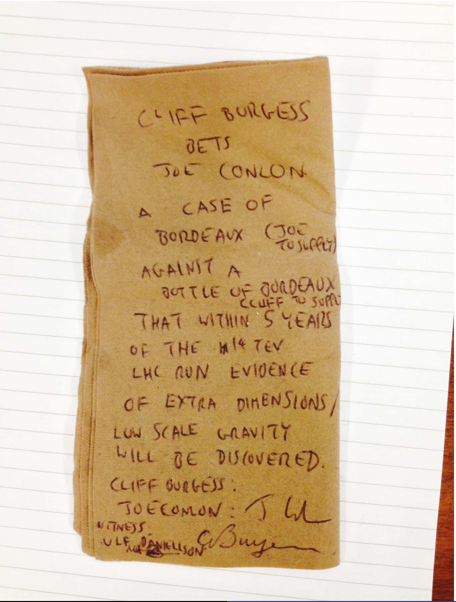
Moduli Stabilisation: the Why and the What?

Joseph Conlon, Oxford University

String Phenomenology 2014, Trieste, 9th July 2014

Thanks in anticipation



Talk Structure

1. Why Stabilise Moduli?
2. Constructions
3. Caveats
4. The Future

I

WHY STABILISE MODULI?

Massless Scalars are Problematic

Supersymmetric compactifications contain many massless scalars (moduli) with gravitational-strength couplings.

These can give rise to 5th forces via couplings

$$\frac{\Phi}{4M_P}F_{\mu\nu}F^{\mu\nu} \qquad \frac{\Phi}{M_P}\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \ldots$$

No such long-range scalar forces are observed, and this world does not contain massless scalars with dilatonic couplings.

Moduli stabilisation allows us to understand the origin and structure of the potential giving mass to these fields.

Consistency of Compactification

The string scale is related to the Planck scale as

$$M_s = \frac{g_s M_P}{\sqrt{\mathcal{V}}}$$

The potential in any string compactification depends on the string scale.

As g_s and $\mathcal{V}$ are part of dilaton and Kähler moduli, we have

$$V_{string} = V_{string}(S, T).$$

By itself, the potential for S and T moduli will run away to decompactification limit.

As potentials are dynamically generated in theories with $N \leq 1$ supersymmetry, to claim any given $N \leq 1$ string compactification exists requires understanding moduli stabilisation.

Breaking Supersymmetry

Unbroken supersymmetry is not a property of this world.

Any string vacuum describing this world must have broken supersymmetry, with $m_{3/2} > 0$.

Moduli potential may lead to susy breaking in the moduli sector

$$V = e^K \left[K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right], \qquad m_{3/2} = e^{K/2}|W|.$$

Supersymmetry breaking occurs if

$$F^i = e^{K/2} K^{i\bar{j}} D_i W \neq 0$$

Moduli potential determines size and structure of supersymmetry breaking and is essential for any understanding of soft terms and mediation mechanisms.

Early Universe Cosmology

Moduli stabilisation is unavoidable when discussing early universe cosmology in string theory

- Inflation: the form of the inflationary potential Nilles, Westphal, Maharana, Burgess
- Inflation: the scale of inflation
- Cosmological moduli problem Marsh
- Overshoot problem
- Reheating and dark radiation Dutta, Marsh

Neglecting moduli stabilisation, it is extremely easy to construct string inflationary models Dvali+Tye 98

Making Observational Predictions

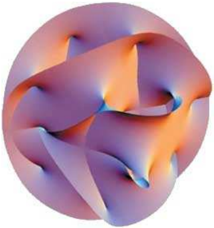
How to turn string compactifications into observational predictions?

It is difficult to single out any preferred extension of the Standard Model as there are so many different approaches to realising the Standard Model. Palti

- Weakly coupled heterotic string Lukas, Vaudevrange
- Free fermionic models
- Rational CFT models (Gepner models)
- IIA intersecting D6 branes
- Branes at singularities
- M-theory on singular G2 manifolds
- IIB magnetised branes with fluxes
- F-theory Palti, Collinucci, Cvetic, Schafer-Nameki, Grimm, Hebecker
- ...

Moduli

The moduli sector is extremely generic:



Closed string sector always present and involves modes (dilaton / volume modulus) always present in compactified string theory.

Such extra-dimensional modes are necessarily present in the spectrum on compactification of 10d theory to four dimensions.

Much of the physics of moduli is universal across compactifications.

Moduli physics is one of the most promising approaches to connect with observations

II
CONSTRUCTIONS OF MODULI STABILISATION

Divide constructions into three kinds:

1. Stabilisation with exponential hierarchies
    - Racetrack stabilisation
    - Large Volume Scenario
2. Stabilisation with power-law hierarchies
    - Heterotic orbifolds
3. Stabilisation without hierarchies
    - KKLT
    - Perturbative stabilisation
    - Kähler uplifting
    - Non-supersymmetric stabilisation

Racetrack stabilisation

Racetrack stabilisation

Two competing exponentials in a superpotential:

$$
\begin{aligned}
W &= Ae^{-aS} + Be^{-bS} \\
K &= -\ln(S + \bar{S})
\end{aligned}
$$

Leads to a supersymmetric minimum with $m_{3/2} \ll M_P$.

Other moduli may be stabilised through threshold corrections and their appearance in A and B - less appealing.

Weakly coupled heterotic compactifications also require stabilisation at the edge of control.

Similar superpotentials are used for G2 MSSM stabilisation.

IIB with fluxes

Consider IIB flux compactifications.

The leading order 4-dimensional supergravity theory is

$$\begin{aligned} K &= -2\ln(\mathcal{V}) - \ln\left(i\int \Omega \wedge \bar{\Omega}\right) - \ln\left(S + \bar{S}\right), \\ W &= \int G_3 \wedge \Omega. \end{aligned}$$

This fixes dilaton and complex structure but is no-scale with respect to the Kähler moduli.

No-scale models have

- Vanishing cosmological constant
- Broken supersymmetry
- Unfixed flat directions

IIB with fluxes

- The effective supergravity theory is

$$\begin{aligned} K &= -2\ln(\mathcal{V}) - \ln\left(i\int \Omega\wedge\bar{\Omega}\right) - \ln(S+\bar{S}) \\ W &= \int (F_3 + iSH_3)\wedge\Omega \equiv \int G_3\wedge\Omega. \end{aligned}$$

- This stabilises the dilaton and complex structure moduli.

$$D_S W = D_U W = 0.$$

$$W = \int G_3\wedge\Omega = W_0.$$

IIb with fluxes

The theory has an important no-scale property.

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T+\bar{T})\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}(U)\right) - \ln\left(S+\bar{S}\right), \\
W &= \int G_3 \wedge \Omega\,(S,U). \\
V &= e^{\hat{K}}\left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + \sum_{T} \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2\right) \\
&= e^{\hat{K}}\left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W}\right) = 0.
\end{aligned}
$$

Moduli Stabilisation: Fluxes/GKP

$$
\begin{aligned}
\hat{K} &= -2 \ln \left(\mathcal{V}(T_i + \bar{T}_i) \right), \\
W &= W_0 . \\
V &= e^{\hat{K}} \left(\sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \\
&= 0
\end{aligned}
$$

No-scale model :

- vanishing vacuum energy
- broken susy
- T unstabilised

No-scale is broken perturbatively and non-pertubatively.

Solution comes from solving higher-dimensional equations of motion

Moduli Stabilisation: KKLT

$$\begin{aligned}\hat{K} &= -2\ln(\mathcal{V}) - \ln\left(i\int \Omega \wedge \bar{\Omega}\right) - \ln\left(S + \bar{S}\right), \\ W &= \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.\end{aligned}$$

Non-perturbative effects (D3-instantons / gaugino condensation) allow the T-moduli to be stabilised by solving $D_T W = 0$.

For consistency, this requires

$$W_0 = \left\langle \int G_3 \wedge \Omega \right\rangle \ll 1.$$

Moduli Stabilisation: KKLT

$$\begin{aligned}\hat{K} &= -2\ln(\mathcal{V}), \\ W &= W_0 + \sum_i A_i e^{-a_i T_i}.\end{aligned}$$

Solving $D_T W = \partial_T W + (\partial_T K) W = 0$ gives

$$\mathrm{Re}(T) \sim \frac{1}{a} \ln(W_0)$$

For Re(T) to be large, W_0 must be <u>enormously</u> small.

Susy breaking: $\bar{D}3$ brane in warped throat? F-terms?

Moduli Stabilisation: Heterotic Orbifolds

$$\hat{K} = -3\ln\left(T+\bar{T}\right),$$
$$W = W_0 + Ae^{-aT}.$$

For Re(T) to be large, W_0 must be enormously small.

One idea: if $W \sim \langle \phi^n \rangle$, with lower powers protected by discrete R-symmetries, then $\langle W \rangle$ may be small even if $\langle \phi \rangle$ is not that small.

Can potentially realise large hierarchies and KKLT-like physics without exponentials

Kappl Nilles Ramos-Sanchez Ratz Schmidt-Hoberg Vaudevrange

Moduli Stabilisation: Kähler Uplifting

$$\hat{K} = -2\ln\left(\mathcal{V} + \xi\right),$$
$$W = W_0 + Ae^{-aT}.$$

Theory can admit de Sitter solution for large W_0.

Plays off non-perturbative corrections to W to perturbative correction to K.

This is attractive, but intrinsic problem is that this is at small volume and with very high scale susy breaking.

Balasubramanian Berglund, Louis Rummel Valandro Westphal

Moduli Stabilisation: Perturbative Stabilisation

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}+\xi\right) + \frac{\alpha}{T+\bar{T}}, \\
W &= W_0.
\end{aligned}
$$

Balancing of leading α' and leading g_s corrections can lead to AdS volume at moderately small volume.

High scale susy breaking and requires some tuning to avoid too small volume.

Berg, Haack, Kors

LARGE Volume Models

Going beyond no-scale the appropriate 4-dimensional supergravity theory is

$$
\begin{aligned}
K &= -2\ln\left(\mathcal{V} + \frac{\xi}{g_s^{3/2}}\right) - \ln\left(i\int \Omega\wedge\bar{\Omega}\right) - \ln\left(S+\bar{S}\right), \\
W &= \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.
\end{aligned}
$$

Key ingredients are:

(1) the inclusion of stringy α' corrections to the Kähler potential
(2) nonperturbative instanton corrections in the superpotential.
(3) multi-moduli and 'Swiss cheese' structure.

Balsubramanian Berglund JC Quevedo, also Angus Berg Cicoli Haack Krippendorf Marsh Pajer Palti Pedro Shukla....

LARGE Volume Models

The simplest model (the Calabi-Yau $\mathbb{P}^4_{[1,1,1,6,9]}$) has two moduli and a 'Swiss-cheese' structure:

$$\mathcal{V} = \left(\tau_b^{3/2} - \tau_s^{3/2}\right).$$

Computing the moduli scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\xi |W|^2}{g_s^{3/2} \mathcal{V}^3}.$$

The minimum of this potential can be found analytically.

Moduli Stabilisation: LARGE Volume

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2}}_{\text{Integrate out heavy mode } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

$$V = -\frac{|W_0|^2 (\ln \mathcal{V})^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

A minimum exists at

$$\mathcal{V} \sim |W_0| e^{a_s\tau_s}, \qquad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

This minimum is non-supersymmetric AdS and at exponentially large volume.

LARGE Volume Models

The locus of the minimum satisfies

$$\mathcal{V} \sim |W| e^{c/g_s}, \qquad \tau_s \sim \ln \mathcal{V}.$$

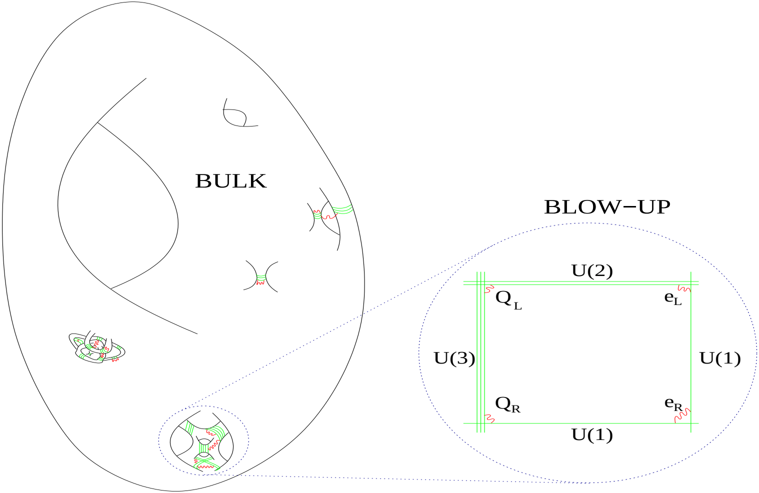
The minimum is at exponentially large volume and non-supersymmetric.

The large volume lowers the string scale and supersymmetry scale through

$$m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} \sim \frac{M_P}{\mathcal{V}}.$$

An appropriate choice of volume will generate TeV scale soft terms and allow a supersymmetric solution of the hierarchy problem.

LARGE Volume Models



Moduli Stabilisation: LARGE Volume

Question: LVS uses an α' correction to the effective action.

If some α' corrections are important, won't all will be?

Truncation is self-consistent because minimum exists at exponentially large volumes.

The inverse volume is the expansion parameter and so it is consistent to only include the leading α' corections.

Moduli Stabilisation: LARGE Volume

Higher α' corrections are suppressed by more powers of volume.

Example:

$$\begin{array}{rcl}
\int d^{10}x\sqrt{g}\mathcal{G}_3^2\mathcal{R}^3 & : & \int d^{10}x\sqrt{g}\mathcal{R}^4 \\
\int d^4x\sqrt{g_4}\left(\int d^6x\sqrt{g_6}\mathcal{G}_3^2\mathcal{R}^3\right) & : & \int d^4x\sqrt{g_4}\left(\int d^6x\sqrt{g_6}\mathcal{R}^4\right) \\
\int d^4x\sqrt{g_4}\left(\mathcal{V}\times\mathcal{V}^{-1}\times\mathcal{V}^{-1}\right) & : & \int d^4x\sqrt{g_4}\left(\mathcal{V}\times\mathcal{V}^{-4/3}\right) \\
\int d^4x\sqrt{g_4}\left(\mathcal{V}^{-1}\right) & : & \int d^4x\sqrt{g_4}\left(\mathcal{V}^{-1/3}\right)
\end{array}$$

Moduli Stabilisation: LARGE Volume

Loop corrections are also suppressed by more powers of volume: there exists an 'extended no scale structure'

$$
\begin{aligned}
W &= W_0, \\
K_{full} &= K_{tree} + K_{loop} + K_{\alpha'} \\
&= -3\ln(T+\bar{T}) + \underbrace{\frac{c_1}{(T+\bar{T})(S+\bar{S})}}_{loop} + \underbrace{\frac{c_2(S+\bar{S})^{3/2}}{(T+\bar{T})^{3/2}}}_{\alpha'}. \\
V_{full} &= V_{tree} + V_{loop} + V_{\alpha'} \\
&= \underbrace{0}_{tree} + \underbrace{\frac{c_2(S+\bar{S})^{3/2}}{(T+\bar{T})^{3/2}}}_{\alpha'} + \underbrace{\frac{c_1}{(S+\bar{S})(T+\bar{T})^2}}_{loop}
\end{aligned}
$$

Nice things about LVS

- Parametric control over expansions: $\mathcal{V} \gg 1$ allows higher α' corrections to be parametrically ignored.
- Distinctive moduli spectrum: volume modulus is by far the lightest Kähler modulus, $m_{\tau_b} \ll m_{3/2}$
- Broken supersymmetry at hierarchically low scales
- No-scale structure inherited from GKP
- Extended no-scale stucture: leading corrections in K are not leading corrections in V

Modulicide

The most extreme way to stabilise moduli is to remove them.

Example: asymmetric orbifolds

Related example: D-term stabilisation of moduli

Personal view: does not count as stabilisation as moduli are never present in low-energy effective field theory

Such modulicide necessarily(?) involves string scale physics and so little chance of generating hierarchies.

Non-supersymmetric moduli stabilisation

Is it possible to stabilise moduli without use of supersymmetric 4d effective field theories?

Certainly possible in principle, but establishing plausible control of construction is much harder.

Considerable back-reaction issues with antibranes, small volumes, etc - restriction to 4-dimensional effective field theory not well justified.

A holy grail? - entirely non-supersymmetric compactifications with hierarchically small scales.

Moduli Stabilisation: IIA with Fluxes

In type IIA flux compactifications, moduli can be stabilised in supersymmetric AdS using fluxes alone. de Wolfe Giryavets Kachru Taylor

Advantage is that by increasing flux quanta volumes can be made larger, and weak coupling can be attained.

However many moduli are tachyonic (although Breitenlohner-Freedman stable) - makes question of turning these into de Sitter solutions more delicate.

Moduli Stabilisation: Non-CalabiYau/ Metric Fluxes / NonGeometric Fluxes

$$W = W(T, U, S)$$

'All moduli' appear in the superpotential.

What is the low-energy effective theory?

To what extent is this stabilisation and to what extent modulicide?

Recent progress in determining the effective action for $SU(3)$ structure manifolds. Grana Louis Theis Waldram

III
MODULI STABILISATION: CAVEATS

The Missing Fields

In most moduli stabilisation constructions, many fields are excluded ab initio

We rely on

- Approximating string theory by 10-d supergravity
- Approximating compactified 10-d supergravity by 4-d effective field theory

Dynamics of Kaluza-Klein modes and string modes are removed from the outset.

Ideally, this is justified due to large scale separations between string scale and KK scale, and KK scale and moduli scale (eg LVS)

Approximations are less justified when potentials are large ($V^{1/4} \sim 10^{16}\mathrm{GeV}$) or the compactification scale is small.

Missing Instabilities?

5-dimensional Kaluza Klein theory compactified on a circle is not obviously unstable.

However famously this system is unstable to nucleation of bubbles of nothing (Witten 1981).

Are there other higher-dimensional instabilities that afflict string vacua?

CdL instantons in 4d EFT are not the only instabilities.

The Flux α' expansion

Fluxes are quantised as

$$\int F_3 = N, \ldots \int H_3 = M.$$

The α' expansion contains terms of higher powers in flux:

$$\int d^{10}x\sqrt{g}\left(\mathcal{R} + \ldots + \alpha'^{,3}\left(G_3\bar{G}_3\right)^3\mathcal{R} + \alpha'^{,5}\left(G_3\bar{G}_3\right)^5\mathcal{R} + \ldots\right)$$

There is an (uncomputed) expansion in terms of

$$\alpha' G_3\bar{G}_3 \sim \frac{\mathcal{O}(N^2) + \mathcal{O}(NM) + \mathcal{O}(M^2)}{\mathcal{V}}$$

for typical flux quanta N and volume $\mathcal{V}$.

For moderate $\mathcal{V}$ and large flux quanta this expansion looks uncontrolled.

Large numbers of species I

Many Calabi-Yaus have $h^{1,1} \gg 1$ or $h^{2,1} \gg 1$, giving many ($\mathcal{O}(100)$) moduli below the KK scale.

There are expected to be corrections proportional to number of species ($M_P \to \frac{M_P}{\sqrt{N}}$)

The only such correction known is the $\alpha'^{,3}\zeta(3)\mathcal{R}^4$ correction involving $\chi(M)$.

No reason to expect this correction to be unique - what others exist? How important are they? When do they matter?

Large numbers of species II

'We assume a non-perturbative superpotential e^{-aT} with $a = \frac{2\pi}{N}$, $N \sim \mathcal{O}(10)$'

Ingredient of many constructions: large-rank gaugino condensation in 4d EFT to generate a non-perturbative superpotential.

However - supersymmetric $SU(N)$ has $4(N^2 - 1) \gg 1$ light degrees of freedom.

These modes are 'hidden' inside the condensing superpotential, but are still in 4-d EFT.

What N^2 effects do they source in Kähler potential? How big are they?

Never computed to my knowledge.

Lack of positive checks

String worldsheet computations contain internal consistency checks that show calculations are correct - ‘string miracles’.

For example:

- Anomaly cancellation and RR tadpole cancellation
- ‘Fortuitous’ appearance of ϑ-function identities

Such computations are often ‘error correcting’.

Not generally true of moduli stabilisation constructions done in 4d EFT - no calculational red flags.

This makes it essential to be highly self-critical

IV
MODULI STABILISATION: THE FUTURE

Open Questions

Great progress has been made in moduli stabilisation.

Moduli stabilisation is not an end in itself - we study it as a means to the end of connecting to the Standard Model or confronting string compactifications with observations.

A clear direction is to make further combination with explicit Standard Model / MSSM constructions.

Excellent progress in type IIB setting.

Cicoli Klevers Krippendorf Mayrhofer Quevedo Valandro

Can this be taken into F-theory?

Open Questions

Personal view: The real world is hierarchical. Moduli stabilisation with hierarchies and small numbers is much more interesting than moduli stabilisation without hierarchies.

There are relatively few ways known to systematically get small numbers and scales:

- Exponentially large volumes - lower string scale and thereby all scales
- Warped regions - lower local string scale
- Gaugino condensation and friends: dynamically generate small infrared scales.

Are there any other ways?

Open Questions

Personal view: Moduli stabilisation is motivated by phenomenological requirements such as supersymmetry breaking or no fifth forces.

We should aim to give back to phenomenology and look at what observational consequences can be extracted.

Cosmology provides probably the best opportunity - eg dark radiation.

Inflation? String theory suggests that during inflation there were $\mathcal{O}(100)$ fields with masses $m \sim H$, and possibly many axion-like particles with $m_a \ll H$.

Does this have any observational consequences? What are they? (cf Baumann Green)

Concluding Thought

'...our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. It is always hard to realise that these numbers and equations we play with at our desks have something to do with the real world.'
Steven Weinberg