Physics of F-theory compactifications without section

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F-theory without section

Most F-theory model building is done for spaces *with* a section.

In this talk I would like to discuss what happens whenever the torus fibration does *not* have a section. I will do this for Calabi-Yau **threefolds**.

- What is the 5d theory of M-theory on such spaces?
- Does a F-theory limit exist?
- How does the 6d theory look like?

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- Does a F-theory limit exist?
- How does the 6d theory look like?

Other interesting questions that I will not discuss: relation to [Morrison Braun, Morrison Taylor], statements in other dimensions, pheno applications, ...

Fibrations with and without section

We study Calabi-Yau threefolds X that are T^2 fibrations:

$$\pi \colon X \to \mathcal{B} \tag{1}$$

with $\mathcal{B}$ a complex surface, and $\pi^{-1}(p)$ topologically a T^2 for generic $p \in S$.

Fibrations with and without section

A section σ is defined by a continuous map $\sigma \colon \mathcal{B} \to X$ of the base $\mathcal{B}$ into the total space X such that

$$\pi(\sigma(p)) = p \qquad (2)$$

for all $p \in \mathcal{B}$.

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for all $p \in \mathcal{B}$. In other words, *a globally defined choice of point for each fiber*. Algebraically, we will want a divisor σ such that $\sigma \; \mathcal{B}$ and $\sigma \cdot F = 1$, with F the curve representing the fiber of the fibration.

Fibrations with and without section

For physics purposes a slight generalization is very important: we allow for **rational** sections: they are generically sections, but at certain loci of the base, where the fiber becomes singular, they may wrap full components of the (resolved) fiber.

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A simple example is the boundary of the Möbius strip, viewed as a $\pi\colon S^1 \to S^1$ fibration. The fiber is $\mathbb{Z}_2$ (two points), and it lacks a section.
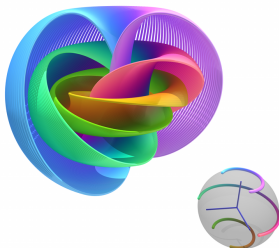
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Another well known example is S^3, with fibration map $\pi\colon S^3 \to S^2$ and generic fiber S^1.

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One section

This is the familiar case. Any fibration with at least one section is birational (i.e., connected by blow-ups) to a possibly singular **Weierstrass model**. [Nakayama]

$$y^2 = x^3 + fxz^4 + gz^6 \tag{3}$$

with f, g appropriate sections of line bundles on the base $\mathcal{B}$, and (x, y, z) coordinates on $\mathbb{P}^{2,3,1}$. More generically, this is a particular representation of T^2 as a degree 6 polynomial in $\mathbb{P}^{2,3,1}$.

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Many sections

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For every free factor of the group of sections (the *Mordell-Weil* group) one gets an extra $U(1)$. [Talk by Cvetič.] Torsion factors give interesting info about the global structure of gauge groups. [Talks by Weigand, Till.]

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Many sections

The previous result still applies: every such fibration is birational to a Weierstrass model. Nevertheless, for studying fibrations with many sections this is not always the most convenient representation. For example, for fibrations with two sections, a better suited representation is as a Calabi–Yau equation on $\mathrm{Bl}_{(0,1,0)} \, \mathbb{P}^{1,1,2}$ [Morrison Park]. More on this soon. [See also Cvetič's talk for progress in cases with more $U(1)$s.]

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No section

In general we cannot find a section, at best we can find an n-section: an holomorphic divisor projecting to the full base, but intersecting the fiber n times, $n > 1$. (In our examples, $n = 2$, but a similar story clearly holds for $n > 2$ too.)

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The subtle features of F-theory on such a manifold, which defines a $6d$ field theory, are best understood upon compactification on a circle, giving a 5d theory $\mathcal{T}_5$. This should be equivalent to studying M-theory on X in a particular limit. We obtain information on the 6d theory by imposing

$$\mathcal{T}_5 = M/X$$

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We determine $\mathcal{T}_5$ by a **fluxed** S^1 reduction of a 6d theory with a massive $U(1)$. We focus on one-loop Chern-Simons terms, which are robust and rather constraining.

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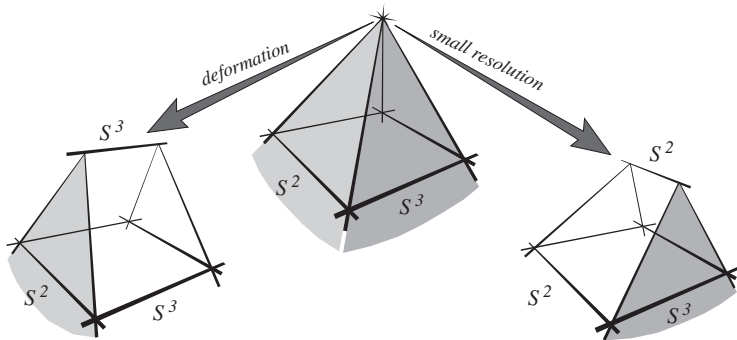
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The M-theory M/X is determined (in the examples we consider) by a **conifold transition** from a compactification on spaces with two sections.

Conifold transitions in M-theory

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The singular conifold is given by

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$$

in $\mathbb{C}^4$. Singularity at $(0,0,0,0)$ ($f = df = 0$).

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Deformation

Change the defining equation to

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \varepsilon\,. \tag{4}$$

$f = df = 0$ has no solution anymore, the space is smooth. There is a finite size S^3 where the singularity was.

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Small resolution

Replace the singularity by a "small resolution", which is a new space with a natural map to the original singular space which is an isomorphism away from the singular point. Rewrite the conifold equation as $xy - zw = 0$. Then the resolved space is

$$\begin{pmatrix} x & z \\ w & y \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0 \tag{5}$$

where $(a,b) \in \mathbb{P}^1$. A full $\mathbb{P}^1$ where the singularity was.

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Local description

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At the singular point there is a 5d $\mathcal{N} = 1$ theory with gauge group $U(1)$ (from reducing C_3 on the two-sphere) and a charged hypermultiplet (from an $M2$ on the contracting two-sphere).

Deformation corresponds to a Higgsing: i.e. giving a vev to the hypermultiplet, so the $U(1)$ vector multiplet becomes massive.

Resolution corresponds to going into a Coulomb branch: giving a vev to the scalar in the $U(1)$ vector multiplet, so the hyper becomes massive.

Conifold transitions for global models

The global picture is similar, but there can be relations between the local cycles, which change the picture slightly [Greene Morrison Strominger, Mohaupt Saueressig].

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Consider the case in which P conifold points are transitioning simultaneously. These curves are typically related by R relations in homology, so this implies that $P - R$ curve classes are transitioning simultaneously.

Studying the low energy theory, one sees that there are R flat directions in which one can Higgs the hypers. Along these directions the $P - R$ $U(1)$ vector multiplets get a mass, by pairing up with the $P - R$ non-flat directions.

Conifold transitions for global models

Starting from the resolved side

$$(n_H, n_V) = \left(h^{2,1}(\mathbb{X}) + 1, h^{1,1}(\mathbb{X}) - 1 \right). \tag{6}$$

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and then, giving a vev to the R flat directions

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From here we can read the Hodge numbers of the deformed side

$$(h^{2,1}(\mathcal{X}), h^{1,1}(\mathcal{X})) = (h^{2,1}(\mathbb{X}) + R, h^{1,1}(\mathbb{X}) - P + R). \qquad (9)$$

Conifold transitions removing the section

We want to consider conifold transitions between an elliptic
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The relevant conifold transition is already implicit in many of the recent works on F-theory with multiple $U(1)$s.

In order to engineer a model with two sections (say), start with a Calabi-Yau hypersurface on $\widehat{\mathbb{P}^{1,1,2}}$

$$gw^2 + wtP(y_1, y_2) + t^2 Q(y_1, y_2) = 0, \tag{10}$$

with

$$P(y_1, y_2) = \alpha y_1^2 + \beta y_1 y_2 + f y_2^2$$

$$Q(y_1, y_2) = y_1 \underbrace{(by_1^3 + cy_1^2 y_2 + dy_1 y_2^2 + ey_2^3)}_{Q'(y_1,y_2)} + ay_2^4. \tag{11}$$

Conifold transitions removing the section

This space will have two sections whenever $a = 0$.

Set $y_1 = 0$. The Calabi-Yau equation reduces to

$$w(gw + tf) = 0 \qquad (12)$$

So our global choice of point in the fiber is given by $y_1 = w = 0$ (one section), and $y_1 = 0$, $(w, t) = (-f, g)$ (another section).

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Each section is associated to a divisor, and together they give rise to $U(1) \times U(1)$ vector bosons in 5d by reduction of C_3 on the Poincare dual two-forms.

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This space is singular. It has conifold singularities at
$y_1 = w = e = f = 0$. (Easy to check that these are solutions of
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We know what to do: resolve or deform!

Conifold transitions removing the section

Resolved side

This is the most familiar side, perhaps. [Morrison Park, . . .]

We **enforce** $a = 0$ by blowing up. Consider the (proper transform of the) hypersurface on an ambient space blown up at a point:

$$
\begin{array}{c|ccccc}
 & y_1 & y_2 & w & t & s \\
\hline
\mathbb{C}_1^* & 1 & 1 & 2 & 0 & 0 \\
\mathbb{C}_2^* & 0 & 0 & 1 & 1 & 0 \\
\mathbb{C}_3^* & 1 & 0 & 1 & 0 & -1
\end{array}
\tag{13}
$$

$$
\widetilde{\phi} \equiv g w^2 s + w t P(s y_1, y_2) + t^2 y_1 Q'(s y_1, y_2) = 0\,.
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Now the singularity is replaced by a finite size $\mathbb{P}^1$, but the two sections remain:

$$
\begin{aligned}
(y_1, y_2, w, t, s) &= (-f, 1, e, 1, 0) \\
(y_1, y_2, w, t, s) &= (0, 1, 1, -g, f) \,,
\end{aligned}
\tag{15}
$$

so we still expect $U(1) \times U(1)$ vector multiplets in 5d.

Conifold transitions removing the section

Simply switch on $a \neq 0$.

The Calabi-Yau equation for $y_1 = 0$ now reduces to

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which no longer factorizes globally. The two sections have recombined into a single object.

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Deformed side

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So we end up a single $U(1)$ vector multiplet in 5d.

In fact, the two sections have recombined into a bi-section. There is **no section anymore**.

Demonstrating absence of a section

Actually proving that there is no section is somewhat subtle. The basic idea is the following. [Oguiso, Morrison Vafa]

First identify the fiber curve. This is typically easy: choose two divisors on the base that intersect over a point, and take their pullbacks to the Calabi-Yau. Their intersection will be the fiber divisor $\mathcal{T}$.

Now we want to show that there is no divisor S such that $S \cdot \mathcal{T} = 1$. We can always parameterize it in terms of some convenient basis of divisors

$$S = \sum a_i D_i \,. \tag{17}$$

In our examples there is always a convenient choice of basis such that

$$S \cdot \mathcal{T} = 2a_2 + 4a_3 \,. \tag{18}$$

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that

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The absence of a section would follow if $a_2, a_3 \in \mathbb{Z}$.

To show this, we study the loci where the T^2 fibers splits into
components $T^2 = \sum C_i$. We find loci where

$$S \cdot C_1 = a_2 \tag{20}$$
$$S \cdot C_2 = a_3 \,. \tag{21}$$

This imposes $a_2, a_3 \in \mathbb{Z}$, so $S \cdot \mathcal{T} \in 2\mathbb{Z}$, and we have (at best) a
bi-section.

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From the low energy point of view, it connects a theory with $U(1) \times U(1)$ vector multiplets with a theory with $U(1)$ vector multiplets. It does this by transitioning at $\deg(e) \cdot \deg(f)$ points in the base, where the singular points $y_1 = w = e = f = 0$ are located.

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A sanity check is then

$$(h^{2,1}(\mathcal{X}), h^{1,1}(\mathcal{X})) = (h^{2,1}(\mathbb{X}) + R, h^{1,1}(\mathbb{X}) - P + R) \qquad (22)$$

with $P = \deg(e) \cdot \deg(f)$, $P - R = 1$.

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This holds in all examples.

Specific examples

We consider Calabi-Yau threefold fibrations with $\mathrm{Bl}_{(0,1,0)} \widehat{\mathbb{P}^{1,1,2}}$
fiber on the resolved side, generic $\widehat{\mathbb{P}^{1,1,2}}$ fiber on the deformed side,
and $\mathbb{P}^2$ base.

There are 16 such examples. On the resolved side we have a (well studied) class of examples with extra $U(1)$ symmetries. The full spectrum of 6d matter can be identified using the techniques in [Grimm Kapfer Keitel, Grimm Hayashi].

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Or more algebraically, by explicitly finding the holomorphic curves in the geometry that go to zero size on the F-theory limit, and computing their $U(1)$ charges (by checking their intersections with the extra section).

Specific examples

(a,b)	$h^{1,1}(\mathbb{X})$	$h^{2,1}(\mathbb{X})$	P	$H(\mathbf{1}_2)$	$H(\mathbf{1}_4)$	$H(\mathbf{2}_1)$	$H(\mathbf{2}_3)$	$H(\mathbf{3}_0)$
$(0,3)$	3	111	18	144	18	0	0	0
$(1,4)$	3	123	10	140	10	0	0	0
$(2,5)$	3	141	4	128	4	0	0	0
$(0,-2)$	4	57	3	64	3	55	15	6
$(0,-1)$	4	60	6	76	6	52	12	3
$(0,0)$	4	67	9	90	9	45	9	1
$(0,1)$	4	78	12	106	12	34	6	0
$(0,2)$	4	93	15	124	15	19	3	0
$(1,0)$	4	68	2	72	2	56	8	3
$(1,1)$	4	76	4	86	4	48	6	1
$(1,2)$	4	88	6	102	6	36	4	0
$(1,3)$	4	104	8	120	8	20	2	0
$(2,3)$	4	104	2	90	2	38	2	0
$(2,4)$	4	121	3	108	3	21	1	0
$(3,6)$	3	165	0	108	0	0	0	0
$(0,-3)$	6	60	0	–	–	–	–	–

Back to the general idea

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We focus on Chern-Simons terms. The CS terms for $\mathbb{X}$ are well understood by now [Grimm Kapfer Keitel, Grimm Hayashi], and following them to the deformed side $\mathcal{X}$ is not hard.

The resulting Chern-Simons terms match perfectly those of a **fluxed** S^1 reduction of a 6d theory with a massive $U(1)$. [See Thomas' talk for more details.]

The origin of the flux

The fact that it is a fluxed reduction is perhaps a little surprising.
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The general metric of a fibration (with or without section) takes the form

$$ds^2 = g_{i\bar{j}}\, du^i d\bar{u}^{\bar{j}} + \frac{v^0}{\mathsf{Im}\tau}|X - \tau Y|^2 \;, \tag{23}$$

with

$$X = dx + \widetilde{X} \;, \qquad Y = dy + \widetilde{Y} \;. \tag{24}$$

Whenever one has a section, one can choose coordinates such that $\widetilde{X} = \widetilde{Y} = 0$, but this is not possible when the fibration has not section, they become connection on a non-trivial bundle.

The origin of the flux

These cross terms give mixed components in the metric, and following the F theory limit (reduction to IIA and T-duality), one easily sees that the effect of the off-diagonal components is the introduction of a flux.

Viewing x as the M-theory circle

$$ds_M^2 = e^{4\phi_{\text{IIA}}/3}(dx + C_1^{\text{IIA}})^2 + e^{-2\phi_{\text{IIA}}/3}ds_{\text{IIA}}^2 \,. \tag{25}$$

one reads

$$C_1^{\text{IIA}} = \text{Re}\,\tau\,dy + \text{Re}\,K\,, \tag{26}$$

$$ds_{\text{IIA}}^2 = \sqrt{\frac{v^0}{\text{Im}\tau}}\Big(\frac{v^0}{\text{Im}\tau}(\text{Im}\,\tau\,dy + \text{Im}\,K)^2 + g_{i\bar{j}}\,du^i d\bar{u}^{\bar{j}}\Big) \tag{27}$$

with $K = \widetilde{X} - \tau\widetilde{Y}$. Then T-dualizing along y

$$\left.\begin{array}{l} C_2^{\text{IIB}} = \widetilde{X} \wedge dy \\ B_2^{\text{IIB}} = \widetilde{Y} \wedge dy \end{array}\right\} \implies (F_3, H_3) \neq (0,0)\,. \tag{28}$$

Conclusions

- F-theory on spaces without section makes perfect sense. The 5d theory requires specification of a flux on the circle.

- There is a nice family of examples closely related to examples with extra $U(1)$s.

- But we also propose (in the paper, not discussed here) an intrinsic recipe, without the need of a conifold transition.

Some questions

- The relation to [Morrison Braun, Morrison Taylor] is not totally clear, although many ingredients seem suggestively related.
- Compactifications to 4d are probably understandable too, and will likely yield interesting structure.
- Applications to pheno?

Workshop:
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