Spontaneously broken Supersymmetries in String Theory

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based on work with A. Kashani-Poor and R. Minasian

arXiv:1301.5031, work in progress
Overview

• 4d supercharges that are spontaneously broken at the Kaluza-Klein scale are present in many string backgrounds.

• In such backgrounds there often exists a formal rewriting of the theory in terms of a gauged supergravity using these supercharges.

• These additional supercharges put strong constraints on 4d string corrections.

• Backgrounds linked by dualities share the same spontaneously broken supercharges.
Calabi-Yau Compactifications (type IIA)

• Simplest $N = 2$ compactifications to 4d:

  ➢ no fluxes ($H = F = 0$)
  ➢ $\nabla\eta = 0$ [SU(3) holonomy]

• Effective action for zero modes is $N = 2$ supergravity with $h^{1,1}$ vector multiplets and $h^{2,1} + 1$ hypermultiplets

• E.g. expand $B$ and $J$ in $h^{1,1}$ harmonic two-forms $\omega_a$ on CY:

$$B + iJ/\alpha' = z^a \omega_a$$

  (vector multiplet scalars)
String Corrections for Calabi-Yau Reductions

- Geometry of $z^a$ described by single holomorphic function

$$ F = d_{abc}(M)z^az^bz^c + \zeta(3) \chi(M) + \sum_{\{k_a\}} n_{k_a}(M) \text{Li}_3\left(e^{2\pi ikaz^a}\right) $$

- All perturbative corrections proportional to Euler number

$$ \chi(M) = 2h^{1,1} - 2h^{2,1} $$

- Non-perturbative corrections include more refined information on the manifold $M$,
  e.g. $n_{k_a}(M)$ is number of rational curves on cycle $k_a \omega_a$
Non-perturbative String Corrections for CYs

- $n_{k_{\alpha}}(M)$ computed usually by:
  - Mirror symmetry:
    - On mirror $\tilde{M}$ prepotential $F$ is classically exact
    - Solve Picard-Fuchs equations for $\Omega$ to find $F$
  - Localisation of 2d path integral

Talk by H. Jockers

Computations very difficult for large Hodge numbers

- Non-perturbative corrections to hypermultiplets are not yet completely understood

Jockers, Kumar, Lapan, Morrison, Romo ’12
Alexandrov, Manschot, Persson, Pioline ’13
String Corrections for Calabi-Yau Reductions

No perturbative corrections for $\chi(M) = 0$

What is special about manifolds with vanishing Euler number?
Calabi-Yau Threefolds of Euler Number $\chi = 0$

- Hopf theorem:
  \[ \chi(M) = 0 \iff \text{Nowhere vanishing vector field } \hat{\nu} \ \text{or one-form } \nu \text{ exists on } M \]

- Therefore, there exist two nowhere vanishing spinors $\eta_i$:
  - Calabi-Yau has spinor $\eta_1$
  - Second spinor $\eta_2 = \nu_m \Gamma^m \eta_1$

- This defines an $\textbf{SU(2)}$ structure on $M$ with
  \[ \nabla \eta_1 = 0 \quad \text{but} \quad \nabla \eta_2 \neq 0 \]

*Why are there no perturbative corrections for $\chi = 0$?*

**Answer:** Because the compactification is secretly $N = 4$!
$N = 4$ Multiplets from SU(2) Structures

- 10d fields organize under $\text{SU}(2)$ into 4d $N = 4$ multiplets
  - $\text{SU}(2)$ singlets+triplets: gravity and vector multiplets
  - $\text{SU}(2)$ doublets: massive gravitino multiplets

- Truncation to “massless” gauged $N = 4$ supergravity:
  - Keep only $\text{SU}(2)$ singlets and triplets
**SU(2)-structure Reduction**

Reid-Edwards, Spanjaard ’08; Louis, Martinez-Pedrera, Micu ’09; Danckaert, Louis, Martinez-Pedrera, Spanjaard, HT ’11; Kashani-Poor, Minasian, HT ’13

• Expand $SU(2)$ structure and form fields in “massless” modes:

2 one-forms $v^i$ and $n$ two-forms $\omega^I$, with

$$dv^i = t^i v^1 \wedge v^2 + t^i_j \omega^j$$

$$d\omega^I = T^I_{jK} v^j \wedge \omega^K$$

$$\omega^I \wedge \omega^J = \eta^{IJ} \text{vol}_4^{(0)}$$

• This gives a reduction to an $N = 4$ gauged supergravity in 4d

• The gaugings are given by parameters $t^i, t^i_j$ and $T^I_{jK}$.

• They induce non-trivial potential for scalars. This spontaneously breaks (part of) supersymmetry.

Scale of gaugings = KK scale
Reorganization of the degrees of freedom in our theory:

- **N=2 massive string excitations**
- **N=2 massive multiplets**
- **N=2 supergravity**

- **N=4 massive string excitations**
- **N=4 massive multiplets**
- **gauged N=4 supergravity**

The diagram illustrates the reorganization of degrees of freedom from N=2 to N=4 supergravity, showing transitions involving different types of excitations and multiplets.
Type II Compactifications on CYs with $\chi = 0$

Calabi-Yau threefold \( \Rightarrow \) SU(2) structure

\( \chi = 0 \)

\( dJ, d\Omega = 0 \) \( \Rightarrow \) \( dK, dJ^a = \) torsion

standard reduction

consistent truncation

solv. gauge group

\( N = 2 \) supergravity \( \Rightarrow \) gauged \( N = 4 \) supergravity

integrate out heavy fields

effective theory

\( N = 2 \) vacuum \( \Rightarrow \) partial super-Higgs
**Partial Super-Higgs**

Ferrara, van Nieuwenhuizen ’83; de Roo, Wagemans ’86; Wagemans ’87; Andrianopoli, D’Auria, Ferrara, Lledo ’02; ...

- **gauged N = 4 SUGRA**
  - gravity multiplet: $2, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 1, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0$
  - $3+n+m$ vector multiplet: $1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0$

- **N = 2 SUGRA**
  - gravity multiplet: $2, \frac{3}{2}, \frac{3}{2}, 1$
  - $n$ vector multiplets: $1, \frac{1}{2}, \frac{1}{2}, 0, 0$
  - $n+1$ hypermultiplets: $\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0$

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**Integrate out:**

- 2 massive gravitino multiplets: $\frac{3}{2}, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0$
- $m$ massive vector multiplets: $1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0$
Partial Super-Higgs

Ferrara, Nieuwenhuizen '83; de Roo, Wagemans '86; Wagemans '87; Andrianopoli, D'Auria, Ferrara, Lledo '02; ...

\[ h^{1,1} = h^{2,1} = n \]

**gauged N = 4 SUGRA**

**N = 2 SUGRA**

Integrate out:

- 2 massive gravitino multiplets \( \frac{3}{2}, 1, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0 \)
- \( n \) vector multiplets
- \( n + 1 \) hypermultiplets
- \( m \) massive vector multiplets \( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0 \)
**$N=2$ Moduli Space from Partial Super-Higgs**

- Super-Higgs mechanism leads to standard $N = 2$ SUGRA
- $N = 2$ moduli space particularly simple:
  - Prepotential $F$ cubic (similar for hypermultiplet scalars)
- For Kähler moduli this is the expected classical result
- For complex structure moduli this is a very strong result, since its classical moduli space is **exact**.

  - Prepotential $F_\Omega$ is cubic (non-perturbatively)
Mirror Symmetry

- Mirror symmetry exchanges Kähler and complex structure moduli spaces

  prepotential $F_J$ is cubic (non-perturbatively)

  i.e. no worldsheet instantons contribute to $F_J$

- If one includes $SU(2)$ doublets, also non-vanishing worldsheets instantons can be computed.

  massive gravitino multiplets $\rightarrow$ non-perturbative corrections

- No statement so far on D-instanton corrections to hypermultiplets
Borcea-Voisin Calabi-Yau

- \((K3 \times T^2)/\mathbb{Z}_2\), then blow singularities up
- Singular limit \(K3 \to T^4/\mathbb{Z}_2\) gives orbifold \(T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)\), with
  \[
  \alpha : (x_i) \mapsto (-x_1, -x_2, -x_3, -x_4, x_5, x_6)
  \]
  \[
  \beta : (x_i) \mapsto (\frac{1}{2} - x_1, -x_2, b_3 + x_3, x_4, -x_5, -x_6)
  \]
- \(b_3 = \frac{1}{2}\): Enriques Calabi-Yau (\(\mathbb{Z}_2\) has no fixed points)
  - self-mirror
  - all string corrections to prepotential vanish
    (still non-zero higher-derivative corrections)
- \(b_3 = 0\): Schön Calabi-Yau (\(\mathbb{Z}_2\) has 8 fixed points)
  - GW invariants vanish for all but one modulus
Enriques and Schön Calabi-Yau

Only one Calabi-Yau mode has an $\textbf{SU}(2)$ doublet component hypersurface exists in moduli space where all GW invariants vanish, explaining Hosono, Saito, Stienstra ’97
M-theory on $G_2$ manifolds

• M-theory compactified on $G_2$ manifolds gives $N = 1$ supergravity in four dimensions

• Compactifications on $G_2$ are much less understood:
  ➢ Only three-form $\phi$, thus no complex structure
  ➢ Classical Kähler potential known only implicitly:
    \[ e^{-K} = \int \phi \wedge \ast \phi \]
  ➢ Curvature corrections to $K$ are very difficult to understand.
$SU(3)$ structure on $G_2$ manifolds

$\chi = 0$

7d $G_2$ manifold \quad $\Rightarrow$ \quad SU(3) structure

$N = 1$ Minkowski vacuum \quad $\Leftrightarrow$ \quad partial super-Higgs

$N = 2$ gauged supergravity

Micu, Palti, Saffin ‘06; Aharony, Berkooz, Louis, Micu ‘08; Cassani, Koerber, Varela ‘12

Louis, Smyth, HT ‘09; ‘10
Consequences from $N = 2$ supergravity

- For an $SU(3)$ structure the classical moduli space is well-known and the (classical) $N = 1$ moduli space can be computed from the super-Higgs effect.

- Curvature corrections to the Kähler potential should be understood as curvature corrections to the holomorphic prepotential in $N = 2$ gauged supergravity.

- Similar arguments for M-theory on Calabi-Yau fourfolds with $\chi = 0$?
Joyce construction of $G_2$ manifolds

- Compact $G_2$ manifolds can be constructed from toroidal orbifolds (not many)  
  Joyce '96

- Example: Blow up $G_2 \to T^7/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$ with involutions
  \[
  \alpha : (x_i) \mapsto (-x_1, -x_2, -x_3, -x_4, x_5, x_6, x_7)
  \]
  \[
  \beta : (x_i) \mapsto (\frac{1}{2} - x_1, b_2 - x_2, x_3, b_4 + x_4, -x_5, -x_6, x_7)
  \]
  \[
  \gamma : (x_i) \mapsto (c_1 - x_1, x_2, c_3 - x_3, x_4, c_5 - x_5, x_6, -x_7)
  \]
  Similarly to Enriques and Schön Calabi-Yaus:
  \[
  T^7/\langle \alpha \beta \rangle \text{ gives solvmanifold, then } \alpha \text{ and } \gamma \text{ project to } SU(3) \text{ structure}
  \]
  These manifolds have heterotic duals on CYs with $\chi = 0$. 

Gauged $N = 2$ Supergravity and Duality

M-theory on Joyce manifold

$\text{SU}(3)$ structure

$N = 2$ gauged supergravity

$N = 1$ Minkowski vacuum

Heterotic string on Enriques/ Schönh CY

$\text{SU}(2)$ structure

Further restrictions on string corrections from this duality?
Conclusions

• For Calabi-Yau manifolds with $\chi = 0$, non-perturbative corrections are constrained to the $SU(2)$ doublet sector.

• If one could include (even classically) include $SU(2)$ doublets/massive gravitino multiplets, non-vanishing GW invariants could be computed.

• It seems that more generally string corrections respect 4d spontaneously broken supersymmetries, and thus can be understood as corrections to gauged supergravity.

• This can help us understand also $N = 1$ backgrounds