# Gauge Data from Chow Groups and Massless Matter in F-Theory

joint work with M. Bies, C. Pehle and T. Weigand: arXiv:1402.5144

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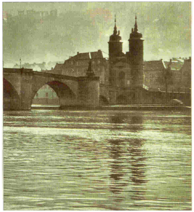
$11^{\text{th}}$ July, 2014



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- Chow groups will give us handle on them;

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$$\int_{\tilde{Y}_4} G_4 \wedge \omega_b = 0$$
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- Supersymmetry: $G_4 \in H^{2,2}(Y_4)$;
- Has to be quantised: $G_4 + \frac{c_2}{2} \in H^4(Y_4, \mathbb{Z})$;

# Chirality

- Type IIB: chirality along curve of intersecting branes given by

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- Matter surfaces, $C_{R_q}$, consist of linear combinations of blow-up $\mathbb{P}^1$'s fibred over enhancement curve $\mathcal{C}_{R_q}$;
- Recall: linear combination such that in dual M-theory picture, M2-brane wrapping this combination is one state of $R_q$ ;

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- Splitting of Pic encoded via:

$$0 \to \mathcal{J}^1(X) \to \mathrm{Pic}(X) \stackrel{c_1}{\to} H^{1,1}_{\mathbb{Z}}(X) \to 0$$

  $c_1(L) = \frac{1}{2\pi}F$ linear map onto $H^{1,1}(X, \mathbb{Z})$,
  $\mathcal{J}^1(X) = H^{0,1}(X, \mathbb{C})/H^1(X, \mathbb{Z})$ Jacobian of $X$ (space of flat connections).

# Beyond the Chiral Index II

- For $A_3$ & $G_4$ in F-theory exists similar decomposition:

$$0 \to \underbrace{\mathcal{J}^2(\hat{Y}_4)}_{\substack{\text{2^{nd} intermediate} \\ \text{Jakobian}}} \to \underbrace{H^4_D(\hat{Y}_4, \mathbb{Z}(2))}_{\substack{\text{4^{th} Deligne} \\ \text{cohomology class}}} \xrightarrow{c_2} H^{2,2}_{\mathbb{Z}}(\hat{Y}_4) \to 0$$

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- But can work indirectly by using Chow groups;

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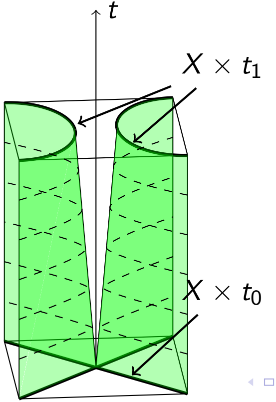
- Rational equivalence: $C_1 \cong C_2 \in Z_n(X)$ if $C_1 - C_2$ is zero/pole of meromorphic function defined on $(n+1)$-dim. irreducible subvariety of $X$; Equivalently: two algebraic cycles $C_1$, $C_2 \in Z_i(X)$ rationally equivalent if $\exists$ rationally parametrised family of cycles interpolating between them;



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$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathrm{CH}^p_{\mathrm{hom}}(X) & \longrightarrow & \mathrm{CH}^p(X) & \xrightarrow{\gamma_p} & H^{p,p}_{alg}(X) & \longrightarrow & 0 \\ & & \downarrow AJ & & \downarrow \hat{\gamma}_p & & \downarrow & & \\ 0 & \longrightarrow & J^p(X) & \longrightarrow & H^{2p}_D(X, \mathbb{Z}(p)) & \xrightarrow{\hat{c}_p} & H^{p,p}_{\mathbb{Z}}(X) & \longrightarrow & 0 \end{array}$$

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  2. Manipulations modulo rational equivalence preserve $C_3$ modulo gauge equivalence;

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- This collection of points $A_{R,G} \in Z_0(\mathcal{C}_R)$ can be used to define line bundle $L_{G,R} = \mathcal{O}_{\mathcal{C}_R}(A_{R,G})$ on $\mathcal{C}_R$;

# Matter II

- **Proposal**: [Bies,CM,Pehle,Weigand]
  massless $\mathcal{N} = 1$ chiral multiplets counted by
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- Can checked proposal for fluxes/gauge data coming from e.g. $U(1)$-symmetries;

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- Take $\mathrm{w}_A \in \mathrm{CH}^1(\hat{Y}_4)$ such that

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$\Rightarrow$ $\gamma(\mathrm{w}_A)$ together with $f \in \mathrm{CH}^1(B_3)$ gives four-form flux (class)

$$G_4^A = \pi^* \gamma(f) \cup \gamma(\mathrm{w}_A) \in H^{2,2}(\hat{Y}_4)$$

which satisfies 'one leg ...' condition and leaves all non-abelian sym. untouched.

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- Have now more then flux, because

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- In many cases, CY four-fold embedded in toric ambient space and $\mathrm{w}_A$ is pullback ($\mathrm{w}_A = j^* \tilde{\mathrm{w}}_A$) then

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- By means of *cohomCalg* and self-write Mathematica code, could work out spectral sequences to obtain cohomologies of line bundles on curves;

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| curve | $h^0(C, \mathcal{L}\|_C)$ | representation | $h^1(C, \mathcal{L}\|_C)$ | representation |
|---|---|---|---|---|
| $C_{\mathbf{10}}$ | 4 | $\mathbf{10}_{-1}$ | 1 | $\overline{\mathbf{10}}_{+1}$ |
| $C_{\overline{\mathbf{5}}_{\mathbf{m}}}$ | 6 | $\overline{\mathbf{5}}_{3}$ | 3 | $\mathbf{5}_{-3}$ |
| $C_{\mathbf{5_H}}$ | 9 | $\mathbf{5}_{2}$ | 9 | $\overline{\mathbf{5}}_{-2}$ |
| $C_{\mathbf{1}}$ | 585 | $\mathbf{1}_{5}$ | 0 | $\overline{\mathbf{1}}_{-5}$ |

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- Apply our methods to cases where intersections have to be done on CY itself;

Thank you for your attention!