Spontaneously broken Supersymmetries in String Theory

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based on work with A. Kashani-Poor and R. Minasian

arXiv:1301.5031, work in progress

Overview

- 4d supercharges that are spontaneously broken at the Kaluza-Klein scale are present in many string backgrounds
- In such backgrounds there often exists a formal rewriting of the theory in terms of a *gauged supergravity* using these supercharges
- These additional supercharges put strong constraints on 4d string corrections
- Backgrounds linked by dualities share the same spontaneously broken supercharges

Calabi-Yau Compactifications (type IIA)

- Simplest $N = 2$ compactifications to 4d:
  - no fluxes ($H = F = 0$)
  - $\nabla\eta = 0$ [SU(3) holonomy]

- Effective action for zero modes is $N = 2$ supergravity with $h^{1,1}$ vector multiplets and $h^{2,1} + 1$ hypermultiplets

- E.g. expand B and J in $h^{1,1}$ harmonic two-forms ω_a on CY:

$$B + \mathrm{i}J/\alpha' = z^a \omega_a \qquad \text{(vector multiplet scalars)}$$

String Corrections for Calabi-Yau Reductions

- Geometry of z^a described by single holomorphic function

$$F = d_{abc}(M) z^a z^b z^c + \zeta(3)\,\chi(M) + \sum_{\{k_a\}} n_{k_a}(M)\,\mathrm{Li}_3\left(e^{2\pi \mathrm{i} k_a z^a}\right)$$

- All perturbative corrections proportional to Euler number

$$\chi(M) = 2h^{1,1} - 2h^{2,1}$$

- Non-perturbative corrections include more refined information on the manifold M,
  e.g. $n_{k_a}(M)$ is number of rational curves on cycle $k_a \omega_a$

Non-perturbative String Corrections for CYs

- $n_{k_a}(M)$ computed usually by:
  - Mirror symmetry:
    - On mirror $\widetilde{M}$ prepotential F is classically exact
    - Solve Picard-Fuchs equations for Ω to find F
  - Localisation of 2d path integral

Jockers, Kumar, Lapan, Morrison, Romo '12

Talk by H. Jockers

Computations very difficult for large Hodge numbers

- Non-perturbative corrections to hypermultiplets are not yet completely understood

Alexandrov, Manschot, Persson, Pioline '13

String Corrections for Calabi-Yau Reductions

No perturbative corrections for $\chi(M) = 0$

What is special about manifolds with vanishing Euler number?

Calabi-Yau Threefolds of Euler Number $\chi = 0$

- **Hopf theorem**:

$\chi(M) = 0$ ⟺ Nowhere vanishing vector field $\hat{v}$ (or one-form v) exists on M

- Therefore, there exist two nowhere vanishing spinors η_i:
  - ➢ Calabi-Yau has spinor η_1
  - ➢ Second spinor $\eta_2 = v_m \Gamma^m \eta_1$

- This defines an **SU(2) structure** on M with

$\nabla \eta_1 = 0$ but $\nabla \eta_2 \neq 0$

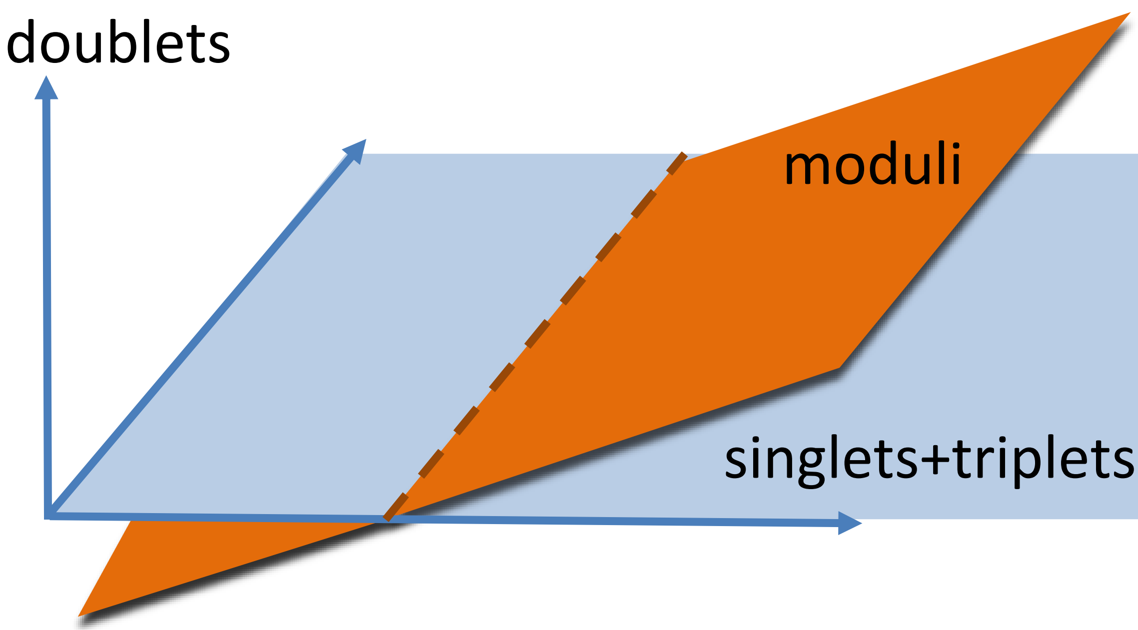
Why are there no perturbative corrections for $\chi = 0$*?*

Answer: Because the compactification is secretly $N = 4$!

$N = 4$ Multiplets from $SU(2)$ Structures

HT, Louis '09

- 10d fields organize under $\mathbf{SU(2)}$ into 4d $\boldsymbol{N = 4}$ multiplets
  - ➢ $\mathbf{SU(2)}$ singlets+triplets: gravity and vector multiplets
  - ➢ $\mathbf{SU(2)}$ doublets: massive gravitino multiplets
- Truncation to "massless" gauged $\boldsymbol{N = 4}$ supergravity:



Keep only $\mathbf{SU(2)}$ singlets and triplets

SU(2)-structure Reduction

Reid-Edwards, Spanjaard '08; Louis, Martinez-Pedrera, Micu '09; Danckaert, Louis, Martinez-Pedrera, Spanjaard, HT '11; Kashani-Poor, Minasian, HT '13

- Expand $SU(2)$ structure and form fields in "massless" modes:

  2 one-forms v^i and n two-forms ω^I, with

$$\mathrm{d}v^i = t^i\, v^1 \wedge v^2 + t^i{}_J\, \omega^J$$
$$\mathrm{d}\omega^I = T^I{}_{jK}\, v^j \wedge \omega^K$$
$$\omega^I \wedge \omega^J = \eta^{IJ}\, \mathrm{vol}_4{}^{(0)}$$

reduction ansatz

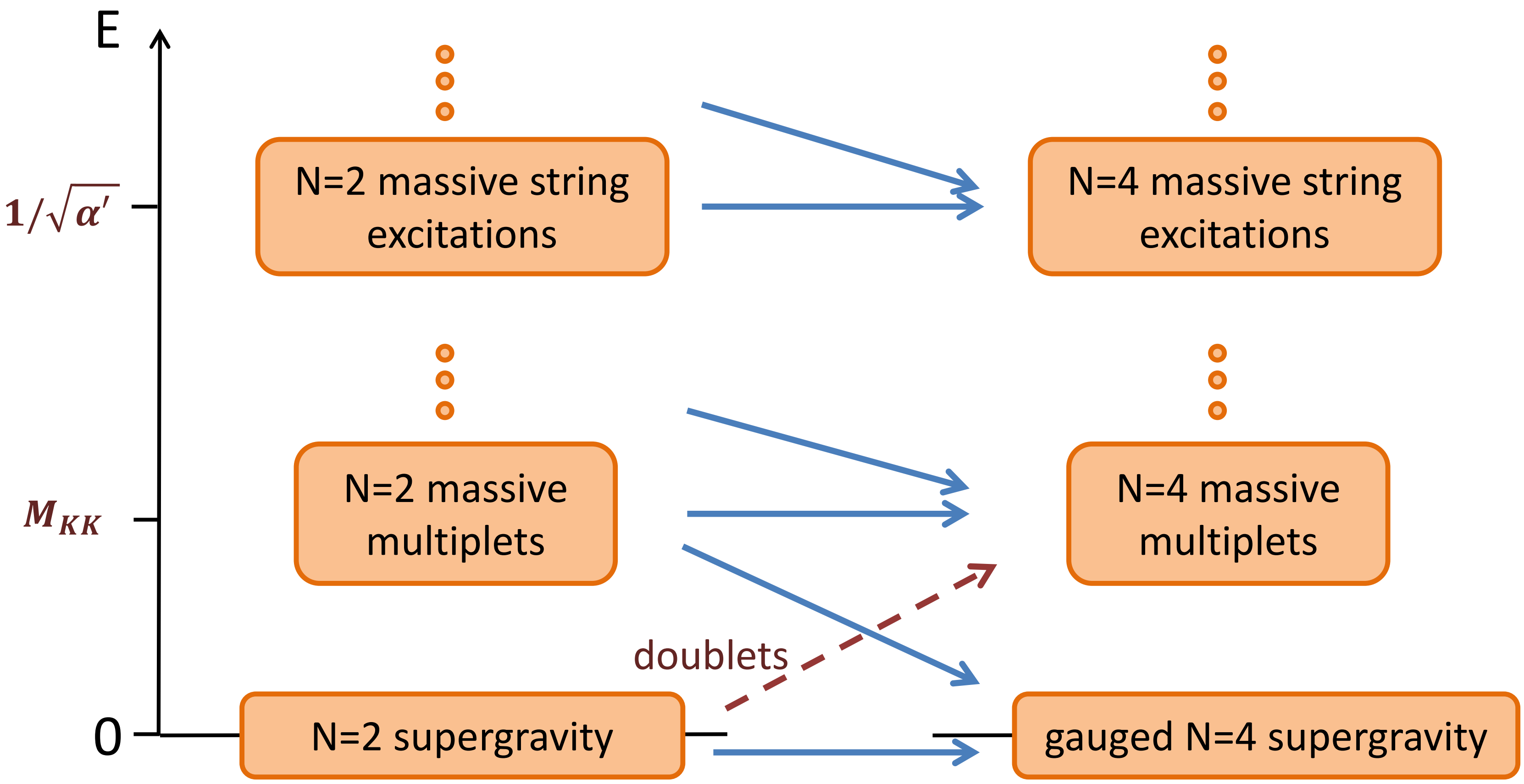
η^{IJ} is SO(3,n-3) metric

- This gives a reduction to an $\mathrm{N} = 4$ gauged supergravity in 4d
- The gaugings are given by parameters t^i, $t^i{}_J$ and $T^I{}_{jK}$.
- They induce non-trivial potential for scalars. This spontaneously breaks (part of) supersymmetry.

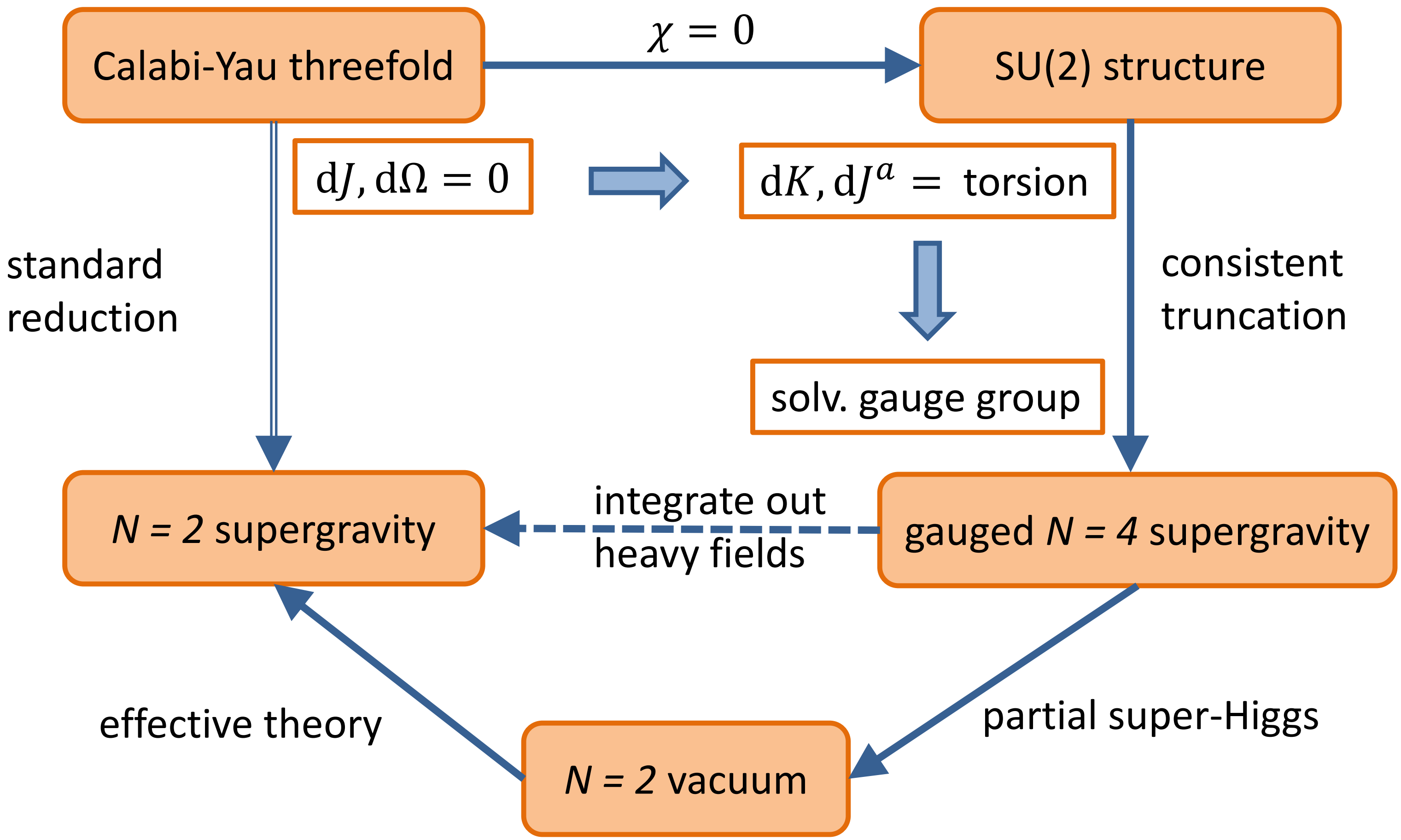
Scale of gaugings = KK scale

N=2 and N=4 Supergravity

Reorganization of the degrees of freedom in our theory:



Type II Compactifications on CYs with $\chi = 0$



Partial Super-Higgs

Ferrara, van Nieuwenhuizen '83; de Roo, Wagemans '86; Wagemans '87; Andrianopoli, D'Auria, Ferrara, Lledo '02; …

gauged N = 4 SUGRA

gravity multiplet $2,\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2},1,1,1,1,1,1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0$

3+n+m vector multiplet $1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0,0,0$

↓

N = 2 SUGRA

gravity multiplet $2,\frac{3}{2},\frac{3}{2},1$

n vector multiplets $1,\frac{1}{2},\frac{1}{2},0,0$

n+1 hypermultiplets $\frac{1}{2},\frac{1}{2},0,0,0,0$

Integrate out:

2 massive gravitino multiplets $\frac{3}{2},1,1,1,1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0$

m massive vector multiplets $1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0,0$

Partial Sup… -Higgs

Fer… Nieuwenhuizen ’83; de Roo, Wagemans ’86; Wagemans ‘87;
An… D’Auria, Ferrara, Lledo ’02; …

gauged N = 4 SUGRA

↓

N = 2 SUGRA

gravity multip… $\ldots,\frac{3}{2},\frac{3}{2},1,1,1,1,1,1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0$

3+n+m vec… $1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0,0,0$

grav… $\ldots,\frac{1}{2},1$

n v… $\ldots,0,0$

…plets $\ldots,0,0$

$$h^{1,1}=h^{2,1}=n$$

Integrate out:

2 …e gravitino multiplets $\frac{3}{2},1,1,\ldots,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0$

m massive vector multiplets $1,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0,0$

N=2 Moduli Space from Partial Super-Higgs

- Super-Higgs mechanism leads to standard $\boldsymbol{N=2}$ SUGRA
- $\boldsymbol{N=2}$ moduli space particularly simple:

  prepotential $\boldsymbol{F}$ cubic (similar for hypermultiplet scalars)

- For Kähler moduli this is the expected classical result
- For complex structure moduli this is a very strong result, since its classical moduli space is **exact**.

⇨ prepotential $\boldsymbol{F_\Omega}$ is cubic (non-perturbatively)

Mirror Symmetry

- Mirror symmetry exchanges Kähler and complex structure moduli spaces

⇒ prepotential $\boldsymbol{F_J}$ is cubic (non-perturbatively)

i.e. *no worldsheet instantons* contribute to $\boldsymbol{F_J}$

- If one includes $\mathbf{SU(2)}$ doublets, also non-vanishing worldsheet instantons can be computed.

massive gravitino multiplets ⇒ non-perturbative corrections

- No statement so far on D-instanton corrections to hypermultiplets

Borcea-Voisin Calabi-Yau

- *(K3 x T^2)/$\mathbb{Z}_2$* , then blow singularities up
- Singular limit ***K3*** $\rightarrow$ ***$T^4/\mathbb{Z}_2$*** gives orbifold *$T^6/(\mathbb{Z}_2$ x $\mathbb{Z}_2)$*, with

$$\alpha : (x_i) \mapsto (-x_1, -x_2, -x_3, -x_4, x_5, x_6)$$

$$\beta : (x_i) \mapsto (½ - x_1, -x_2, b_3 + x_3, x_4, -x_5, -x_6)$$

- $b_3 = ½$: Enriques Calabi-Yau ($\mathbb{Z}_2$ has no fixed points)
  - self-mirror
  - all string corrections to prepotential vanish (still non-zero higher-derivative corrections)

Ferrara, Harvey, Strominger, Vafa ’95

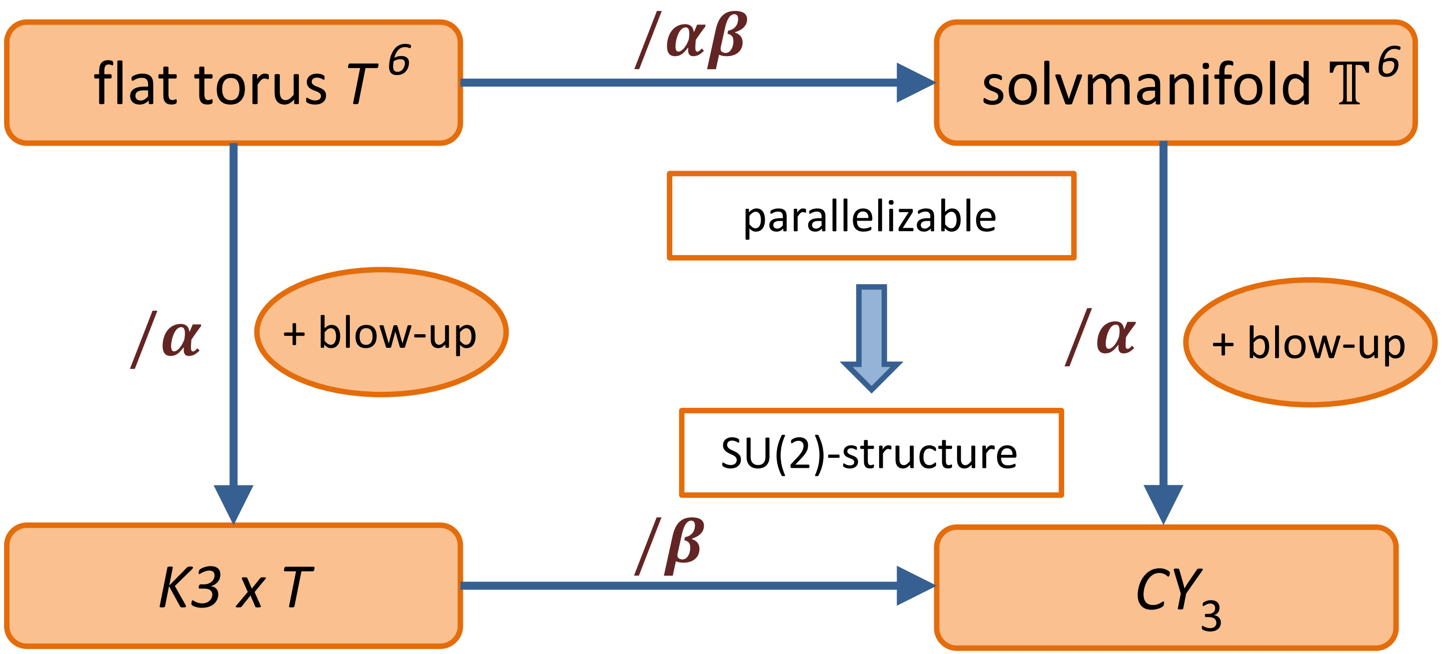
Klemm, Marino ’05

- $b_3 = 0$: Schön Calabi-Yau ($\mathbb{Z}_2$ has 8 fixed points)
  - GW invariants vanish for all but one modulus

Hosono, Saito, Stienstra ’97

Enriques and Schön Calabi-Yau

Kashani-Poor, Minasian, HT '13



Only one Calabi-Yau mode has an $\mathbf{SU(2)}$ doublet component

⇒ hypersurface exists in moduli space where all GW invariants vanish, explaining Hosono, Saito, Stienstra '97

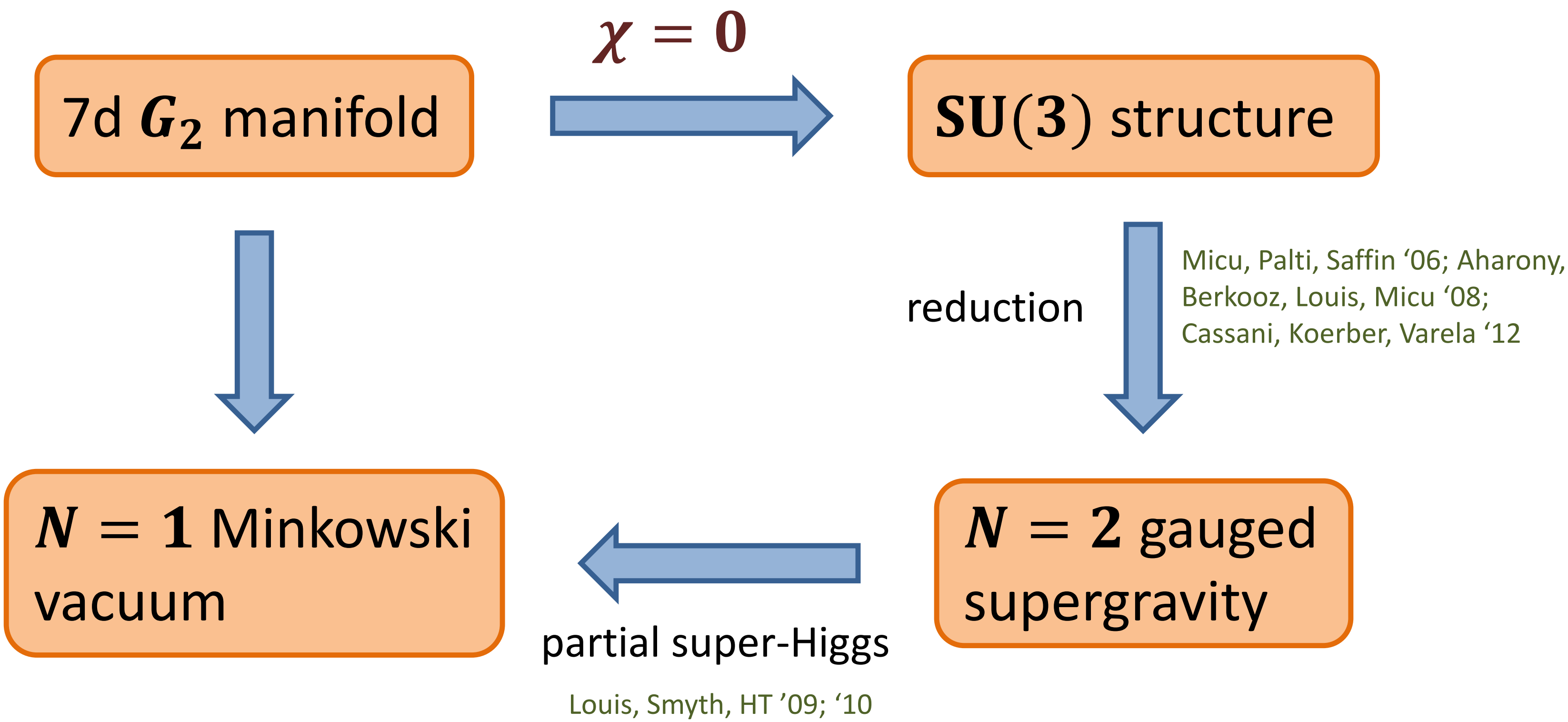
M-theory on G_2 manifolds

- M-theory compactified on $\boldsymbol{G_2}$ manifolds gives $\boldsymbol{N=1}$ supergravity in four dimensions
- Compactifications on $\boldsymbol{G_2}$ are much less understood:
  - Only three-form $\boldsymbol{\phi}$, thus no complex structure
  - Classical Kähler potential known only implicitly:

$$e^{-\mathbf{K}} = \int \boldsymbol{\phi} \wedge * \boldsymbol{\phi}$$

  - Curvature corrections to $\mathbf{K}$ are very difficult to understand.

SU(3) structure on G_2 manifolds



Consequences from N = 2 supergravity

- For an $\mathbf{SU(3)}$ structure the classical moduli space is well-known and the (classical) $\boldsymbol{N = 1}$ moduli space can be computed from the super-Higgs effect.

- Curvature corrections to the Kähler potential should be understood as curvature corrections to the holomorphic prepotential in $\boldsymbol{N = 2}$ gauged supergravity.

- Similar arguments for M-theory on Calabi-Yau fourfolds with $\boldsymbol{\chi = 0}$?

Joyce construction of G_2 manifolds

- Compact G_2 manifolds can be constructed from toroidal orbifolds (not many) Joyce '96

- Example: ➢ Blow up $G_2 \to T^7/(\mathbb{Z}_2 x \mathbb{Z}_2 x \mathbb{Z}_2)$ with involutions

$$\alpha : (x_i) \mapsto (-x_1, -x_2, -x_3, -x_4, x_5, x_6, x_7)$$

$$\beta : (x_i) \mapsto (½ - x_1, b_2 - x_2, x_3, b_4 + x_4, -x_5, -x_6, x_7)$$

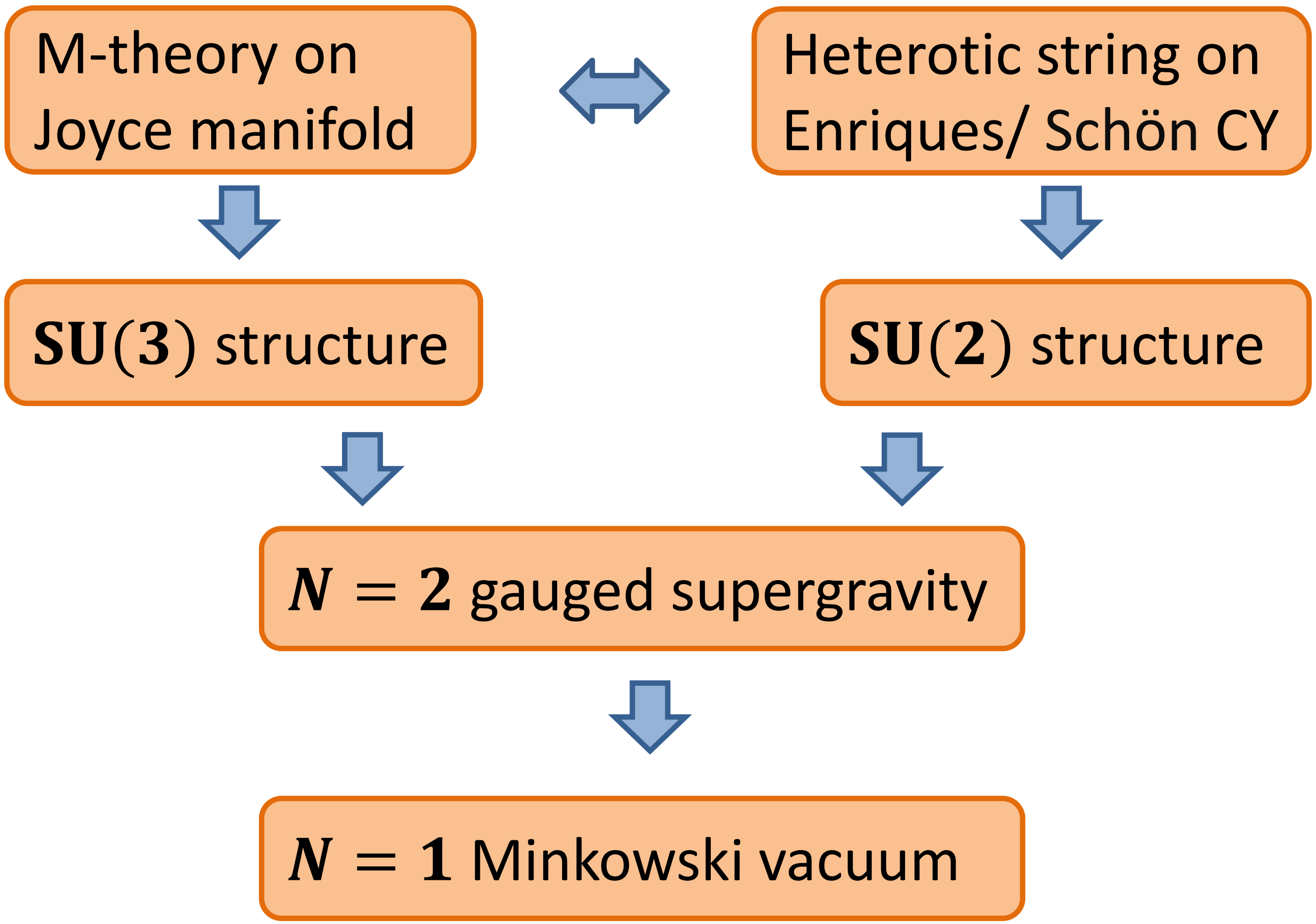
$$\gamma : (x_i) \mapsto (c_1 - x_1, x_2, c_3 - x_3, x_4, c_5 - x_5, x_6, -x_7)$$

➢ Similarly to Enriques and Schön Calabi-Yaus:

$T^7/<\alpha\beta>$ gives solvmanifold, then
α and γ project to $\mathrm{SU}(3)$ structure

➢ These manifolds have heterotic duals on CYs with $\chi = 0$.

Gauged $N = 2$ Supergravity and Duality



Further restrictions on string corrections from this duality?

Conclusions

- For Calabi-Yau manifolds with $\chi = 0$, non-perturbative corrections are constrained to the $SU(2)$ doublet sector.

- If one could include (even classically) include $SU(2)$ doublets/massive gravitino multiplets, non-vanishing GW invariants could be computed.

- It seems that more generally string corrections respect 4d spontaneously broken supersymmetries, and thus can be understood as corrections to gauged supergravity.

- This can help us understand also $N = 1$ backgrounds