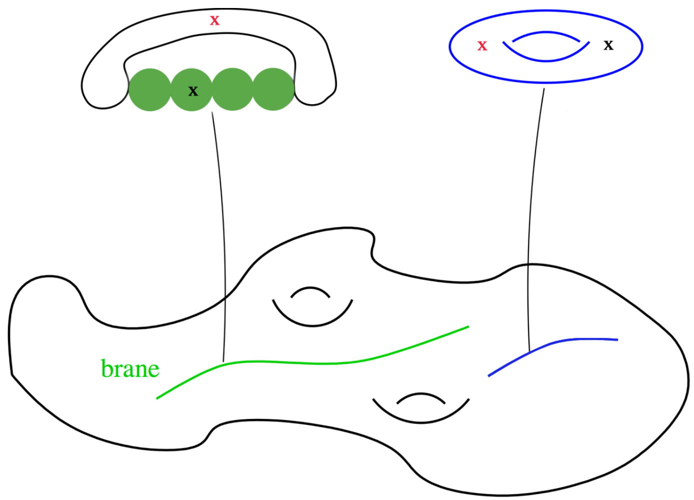
  [Aspinwall,Morrison'98], [Aspinwall,Katz,Morrison'00], ..., [Mayrhofer,Till,Morrison,TW'14]

Systematic recent study of U(1)s via rational sections:

✓ extra selection rules, e.g. crucial in F-theory GUTs
✓ window to gauge fluxes and chirality
✓ general interest in any theory with massless U(1)s (landscape studies, ...)

Antoniadis,Anderson,Bizet,Borchmann,Braun,Braun,Choi,Collinucci,Cvetič,Etxebarria,Grassi,Grimm, Hayashi, Keitel,Klevers,Küntzler,Krippendorf,Oehlmann,Klemm,Leontaris,Lopes,Mayrhofer, Mayorga, Morrison, Park,Palti,Piragua,Rühle,S-Nameki,Song,Valandro,Taylor,TW,...

Mordell-Weil group



Divisors on elliptic 4-fold $\hat{Y}_4 \to \mathcal{B}$:

- zero-section Z
- pullback from base $\pi^{-1}(D_\mathrm{b})$
- resolution divisors F_m, $m = 1, \ldots, \mathrm{rk}(G)$
- **rational sections** S_i

Shioda map

- homomorphism $\varphi : \underbrace{E(K)}_{\text{group of sections}} \to \underbrace{NS(\hat{Y}_4) \otimes \mathbb{Q}}_{\text{group of divisors}}$ [Shioda'89]

$$\varphi(S - Z) = S - Z - \pi^{-1}(\delta) + \sum l_i F_i, \qquad l_i \in \mathbb{Q}$$

- transversality

$$\int_{\hat{Y}_4} [\varphi(S - Z)] \wedge [X] \wedge [\pi^{-1}\omega_4] = 0 \qquad X \in \{Z, F_l, \pi^{-1}(D_\mathrm{b})\}$$

Non-torsional sections

- $[\mathcal{S}] \equiv [\varphi(S - Z)]$ is non-trivial in $H^{1,1}(\hat{Y}_4)$
- $[\mathcal{S}]$ serves as the generator of $U(1)$ gauge group

$$C_3 = A \wedge [\mathcal{S}] \qquad A\text{: } U(1) \text{ gauge potential}$$

Engineering extra sections via restriction of complex structure of fibration

- Non-generic Weierstrass models, simplest example: [Grimm,TW'10]

  $y^2 - a_1 x\, y\, z - a_3\, y\, z^3 = x^3 + a_2\, x^2\, z^2 + a_4\, x\, z^4 + \cancel{a_6\, z^6}, \quad [x : y : z] \in \mathbb{P}_{2,3,1}$

  zero-section: $[1 : 1 : 0]$ extra section: $[0 : 0 : 1]$

  Generalisations in [Mayrhofer,Palti,TW'12]

- Systematic approach via different fiber representations
  - rank 1: $\mathbb{P}_{1,1,2}[4]$ [Morrison,Park'12]
  - rank 2: $\mathbb{P}^2[3]$ [Borchmann,Mayrhofer,Palti,TW][Cvetič,(Grassi),Klevers,Piragua]'13
  - rank 3: complete intersections [Cvetič,Klevers,Piragua,Song'13]
  - toric hypersurfaces [Braun,Grimm,Keitel'13][Grassi,Perduca'12]

Fibrations with MW torsion

Hypersurface fibrations with MW torsion

- $\mathbb{Z}_k$ for $k = 2, 3, 4, 5, 6$, $\mathbb{Z}_2 \oplus \mathbb{Z}_n$ with $n = 2, 4$ and $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ [Morrison,Aspinwall'98]
- Toric models: $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z} \oplus \mathbb{Z}_2$ [Braun,Grimm,Keitel'13]
- Previous analysis from perspective of $SL(2, \mathbb{Z})$-monodromies:
  [Berglund,Klemm,Mayr,Theisen'98]
- General structure + exemplification for toric models [Mayrhofer,Morrison,Till,TW'14]

General pattern

cf. talk by Oskar Till

- Codimension-one fibers:
  $\mathbb{Z}_k$-torsion automatically produces non-ab. gauge group factor $G_0/\mathbb{Z}_k$
- Codimension-two fibers:
  matter spectrum greatly restricted due to reduced center of G
- Codimension-three fibers:
  no further selection rules at the level of Yukawa couplings

$U(1) \times U(1)$ fibrations - details

[Borchmann,Mayrhofer,Palti,TW][Cvetič,(Grassi),Klevers,Piragua]'13 cf talk by Mirjam Cvetič

Step 1: Elliptic curve with 3 rational points $\leftrightarrow$ non-generic cubic in $\mathbb{P}^2$

- $P_T = \mathrm{v}\,\mathrm{w}(c_1\,\mathrm{w} + c_2\,\mathrm{v}) + \mathrm{u}\,(b_0\,\mathrm{v}^2 + b_1\,\mathrm{v}\,\mathrm{w} + b_2\,\mathrm{w}^2) + \mathrm{u}^2(d_0\,\mathrm{v} + d_1\,\mathrm{w} + d_2\,\mathrm{u})$
- $\mathrm{Sec}_0 : [\mathrm{u} : \mathrm{v} : \mathrm{w}] = [0 : 0 : \mathrm{w}]$, $\mathrm{Sec}_1 : [\mathrm{u} : \mathrm{v} : \mathrm{w}] = [0 : \mathrm{v} : 0]$,
  $\mathrm{Sec}_2 : [\mathrm{u} : \mathrm{v} : \mathrm{w}] = [0 : -c_1 : c_2]$

Step 2: Resolve singularities over curves

- $\mathrm{u} = \mathrm{v} = c_1 = b_2 = 0 \quad \Longrightarrow \quad \mathrm{u} \to \mathrm{u}\,s_0, \quad \mathrm{v} \to \mathrm{v}\,s_0$
- $\mathrm{u} = \mathrm{w} = c_2 = b_0 = 0 \quad \Longrightarrow \quad \mathrm{u} \to \mathrm{u}\,s_1, \quad \mathrm{w} \to \mathrm{w}\,s_1$

$$\begin{aligned} 0 \quad = \quad & \mathrm{v}\,\mathrm{w}(c_1\,\mathrm{w}\,s_1 + c_2\,\mathrm{v}\,s_0) + \mathrm{u}\,(b_0\,\mathrm{v}^2\,{s_0}^2 + b_1\,\mathrm{v}\,\mathrm{w}\,s_0\,s_1 + b_2\,\mathrm{w}^2\,{s_1}^2) + \\ & + \mathrm{u}^2(d_0\,\mathrm{v}\,{s_0}^2\,s_1 + d_1\,\mathrm{w}\,s_0\,{s_1}^2 + d_2\,\mathrm{u}\,{s_0}^2\,{s_1}^2) \end{aligned}$$

Step 3: 3 independent rational sections

$S_0 : \{s_0 = 0\}$, $S_1 : \{s_1 = 0\}$, and $S_2 : \{\mathrm{u} = 0\}$

$SU(3) \times SU(2) \times U(1) \times U(1)$

$$0 = \mathrm{v}\,\mathrm{w}(c_1\,\mathrm{w}\,s_1 + c_2\,\mathrm{v}\,s_0) + \mathrm{u}\,(b_0\,\mathrm{v}^2\,{s_0}^2 + b_1\,\mathrm{v}\,\mathrm{w}\,s_0\,s_1 + b_2\,\mathrm{w}^2\,{s_1}^2) + $$
$$+\mathrm{u}^2(d_0\,\mathrm{v}\,{s_0}^2\,s_1 + d_1\,\mathrm{w}\,s_0\,{s_1}^2 + d_2\,\mathrm{u}\,{s_0}^2\,{s_1}^2)$$

Combine with non-abelian gauge group: [Lin,TW'14]

$SU(2) \leftrightarrow W_2 = \{w_2 = 0\}$ $SU(3) \leftrightarrow W_3 = \{w_3 = 0\}$

'toric enhancements':

$$g_m = g_{m,k,l}\, w_2^k\, w_3^l \qquad g_m \in \{b_i, c_j, d_k\}$$

Note: This is not the complete list - 'non-generic' enhancements by relations possible

for single U(1): [Mayrhofer,Palti,TW'12][Küntzler,Schäfer-Nameki'14] cf. S.-Nameki's talk

Classified by toric tops [Bouchard,Skarke'03]:

for SU(2) and SU(3) each: 2 types of toric enhancements

=2 types of vanishing orders

Combination gives 4 inequivalent toric $SU(3) \times SU(2) \times U(1) \times U(1)$ fibrations over generic basis

$SU(3) \times SU(2) \times U(1) \times U(1)$

Example: Combination $I \times A$

$$b_0 = b_{0;1,1}\, w_2\, w_3, \quad c_2 = c_{2;0,1}\, w_3,$$

$$d_0 = d_{0;1,1} w_2\, w_3, \quad d_2 = d_{2;1,1}\, w_2\, w_3$$

Resolution of fibration requires:

E_1: Resolution divisor of SU(2) F_1, F_2: Resolution divisors of SU(3)

Shioda map for $U(1)_1 \times U(1)_2$

$$\omega_1 = S_1 - S_0 - \overline{\mathcal{K}} + \frac{1}{2}E_1 + \frac{2}{3}F_1 + \frac{1}{3}F_2,$$

$$\omega_2 = U - S_0 - \overline{\mathcal{K}} - [c_{1;0,0}] + \frac{2}{3}F_1 + \frac{1}{3}F_2$$

$$C_3 = A_1 \wedge \omega_1 + A_2 \wedge \omega_2 + \ldots$$

Matter and Yukawas

see talk by Ling Lin for more details

1) Classification of matter curves in fully resolved fibration :

- at $W_3 \cap W_2$: $(\mathbf{3},\mathbf{2})_{q_1,q_2}$ - only 1 such curve!
- $3 \times (\mathbf{1},\mathbf{2})_{q_1,q_2}$ $5 \times (\overline{\mathbf{3}},\mathbf{1})_{q_1,q_2}$ $6 \times (1,1)_{q_1,q_2}$

2) Classification of geometrically realized Yukawas:

All couplings allowed by gauge symmetries are realized:

- $\mathbf{1}-\mathbf{1}-\mathbf{1}$: affine SU(3) $\mathbf{2}-\mathbf{2}-1$: affine SU(4)
- $\mathbf{3}-\overline{\mathbf{3}}-1$: affine SU(5) $\mathbf{3}-\mathbf{3}-\mathbf{3}$: affine SO(8)
- $(\mathbf{3},\mathbf{2})-\overline{\mathbf{3}}-\mathbf{2}$: affine SU(6): no exceptional symmetry needed for top mass
- $(\mathbf{3},\mathbf{2})-(\mathbf{3},\mathbf{2})-\mathbf{3}$ non-standard fiber!

(N)MSSM identifications

Identification of (N)MSSM representations:

1. Only one type of $(\mathbf{3},\mathbf{2})_{(q_1,q_2)}$ → all families of Q_L on single curve
2. 3 types of $(1,\mathbf{2})_{(q_1,q_2)}$
   - choose H_u and H_d $\Longrightarrow$ fixes $U(1)_Y = a\,U(1)_1 + b\,U(1)_2$
   - list possible identifications for L compatible with $U(1)_Y$-charge
   - Oftentimes several allowed identifications, each distinguished by $U(1)_\perp$ charge ↔ family symmetry
3. 5 types of $\bar{\mathbf{3}}_{(q_1,q_2)}$ → identify candidates for u_R^c and d_R^c
4. 6 types of singlets $\mathbf{1}_{(q_1,q_2)}$ → identify candidates for ν_R^c and e_R^c

Exclusion principle:

If no identification for e_R^c or u_R^c or d_R^c can be made, then discard this particular choice of $U(1)_Y$

Coupling analysis

Dimension-four couplings in NMSSM:

$$\begin{aligned} W &= W_1 + W_2 + W_{\rm singlet}\,, \\ W_1 &= Y_u\, Q\, H_u\, u_R^c + Y_d\, Q\, H_d\, d_R^c + Y_e\, L\, H_d\, e_R^c + Y_\nu\, L\, H_u\, \nu_R^c + \mu\, H_u\, H_d \end{aligned}$$

W_1: Yukawa couplings

- If a specific family enjoys such a coupling, it is 'heavy'.
  Caveat: In presence of only a single intersection point, coupling rank is 1
  $\rightarrow$ non-pert. effects needed see talk by Zoccarato
  [Marchesano,Martucci'10],[Font,Marchesano,Regalado,Zoccarato'13]
- If coupling is absent, family is 'light', and need
  - either charged instanton effects (model dependent)
  - or higher operator couplings to singlets via Froggat-Nielsen

  $\rightarrow$ future work (including analysis of unwanted extra couplings along the ride)

  Type II quiver analysis: [Richter,Ibanez'08][Cvetič,Halverson,Richter'09]
  [Cvetič,Halverson,Langacker'10],[Anastasopoulos et al.'12]
- μ-term: if forbidden by $U(1)_\perp$ potentially generated by extra singlet $1_\mu H_u H_d$ if $\langle 1_\mu \rangle \neq 0$ $\rightarrow$ list possible 1_μ

Standard Model identifications

$$\begin{aligned} W &= W_1 + W_2 + W_{\text{singlet}}, \\ W_2 &= \alpha\, Q\, L\, d^c_R + \beta\, u^c_R\, d^c_R\, d^c_R + \gamma\, L\, L\, e^c_R + \kappa\, L\, H_u, \\ W_{\text{singlet}} &= \delta_{3,0}\, \mathbf{1}_\mu\, \mathbf{1}_\mu\, \mathbf{1}_\mu + \delta_{2,1}\, \mathbf{1}_\mu\, \mathbf{1}_\mu\, \nu^c_R + \delta_{1,2}\, \mathbf{1}_\mu\, \nu^c_R\, \nu^c_R + \delta_{0,3}\, \nu^c_R\, \nu^c_R\, \nu^c_R, \end{aligned}$$

W_2: R-partiy violating $R = (-1)^{2S+3(B-L)}$

We list all non-zero couplings in specific model

- proton decay only if $\alpha \times \beta \neq 0$
- κ requires H_u and L on same curve (depends on spectrum) or via singlets

Dimension-five couplings:

$$\begin{aligned} W_3 = \lambda_1\, Q\, Q\, Q\, L + \lambda_2\, u^c_R\, u^c_R\, d^c_R\, e^c_R + \lambda_3\, Q\, Q\, Q\, H_d + \lambda_4\, Q\, u^c_R\, e^c_R\, H_d \\ + \lambda_5\, L\, L\, H_u\, H_u + \lambda_6\, L\, H_d\, H_u\, H_u, \\ K \supset \lambda_7\, u^c_R\, (d^c_R)^*\, e^c_R + \lambda_8\, H^*_u\, H_d\, e^c_R + \lambda_9\, Q\, u^c_R\, L^* + \lambda_{10}\, Q\, Q\, (d^c_R)^*. \end{aligned}$$

We list all couplings on basis of U(1) charges (necessary for existence)

Standard Model identifications

Have not discarded any model on basis of dim-4 or dim-5 couplings

- Stringent constraints due to proton stability apply in context of low-scale SUSY
- Detailed analysis for intermediate or high-scale models more involved and dependent on ~~SUSY~~ scale

Examples exist where $U(1)_\perp$ forbids all dim-4 and important dim-5 operators $\lambda_1\, Q\, Q\, Q\, L + \lambda_2\, u^c_R\, u^c_R\, d^c_R\, e^c_R$

Model No 5:

- $U(1)_Y = U(1)_1, \qquad U(1)_\perp = U(1)_2$
- all families on one curve and 'heavy'

$$Q = (\mathbf{3},\mathbf{2})_{\frac{1}{6},-\frac{1}{3}} \qquad H_u = (1,\mathbf{2})_{\frac{1}{2},1} \qquad H_d = (1,\bar{\mathbf{2}})_{-\frac{1}{2},1} \qquad L = (1,\bar{\mathbf{2}})_{-\frac{1}{2},0}$$
$$u^c_R = (\bar{\mathbf{3}},1)_{-\frac{2}{3},-\frac{2}{3}} \qquad d^c_R = (\bar{\mathbf{3}},1)_{\frac{1}{3},-\frac{2}{3}} \qquad e^c_R = (1,1)_{1,-1} \qquad \nu^c_R = (1,1)_{0,-1}$$

Different assignment of families can realise 'family' symmetries

Standard Model identifications

[Lin,TW'14]

Matter spectrum	*Baryon- and Lepton number violation*
possibility no. 5 $a=1, b=0$; $(H_u, H_d) = (\mathbf{2}^{\mathrm{I}}_2, \overline{\mathbf{2}}^{\mathrm{I}}_1)$ heavy (u^c_R, d^c_R): $(\overline{\mathbf{3}}^{\mathrm{A}}_4, \overline{\mathbf{3}}^{\mathrm{A}}_3)$ light u^c_R: $\overline{\mathbf{3}}^{\mathrm{A}}_1$; light d^c_R: $\overline{\mathbf{3}}^{\mathrm{A}}_2$, $\overline{\mathbf{3}}^{\mathrm{A}}_5$ heavy generations of (L, ν^c_R, e^c_R): $(\overline{\mathbf{2}}^{\mathrm{I}}_1, \overline{\mathbf{1}}^{(5)}, -)$, $(\overline{\mathbf{2}}^{\mathrm{I}}_2, -, \mathbf{1}^{(2)})$, $(\overline{\mathbf{2}}^{\mathrm{I}}_3, \overline{\mathbf{1}}^{(6)}, \mathbf{1}^{(1)})$ light ν^c_R: $\mathbf{1}^{(5)}$, $\mathbf{1}^{(6)}$; light e^c_R: $\mathbf{1}^{(3)}$, $\mathbf{1}^{(4)}$ $\mathbf{1}_\mu$: $\overline{\mathbf{1}}^{(5)}$	α: $(\overline{\mathbf{2}}^{\mathrm{I}}_1, \overline{\mathbf{3}}^{\mathrm{A}}_3)$, $(\overline{\mathbf{2}}^{\mathrm{I}}_2, \overline{\mathbf{3}}^{\mathrm{A}}_2)$, $(\overline{\mathbf{2}}^{\mathrm{I}}_3, \overline{\mathbf{3}}^{\mathrm{A}}_5)$; β: $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_3\overline{\mathbf{3}}^{\mathrm{A}}_2$, $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_5\overline{\mathbf{3}}^{\mathrm{A}}_5$, $\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_3\overline{\mathbf{3}}^{\mathrm{A}}_5$; δ: $\overline{\mathbf{1}}^{(5)}\mathbf{1}^{(6)}\mathbf{1}^{(6)}$, $\mathbf{1}^{(5)}\overline{\mathbf{1}}^{(6)}\overline{\mathbf{1}}^{(6)}$; γ: $\overline{\mathbf{2}}^{\mathrm{I}}_1\overline{\mathbf{2}}^{\mathrm{I}}_2\mathbf{1}^{(2)}$, $\overline{\mathbf{2}}^{\mathrm{I}}_1\overline{\mathbf{2}}^{\mathrm{I}}_3\mathbf{1}^{(1)}$, $\overline{\mathbf{2}}^{\mathrm{I}}_2\overline{\mathbf{2}}^{\mathrm{I}}_2\mathbf{1}^{(3)}$, $\overline{\mathbf{2}}^{\mathrm{I}}_2\overline{\mathbf{2}}^{\mathrm{I}}_3\mathbf{1}^{(4)}$, $\overline{\mathbf{2}}^{\mathrm{I}}_3\overline{\mathbf{2}}^{\mathrm{I}}_3\mathbf{1}^{(2)}$; λ_1: $\mathbf{2}^{\mathrm{I}}_1$; λ_3: $\checkmark$; λ_6: $-$; λ_8: $\mathbf{1}^{(2)}$; λ_{10}: $(\overline{\mathbf{3}}^{\mathrm{A}}_3)^*$; λ_4: $(\overline{\mathbf{3}}^{\mathrm{A}}_4, \mathbf{1}^{(2)})$, $(\overline{\mathbf{3}}^{\mathrm{A}}_1, \mathbf{1}^{(1)})$; λ_5: $(\overline{\mathbf{2}}^{\mathrm{I}}_2, \overline{\mathbf{2}}^{\mathrm{I}}_2)$; λ_9: $(\overline{\mathbf{3}}^{\mathrm{A}}_4, (\overline{\mathbf{2}}^{\mathrm{I}}_2)^*)$, $(\overline{\mathbf{3}}^{\mathrm{A}}_1, (\overline{\mathbf{2}}^{\mathrm{I}}_3)^*)$; λ_7: $\overline{\mathbf{3}}^{\mathrm{A}}_4(\overline{\mathbf{3}}^{\mathrm{A}}_3)^*\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4(\overline{\mathbf{3}}^{\mathrm{A}}_2)^*\mathbf{1}^{(3)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4(\overline{\mathbf{3}}^{\mathrm{A}}_5)^*\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_1(\overline{\mathbf{3}}^{\mathrm{A}}_3)^*\mathbf{1}^{(1)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_1(\overline{\mathbf{3}}^{\mathrm{A}}_2)^*\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_1(\overline{\mathbf{3}}^{\mathrm{A}}_5)^*\mathbf{1}^{(2)}$; λ_2: $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_3\mathbf{1}^{(3)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_2\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_5\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_3\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_2\mathbf{1}^{(1)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_4\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_5\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_3\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_1\overline{\mathbf{3}}^{\mathrm{A}}_5\mathbf{1}^{(1)}$

Red choice: no dim-4 ~~R-parity~~, no $Q\,Q\,Q\,L$, $u^c_R\,u^c_R\,d^c_R\,e^c_R$

Next steps include:

- Analysis of chiral spectrum via gauge fluxes (see later)
- Analyse distinctive non-pert. features

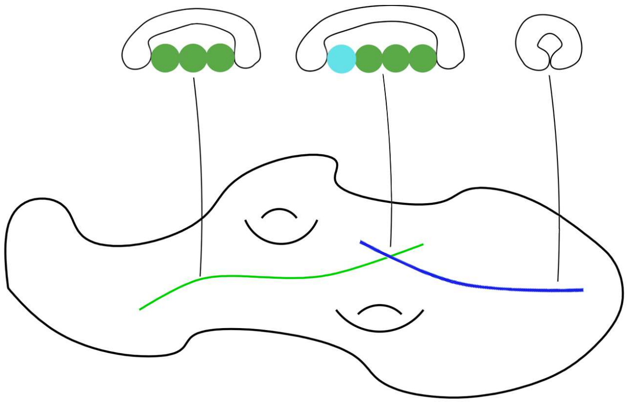
Matter in F-theory

of charged zero modes $\leftrightarrow$ background gauge field C_3 with $G_4 = dC_3$

- chiral index:

$$\nu_+ - \nu_- = \int_{\mathcal{C}_4} G_4$$

[Donagi,Wijnholt'09], [Braun,Collinucci,Valandro] [Marsano,S-Nameki], [Krause,Mayrhofer,TW], [Grimm,Hayashi]'11 ...



- What is the spectrum of states beyond the chiral index?

$\Longrightarrow$ need C_3 beyond its field strength [(Curio),Donagi'98], ...

$$0 \longrightarrow \underbrace{J^2(\hat{Y}_4)}_{\oint C_3 \text{ 'Wilson lines'}} \longrightarrow \underbrace{H^4_D(\hat{Y}_4, \mathbb{Z}(2))}_{\textbf{Deligne cohomology}} \xrightarrow{\hat{c}_2} \underbrace{H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)}_{\text{field strength } G_4} \longrightarrow 0$$

Framework for computation of non-chiral states: [Bies,Mayrhofer,Pehle,TW'14]

[cf. talk by Christoph Mayrhofer]

Fluxes

Constraint: Massless hypercharge $U(1)_Y = a\,U(1)_1 + b\,U(1)_2$:

$$\int_{\hat{Y}_4} G_4 \wedge \mathrm{w}_Y \wedge \pi^{-1} D \overset{!}{\equiv} 0 \qquad \forall D_a \in H^{1,1}(\mathcal{B}), \quad \text{where} \quad \omega_Y = a\,\omega_1 + b\,\omega_2$$

Types of fluxes:

- $U(1)$-flux: $G_4^{(i)} = \pi^{-1} F_i \wedge \omega_i, \qquad F_i \in H^{1,1}(\mathcal{B}), \qquad i = 1, 2$
- extra fluxes from matter surfaces: [Cvetič,Grassi,Klevers,Piragua],

  $G_4^{\gamma} = \pi^{-1}[b_2] \wedge \pi^{-1}[c_1] - [b_2 \cap c_1 \cap s_0]$ [Borchmann,Mayrhofer,Palti,TW]'13

  more fluxes may exist as independent fluxes - work in progress

Constraint from massless $U(1)_Y$: $G_4 = G_4^1 + G_4^2 + \alpha G_4^{\gamma}$

$$\int_{\mathcal{B}} \Big[\quad F_1 \wedge \Big(a(-2\overline{\mathcal{K}} + \tfrac{1}{2} W_2 + \tfrac{2}{3} W_3) + b(-2\overline{\mathcal{K}} - 2[c_1] + \tfrac{2}{3} W_3) \Big) - \alpha(a+2b)\,[b_2] \wedge [c_1] +$$
$$F_2 \wedge \Big(a(-2\overline{\mathcal{K}} - 2[c_1] + \tfrac{2}{3} W_3) + b(-\overline{\mathcal{K}} + [c_2] - [c_1] + \tfrac{2}{3} W_3) \Big) \Big] \wedge D_a \overset{!}{\equiv} 0$$

to be fulfilled in agreement with quantization!

Summary and Outlook

Overheard after a long Monday on F-theory U(1)s at String Pheno 2013:

'What are all these U(1)s good for?

Please stop producing more and turn them into something useful!'

We have taken this seriously and tried our best:

Mordell-Weil group of rational sections $\longleftrightarrow$ Standard Model fibrations for F-theory

Philosophy of pursuing this in F-theory:

Exploration of most generic class of brany SM constructions

So far: Classification of toric $SU(3) \times SU(2) \times U(1)_Y \times U(1)_\perp$ fibrations

- matter representations
- couplings
- identifications with NMSSM and coupling analysis

Summary and Outlook

Next steps:

1) Fluxes and spectrum:

- Which of these setups admit 3 chiral generations?
- Is $U(1)_{PQ}$ compatible with absence of exotics?
- How hard is it to get precisely the SM spectrum?

2) Gauge couplings and constraints on moduli

3) Type IIB limit and comparison with perturbative quivers

4) Detailed phenomenology with high/low-scale SUSY (couplings!)

5) ...

Appendix: Torsional sections

[Mayrhofer,Morrison,Till,TW'14]

- If R is $\mathbb{Z}_k$-torsional section: $k(R-Z)=0$
- Shioda map is homomorphism: $0=\varphi(k(R-Z))=k\varphi(R-Z)\in NS(\hat{Y}_4)\otimes\mathbb{Q}$
- Since $NS(\hat{Y}_4)\otimes\mathbb{Q}$ is torsion free: $\varphi(R-Z)=R-Z-\pi^{-1}(\delta)+\sum_i \frac{a_i}{k}F_i$ is trivial

$$\Xi_k \equiv R-Z-\pi^{-1}(\delta) = -\mathbf{\frac{1}{k}}\sum_i a_i F_i, \quad a_i\in\mathbb{Z} \quad \text{is \textbf{integer} divisor}$$

Relation to group theory:

- Lie algebra $\mathfrak{g} \leftrightarrow$ coroot lattice $Q^\vee = \langle F_i\rangle_{\mathbb{Z}}$ F_i: resol. divisors $\leftrightarrow$ codim.-1
- Representation content $\leftrightarrow$ weight lattice $\Lambda \leftrightarrow$ codimension-2 fibers
- Integer pairing $\Lambda^\vee\times\Lambda\to\mathbb{Z}$ gives weights coweight lattice $\Lambda^\vee\supseteq Q^\vee$
- If $\Lambda^\vee = Q^\vee = \langle F_i\rangle_{\mathbb{Z}} \to$ gauge group G_0: $\pi_1(G_0)\approx\frac{\Lambda^\vee}{Q^\vee}=\emptyset$
- Integer $\Xi_k = -\frac{1}{k}\sum_i a_i F_i \to$ **'k-fractional refinement' of $\Lambda^\vee$**
- dual weight lattice becomes coarser $\to$ **smaller representation content**

gauge group $G=G_0/\mathbb{Z}_k$ with $\pi_1(G)=\mathbb{Z}_k$