Top quark mass coupling and the classification of $SO(10)$ heterotic superstring vacua

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The Standard Model from Strings

String theory, as a theory of all interactions, should reproduce the Standard Model at low energies. However, String Theory is formulated in higher dimensions and it contains a huge number of vacua in four dimensions. Although, various low energy models with (semi)realistic features have been constructed in the past, we have recently started a systematic exploration of string vacua in 4-dimensions.
In this talk I will survey some recent progress towards classification of heterotic superstring vacua in the Free Fermionic Formulation.

Work in collaboration with
C. Kounnas (ENS), A. Faraggi (Univ. of Liverpool), S. Abel (Durham U.)
and
T. Catelin-Jullien, S.E.M. Nooij, B. Assel, K. Christodoulides, Laura Bernard, Ivan Glasser and Hasan Sonmez

The Free Fermionic Formulation of the heterotic superstring

In the Free Fermionic Formulation of the heterotic superstring we can reduce the critical dimension and construct models in $D = 4$ by fermionizing the left movers and introducing non-linear supersymmetry among them. A model is defined in terms of :

- a set of boundary condition basis vectors $B = \{v_1, v_2, \dots v_n\}$ related to the parallel transportation properties of the fermionised world-sheet degrees of freedom along the non-contractable torus loops

$$v_i = \{\alpha_i(f_1), \alpha_i(f_1), \dots, \alpha_i(f_N)\}\ ,\ \ f_i \to -e^{-i\,\pi\,\alpha(f_i)}\, f_i$$

- a set of $n(n-1)$ phases $c\begin{bmatrix} v_i \\ v_j \end{bmatrix}, i > j$ related to generalised GSO projections (GGSO).

Apart from mild constraints due to modular invariance and string amplitude factorization the basis vectors and GGSO phases are arbitrary, giving rise to a huge number of 4D models.

Model classification: Basic Phenomenological requirements

Some of the basis phenomenological requirements are

- $N = 1$ space-time supersymmetry
- Gauge group $G \subset SO(10)$
- 3 generations ($\mathbf{16}$)
- Presence of Higgs to break MSSM (and GUT) ($\mathbf{10} + / \mathbf{16} + \overline{\mathbf{16}}$)
- Fractional Charge Exotics
- Fermion masses

X. G. Wen and Witten (1985), G.G. Athanasiou, J.J. Atick, M. Dine and W. Fishler (1988), A. N. Schellekens (1989)
Paul Langacker and Gary Steigman, (2011)

Classification strategy

The number of vacua is huge, so we need a classification strategy

- Restrict to a subpace of models with interesting phenomenology. Choose a basis set that singles out an $SO(10)$ gauge factor to play the role of the observable gauge group.
- Use arbitrary GGSO coefficients
- Identify models by few characteristic properties:e.g. # of spinorials, # of antispinorials, # of vectorials
- Derive general analytic formulae for the above characteristics in terms of the GGSO phases
- Use a fast computer program to scan for models (10^5 models/second)
- Employ one or two additional basis vectors to break $SO(10)$ to a subgroup containing the SM (and restart from the second step)

The class of $Z_2 \times Z_2$ $SO(10)$ heterotic models

The world-sheet fermions in the light-cone gauge in the traditional notation are:

left: $\psi^\mu, \chi^{1,\dots,6}, \quad y^{1,\dots,6}, \omega^{1,\dots,6}$

right: $\bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \quad \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \quad \bar{\phi}^{1,\dots,8}$

Basis vectors $B = \{v_1, v_2, \dots, v_{12}\}$: where

$$
\begin{aligned}
v_1 = 1 \quad &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \\
&\qquad \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
v_2 = S \quad &= \{\psi^\mu, \chi^{1,\dots,6}\} \\
\textit{shifts}: \quad v_{2+i} = e_i \quad &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6 \\
Z_2\ \textit{twist}: v_9 = b_1 \quad &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\} \\
Z_2\ \textit{twist}: v_{10} = b_2 \quad &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\} \\
v_{11} = z_1 \quad &= \{\bar{\phi}^{1,\dots,4}\} \\
v_{12} = z_2 \quad &= \{\bar{\phi}^{5,\dots,8}\}
\end{aligned}
$$

and a set of $12(12-1)/2 = 66$ phases $c[v_i, v_j] = \pm 1$, $j < i = 1, \dots, 12$

Gauge Group

$$1, S, e_i, b_1, b_2, z_1, z_2 \qquad \overbrace{\underbrace{SO(10)}_{\Downarrow} \times U(1)^3}^{E_6 \times U(1)^2} \overbrace{\times SO(8)^2}^{E_8}$$

$$\alpha, \beta, \ldots \qquad G \quad \times U(1)^3 \times G'$$

where
$G =$
$SU(4) \times SU(2)_L \times SU(2)_R$ (Pati–Salam)
$SU(5) \times U(1)$ (Flipped $SU(5)$)
$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ (left-right)
$SU(3) \times SU(2) \times U(1)$ (SM)

$G' = SU(2)^n \times SO(8)^m$

Massless spectrum

Untwisted sector matter spectrum (universal)
6 pairs of $SO(10)$ vectorials and a number of $SO(10)$ singlets.
The twisted sectors are generated by $b_1, b_2, b_1 + b_2$ (three $Z_2 \times Z_2$ orbifold planes), labeled by the shifts ($e_i, i = 1, \ldots, 6$),they contain

Spinorial $SO(10)$ representations $(\mathbf{16} + \mathbf{\bar{16}})$:

$$\begin{aligned} B^{(1)}_{pqrs} &= S + b_1 + p^1\, e_3 + q^1\, e_4 + r^1\, e_5 + s^1\, e_6 \\ B^{(2)}_{pqrs} &= S + b_2 + p^2\, e_1 + q^2\, e_2 + r^2\, e_5 + s^2\, e_6 \\ B^{(3)}_{pqrs} &= S + b_3 + p^3\, e_1 + q^2\, e_2 + r^3\, e_3 + s^3\, e_4 \end{aligned}$$

where $b_3 = b_1 + b_2 + x,\ p^i, q^i, r^i, s^i = \{0, 1\}$.

Vectorial $SO(10)$ representations $(\mathbf{10})$

$$V^{(I)}_{pqrs} = B^{(I)}_{pqrs} + x$$

where $x = 1 + S + \sum_{i=1}^{6} e_i + \sum_{i=1}^{2} z_i$.

$SO(10)$ singlets

Analytic formulae for model characteristics

Number of Spinorials ($\mathbf{16}/\overline{\mathbf{16}}$)

$$\#(S^{(I)}) = \begin{cases} 2^{4-\mathrm{rank}\left(\Delta^{(I)}\right)} & \mathrm{rank}\left(\Delta^{(I)}\right) = \mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \\ 0 & \mathrm{rank}\left(\Delta^{(I)}\right) < \mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \end{cases}$$

Number of Vectorials ($\mathbf{10}$)

$$\#(V^{(I)}) = \begin{cases} 2^{4-\mathrm{rank}\left(\Delta^{(I)}\right)} & \mathrm{rank}\left(\Delta^{(I)}\right) = \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \\ 0 & \mathrm{rank}\left(\Delta^{(I)}\right) < \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \end{cases}$$

$\Delta^{(I)}$, are 4x4 and $Y^{(I)}$ $I = 1, 2, 3$ are 4x1 GGSO coefficient matrices

Analytic formulae for model characteristics

The chirality of the surviving spinorials is given by

$$X^{(1)}_{pqrs} = c\begin{bmatrix} b_2 + (1-r)e_5 + (1-s)e_6 \\ B^{(1)}_{pqrs} \end{bmatrix}$$
$$X^{(2)}_{pqrs} = c\begin{bmatrix} b_1 + (1-r)e_5 + (1-s)e_6 \\ B^{(2)}_{pqrs} \end{bmatrix}$$
$$X^{(3)}_{pqrs} = c\begin{bmatrix} b_1 + (1-r)e_3 + (1-s)e_4 \\ B^{(3)}_{pqrs} \end{bmatrix}$$

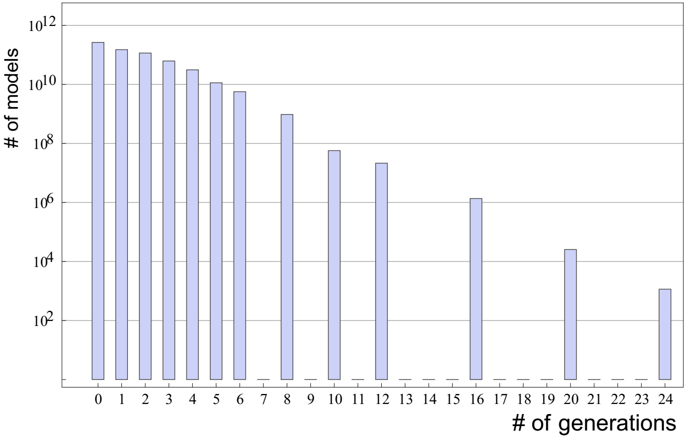
where $X^i_{pqrs} = +1$ corresponds to a **16** of $SO(10)$($X^i_{pqrs} = -1$ corresponds to a $\overline{\mathbf{16}}$). The net number of families is given by

$$N_F = \sum_{i=1}^{3} \sum_{p,q,r,s=0}^{1} X^{(i)}_{pqrs} P^{(i)}_{pqrs}$$

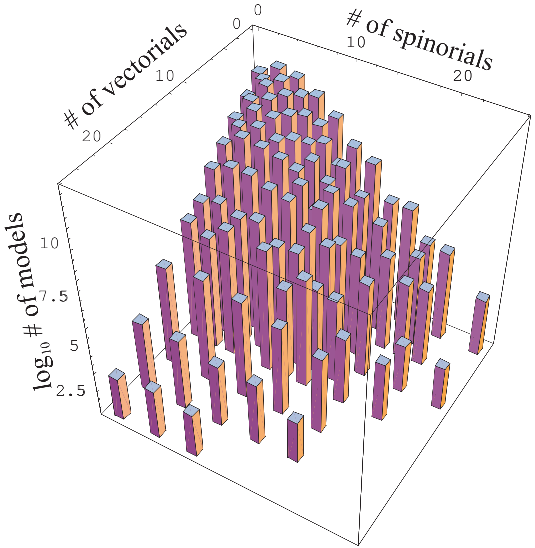
Useful formulae but they cannot be inverted (i.e solve for number of generations)

Results of a comprehensive computer scan

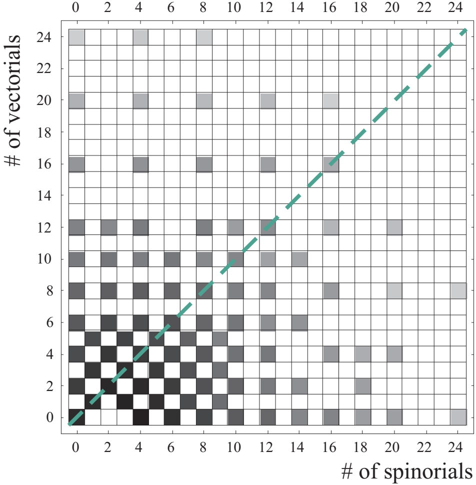
Total number of configurations in this class: 1.016.808.865.792 $\sim 10^{12}$



Spinorial-Vectorial symmetry



Spinorial-Vectorial symmetry



Density plot of the number of models as a function of the number of spinorials ($\mathbf{16} + \overline{\mathbf{16}}$) and the number of vectorials ($\mathbf{10}$) $SO(10)$ representations.

Results of $SO(10)$ model classification

1. Three generation models are quite abundant 15% of the class.
2. Models in this class appear in pairs related with spinor-vector symmetry:

$$(S = \text{spinorinals}, V = \text{vectorials})$$

$$\Updownarrow$$

$$(V = \text{spinorinals}, S = \text{vectorials})$$

The map has been derived analytically and holds to each orbifold plane separately. Self-dual models under this symmetry appear to be anomaly free (no anomalous $U(1)$)

Towards the Standard Model

$$
\begin{aligned}
SO(10) &\Rightarrow SU(4)\times SU(2)_L\times SU(2)_R \quad \alpha=\{\bar\psi^{4,5},\bar\phi^{1,2}\}\\
&\Rightarrow SU(5)\times U(1) \quad \alpha=\left\{\begin{matrix}\bar\psi^1\dots\bar\psi^5\\ \frac12\dots\frac12\end{matrix},\begin{matrix}\bar\eta^1\bar\eta^2\bar\eta^3\\ \frac12\frac12\frac12\end{matrix},\begin{matrix}\bar\phi^1\dots\bar\phi^4\\ \frac12\dots\frac12\end{matrix},\begin{matrix}\bar\phi^5\\ 1\end{matrix}\right\}\\
&\Rightarrow SU(3)\times SU(2)_L\times SU(2)_R\times U(1)\\
&\Rightarrow SU(3)\times SU(2)_L\times U(1)
\end{aligned}
$$

The supersymmetric Pati-Salam model

The supersymmetric Pati–Salam $G = SU(4) \times SU(2)_L \times SU(2)_R$, spectrum comprises

$$(\mathbf{4},\mathbf{2},\mathbf{1}) = Q\left(\mathbf{3},\mathbf{2},+\frac{1}{6}\right) + L\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$$

$$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = u^c\left(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3}\right) + d^c\left(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3}\right) + e^c\,(\mathbf{1},\mathbf{1},1) + \nu^c\,(\mathbf{1},\mathbf{1},0)$$

$$(\mathbf{1},\mathbf{2},\mathbf{2}) = H_u\left(\mathbf{1},\mathbf{2},+\frac{1}{2}\right) + H_d\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$$

$$(\mathbf{6},\mathbf{1},\mathbf{1}) = \delta\left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right) + \delta^c\left(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3}\right)$$

plus a number of singlets. In string realisations we also have Fractional Charge Exotics

$$(\mathbf{4},\mathbf{1},\mathbf{1}) + (\bar{\mathbf{4}},\mathbf{1},\mathbf{1})\ ,\ (\mathbf{1},\mathbf{2},\mathbf{1})\ ,\ (\mathbf{1},\mathbf{1},\mathbf{2})$$

J. Pati and A. Salam, *Lepton number as the fourth color* (1974)
I. Antoniadis and G. Leontaris (1988) (SUSY version)
I. Antoniadis, G. Leontaris and J. Rizos (1990) (heterotic superstring version)

The supersymmetric Pati–Salam model

Symmetry breaking The PS symmetry can be broken to the Standard Model by $\langle\nu_H^c\rangle, \langle\nu_H\rangle$

$$(\mathbf{4},\mathbf{1},\mathbf{2})_H = u_H\left(\mathbf{3},\mathbf{1},+\frac{2}{3}\right) + d_H\left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right) + e_H\left(\mathbf{1},\mathbf{1},-1\right) + \nu_H\left(\mathbf{1},\mathbf{1},0\right)$$

$$\left(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}\right)_H = u_H^c\left(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3}\right) + d_H^c\left(\bar{\mathbf{3}},\mathbf{1},+\frac{1}{3}\right) + e_H^c\left(\mathbf{1},\mathbf{1},+1\right) + \nu_H^c\left(\mathbf{1},\mathbf{1},0\right)$$

Triplet mass

$$(\mathbf{4},\mathbf{1},\mathbf{2})_H^2\,(\mathbf{6},\mathbf{1},\mathbf{1}) + \left(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}\right)_H^2\,(\mathbf{6},\mathbf{1},\mathbf{1}) = d_H\,\delta^c\,\langle\nu_H\rangle + d_H^c\,\delta\,\langle\nu_H^c\rangle + \dots$$

We need at least one $(\mathbf{6},\mathbf{1},1)$ to realize this mechanism.

Fermion masses

$$(\mathbf{4},\mathbf{2},\mathbf{1})\,\left(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}\right)\,(\mathbf{1},\mathbf{2},\mathbf{2}) = Q\,u^c\,H_u + Q\,d^c\,H_d + L\,e^c\,H_d + L\,\nu^c\,H_u\,.$$

Classification of PS heterotic string models

The spectrum of a PS model can be summarised as follows

n_g	fermion generations in $(\mathbf{4},\mathbf{2},\mathbf{1})+(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$
k_R	PS Higgs pairs transforming as $(\mathbf{4},\mathbf{1},\mathbf{2})+(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$
n_h	SM Higgs doublet pairs in $(\mathbf{1},\mathbf{2},\mathbf{2})$
n_6	additional triplet pairs in $(\mathbf{6},\mathbf{1},\mathbf{1})$
k_L	additional (left) vector-like fields, $(\mathbf{4},\mathbf{2},\mathbf{1})+(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$
n_e	exotic $(\mathbf{4},\mathbf{1},\mathbf{1})$, $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$ $(\mathbf{1},\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\mathbf{2})$

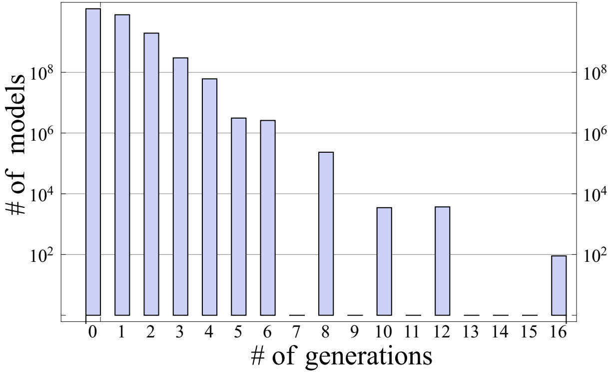
Obviously for a realistic model

$$n_g = 3\,,\; k_R \geq 1\,,\; k_L \geq 0\,,\; n_h \geq 1\,,\; n_6 \geq 1$$

with equalities corresponding to the minimal models.

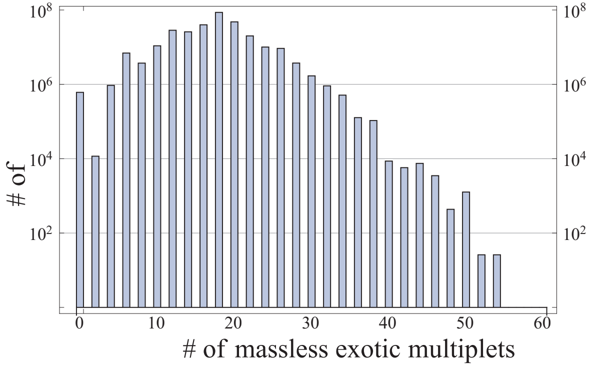
The models in this class are of the order of $2^{51} \sim 10^{15}$. A comprehensive scan would require 300 years (in a single CPU) however we can perform a random scan of a sample of 10^{11} models in a few days.

Generation structure of PS models



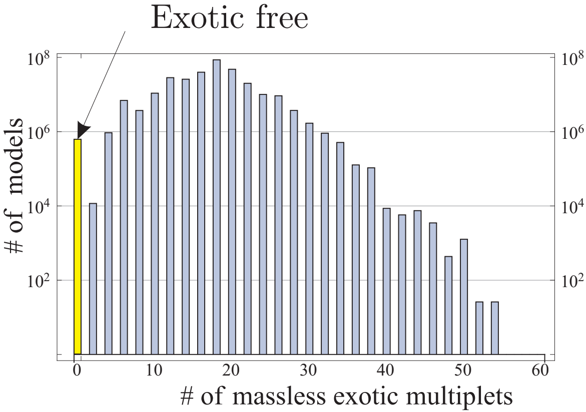
Number of models versus number of generations (n_g) in a random sample of 10^{11} vacua.

Exotic Fractional Charge States



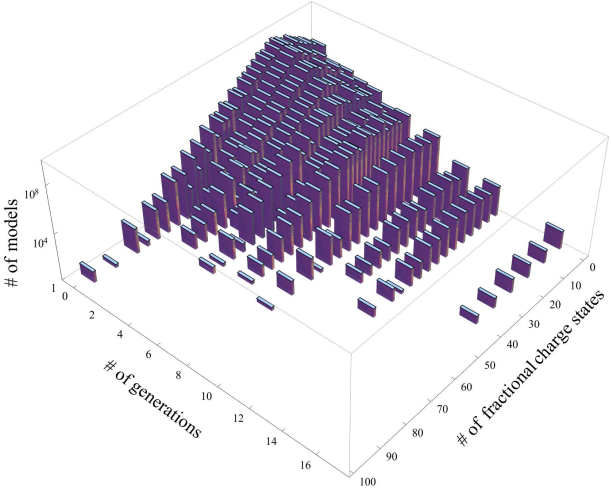
Number of 3 generation models versus total number of exotic multiplets in a random sample of 10^{11} *vacua.*

Exotic Fractional Charge States



Number of 3 generation models versus total number of exotic multiplets in a random sample of 10^{11} *vacua.*

Generation structure of exotic free models



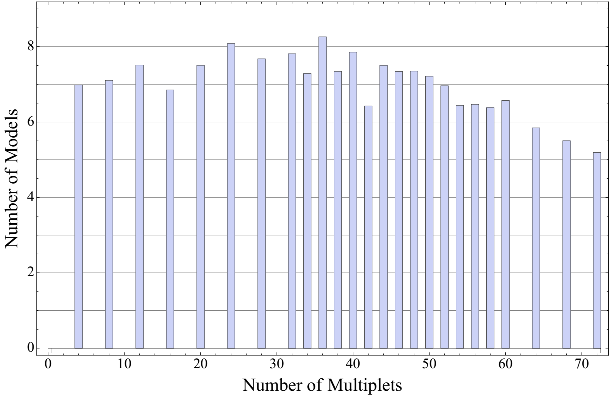
Number models versus number of generations total number of exotic multiplets in a random sample of 10^{11} *vacua.*

Summary of the Pati-Salam model "landscape"

constraint	# of models in sample	probability	estimated # of models in class
None	10^{11}	1	2.25×10^{15}
+ No group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete generations	22 497 003 372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298 140 621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23 694 017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19 191 088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121 669	1.22×10^{-6}	2.74×10^{9}

Classification of flipped $SU(5)$ models

A similar analysis has been performed for $SU(5) \times U(1)$ vacua.
see talk H. Sonmez
Here the possible configurations are $2^{44} \sim 2 \times 10^{13}$



No exotic free models (exophobic) with odd number of generations.

The top quark mass coupling criterion

Criteria associated with interactions in the effective low energy theory, are hard to implement since they entail, model dependent, detailed calculations of string amplitudes. In the case of the models under consideration the candidate top mass Yukawa coupling will have the form

$$\lambda_t \, \mathbf{S}^{Q_L} \, \mathbf{S}^{u_R} \, \mathbf{V}^{H_u}$$

where **S** is the "spinorial" and **V** the "vectorial" representation of $G \subset SO(10)$.
This coupling can be calculated for a generic model in the class under consideration

$$\lambda_t \sim \left\langle S^F_{-1/2} \, S^F_{-1/2}, V^B_{-1} \right\rangle$$

It is proportional to some Ising correlators of the left/right real fermion pairs.

Existence of top quark mass requirements

The top mass coupling constraint can be translated to relations among the GGSO phases

$$c\begin{bmatrix} b_1 \\ e_1 \end{bmatrix} = c\begin{bmatrix} b_1 \\ e_2 \end{bmatrix} = c\begin{bmatrix} b_2 \\ e_3 \end{bmatrix} = c\begin{bmatrix} b_2 \\ e_4 \end{bmatrix} = 1$$

$$c\begin{bmatrix} b_1 \\ z_1 \end{bmatrix} = c\begin{bmatrix} b_1 \\ z_2 \end{bmatrix} = c\begin{bmatrix} b_2 \\ z_1 \end{bmatrix} = c\begin{bmatrix} b_2 \\ z_2 \end{bmatrix} = +1$$

$$c\begin{bmatrix} b_1 \\ e_5 \end{bmatrix} c\begin{bmatrix} b_2 \\ e_5 \end{bmatrix} = c\begin{bmatrix} b_1 \\ e_6 \end{bmatrix} c\begin{bmatrix} b_2 \\ e_6 \end{bmatrix} = +1$$

$$\prod_{i=3,4,5,6} c\begin{bmatrix} b_1 \\ e_i \end{bmatrix} = \prod_{i=1,2,5,6} c\begin{bmatrix} b_2 \\ e_i \end{bmatrix} = 1$$

plus additional relations for phases associated with the $SO(10)$ breaking vectors.

Top quark mass coupling and the PS vacua

In the case of the PS vacua quark and lepton masses arise, as in the case of $SO(10)$ model, from a single superpotential term

$$F_L\left(\mathbf{4},\mathbf{2},\mathbf{1}\right)\ \bar{F}_R\left(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}\right)\ h\left(\mathbf{1},\mathbf{2},\mathbf{2}\right)$$

The additional constraints are

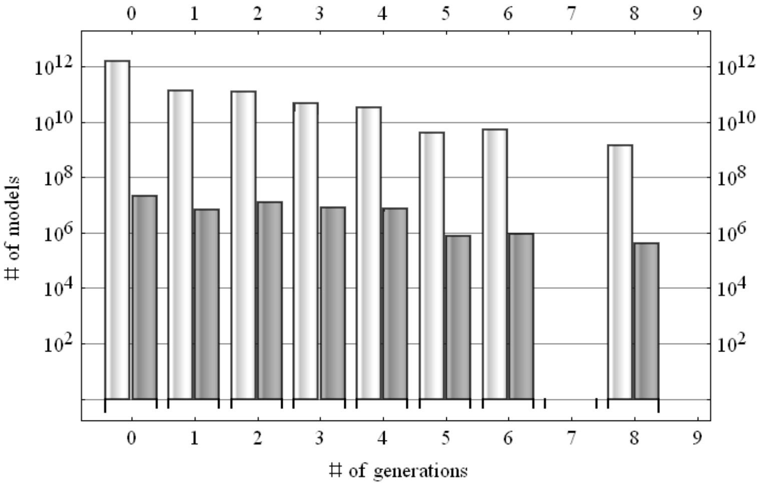
$$c\begin{bmatrix} b_1 \\ a \end{bmatrix} = -c\begin{bmatrix} b_2 \\ a \end{bmatrix} = +1\,.$$

leading to 14 relations among GGSO phases. That is

$$1:2^{14}=16384$$

models in this class has at least one candidate coupling for quark and lepton masses.

Top quark mass coupling and the PS vacua



Estimated number of Pati–Salam models, that meet certain phenomenological criteria, versus number of generations, before (light-gray) and after (dark-gray) the application of the top mass coupling constraints (based on a random sample of 10^{11} *models).*

Heuristic search methods: The GA algorithm

The methods we have presented turn out to be quite efficient in the case of technically simple models, however the exploration of models with more complicated structure requires the development of new tools that go beyond the simple deterministic search algorithms, such as exhaustive scanning or simple randomised scan used here.
In a recent work we have explored the use of heuristic search methods and more particularly a method based on the Genetic Algorithm (GA).
It is an open question whether such methods are applicable to the string landscape. However, the heuristic algorithms have been proved very efficient in finding approximate solutions to problems considered as "NP-complete".

B. C. Allanach, D. Grellscheid and F. Quevedo, "Genetic algorithms and experimental discrimination of SUSY models," JHEP **0407**, 069 (2004) [hep-ph/0406277].
J. Blbck, U. Danielsson and G. Dibitetto, "Fully stable dS vacua from generalised fluxes," JHEP **1308** (2013) 054 [arXiv:1301.7073 [hep-th]];
C. Damian, L. R. Diaz-Barron, O. Loaiza-Brito and M. Sabido, "Slow-Roll Inflation in Non-geometric Flux Compactification," JHEP **1306**, 109 (2013) [arXiv:1302.0529 [hep-th]].

Exploring the PS vacua with GA

As a benchmark case we have used the class of Pati-Salam vacua analysed here. They are parametrised by 51 binary parameters

$$\ell_i = \{0,1\}\,,\ i = 1,\dots 51$$

that correspond to GGSO phases. This immediately solves the most difficult step in the application of the GA algorithm known as "encoding".
We select randomly an initial population of models

$p_1 =$	ℓ_1	ℓ_2	ℓ_3	$\cdots$	ℓ_{49}	ℓ_{50}	ℓ_{51}
$p_2 =$	ℓ_1^2	ℓ_2^2	ℓ_3^2	$\cdots$	ℓ_{49}^2	ℓ_{50}^2	ℓ_{51}^2
$\vdots$				$\vdots$			
$p_k =$	ℓ_1^k	ℓ_2^k	ℓ_3^k	$\cdots$	ℓ_{49}^k	ℓ_{50}^k	ℓ_{51}^k

and define a fitness function.

Exploring the PS vacua with GA

As a benchmark case we have used the class of Pati-Salam vacua analysed here. They are parametrised by 51 binary parameters $\ell_i = \{0, 1\}$, $i = 1, \ldots 51$. that correspond to GGSO phases. This immediately solves the most difficult step in the application of the GA algorithm known as "encoding".
We select randomly an initial population of models ("chromosomes")

$$p_1 = \begin{array}{|c|c|c|c|c|c|c|c|}\hline \ell_1 & \ell_2 & \ell_3 & & \cdots & \ell_{49} & \ell_{50} & \ell_{51} \\ \hline \end{array}$$

$$p_2 = \begin{array}{|c|c|c|c|c|c|c|c|}\hline \ell_1^{\,2} & \ell_2^{\,2} & \ell_3^{\,2} & & \cdots & \ell_{49}^{\,2} & \ell_{50}^{\,2} & \ell_{51}^{\,2} \\ \hline \end{array}$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$p_k = \begin{array}{|c|c|c|c|c|c|c|c|}\hline \ell_1^{\,k} & \ell_2^{\,k} & \ell_3^{\,k} & & \cdots & \ell_{49}^{\,k} & \ell_{50}^{\,k} & \ell_{51}^{\,k} \\ \hline \end{array}$$

and define a fitness function.
In every step we try to "improve" this population by two operations:
Crossover: Cut the chromosomes of a breeding pair at the same two randomly chosen positions and swap the middle section.
Mutation: Reassign a tiny fraction of the digits in the offspring (less than a percent usually) to arbitrary random values (here just swap 1 and 0).

Exploring the PS vacua with GA

We define our fitness function according to the following phenomenological criteria

(a) 3 complete family generations, $n_g = 3$

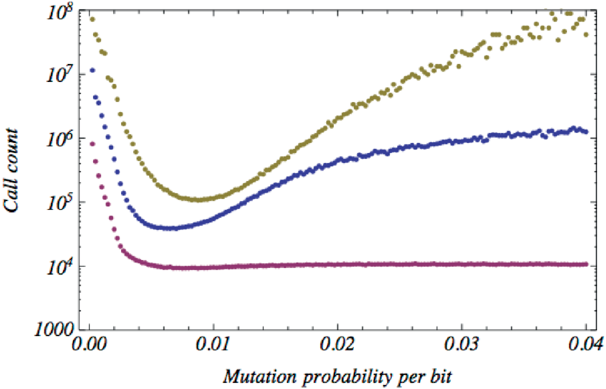
(b) Existence of PS breaking Higgs, $k_R \geq 1$

(c) Existence of SM Higgs doublets, $n_h \geq 1$

(d) Absence of exotic fractional charge states, $n_e = 0$

(e) Existence of top quark mass coupling.

Results of the GA search



Call count the for three different classes of solutions with increasing search difficulty. (i.e. the mean number of models one has to construct before finding a solution.) Bottom/purple: solutions with three generations and Higgses for the Standard Model and Pati-Salam sectors. Middle/blue: solutions with three generations, Higgses for the Standard Model and Pati-Salam sectors, and in addition no exotics. Top/yellow: solutions with three generations, Higgses for the Standard Model and Pati-Salam sectors, no exotics and a top-Yukawa. The search difficulties are respectively one in 10^4*, one in* 2.5×10^6*, one in* 10^{10}.

Conclusions

- We have developed tools that allow the systematic exploration of $SO(10)$ heterotic models.
- We have classified a set of approximately 10^{15} Pati-Salam heterotic and a set of approximately 10^{13} flipped $SU(5)$ vacua in this class.
- This class of vacua seems to be very rich, the include models with 3 generations, PS breaking Higgs, SM breaking Higgs, $(1 : 10^4)$.
- We have identified an interesting subclass of realistic models, (3×10^9) where the massless string spectrum is free of exotic fractionally charged states (exophobic).
- We also considered criteria related to top quark mass coupling and shown that they can be successfully implemented in this framework and reduce significantly the number of acceptable models (10^7 in the case of PS vacua)
- We have also explored this class of vacua using heuristic methods and we have shown that the GA algorithm performs very well in identifying phenomenologicaly interesting vacua.